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Discrete structures model a vast array of objects ranging from DNA sequences to internet networks. The theory of generating functions provides an algebraic framework for discrete structures to be enumerated using mathematical tools. This book is the result of 25 years of work developing analytic machinery to recover asymptotics of multivariate sequences from their generating functions, using multivariate methods that rely on a combination of analytic, algebraic, and topological tools. The resulting theory of analytic combinatorics in several variables is put to use in diverse applications from mathematics, combinatorics, computer science, and the natural sciences. This new edition is even more accessible to graduate students, with many more exercises, computational examples with Sage worksheets to illustrate the main results, updated background material, additional illustrations, and a new chapter providing a conceptual overview.

Robin Pemantle is Merriam Term Professor of Mathematics at the University of Pennsylvania, working in the fields of probability theory and combinatorics. He received his bachelor's degree from Berkeley and his PhD from MIT. He is a Fellow of the AMS and IMS and a winner of the Rollo Davidson Prize.

Mark C. Wilson is Senior Teaching Faculty at the College of Information and Computer Sciences at the University of Massachusetts, Amherst. He received his PhD in mathematics from the University of Wisconsin–Madison. He is Editor-in-Chief of *Notices of the American Mathematical Society* and life member of the Combinatorial Mathematics Society of Australasia.

Stephen Melczer is Assistant Professor in the Department of Combinatorics and Optimization at the University of Waterloo. He received doctorates from the École normale supérieure de Lyon and the University of Waterloo. He is a recipient of a Governor General Silver Academic Medal and previously published the textbook *An Invitation to Analytic Combinatorics*.

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