cians to bridge the (largely notational) gulf between themselves and others who handle the same basic material on crystallography rather differently.

Throughout the book the material is treated in a fairly geometric way but quite a lot of linear algebra, elementary group theory and topology is used. Students who attended this course saw some down to earth questions answered using techniques that they had probably seen in a more abstract format earlier in their courses. Their knowledge of some basic material (linear algebra etc.) would be both consolidated and further motivated by this course. Personally, I feel that more of our final year mathematics courses should 'round off' earlier courses in the way that this course did.

The first chapter discusses groups of motion of the plane. Almost everything that is done later in the book is done in this chapter for the special case of the plane.

The second chapter introduces the language that is used throughout the book and studies affine groups acting on $\mathbb{R}^n$. The importance of the subgroup of translations is stressed and short exact sequences are consistently used to describe the relationship between a group, its translation subgroup and its quotient group. This chapter also has a careful explanation of the equivalence used between two affine groups. This important but slightly subtle point has often been omitted in the 'applied' texts, indeed different equivalences have sometimes been used as if they were the same—leading to mistakes.

Space groups are treated in the third chapter. There is a topological as well as an algebraic discussion. To me the most fascinating elementary aspect of the study of space groups is the crystallographic restriction; in most books this is done for $\mathbb{R}^2$ and $\mathbb{R}^3$. Here it is done for $\mathbb{R}^n$ but the theorem stated only applies if the action of the group is irreducible (however the reviewer must admit that in a course a few years ago he also 'proved' the theorem as stated in this book). There is a correct version but is slightly complicated to state.

The next three chapters give a full, detailed account of the classification of n-dimensional space groups for low values of n and also a discussion of the classification programme for large n.

The final chapter discusses deformations; this topic does not usually make an appearance in books on crystallography. The author makes a convincing case for its inclusion. Most of the material in this chapter has been developed by the author himself.

The book ends with a very interesting historical and bibliographical note. This is particuarly valuable in a book that treats material, some of which is quite old and much of it on the border of several subjects.

The book is produced from typescript but it is quite easy to read and has relatively few misprints. It suffers a little by not having an index; the discussion sometimes relies on a precise definition and it would be useful to be able to find the exact definition quickly. My overall impression is that lecturers contemplating giving a final year university course of a geometrical nature and hoping to consolidate some of the concepts that the students have already met would find a lot of useful material in this book. Nevertheless they would probably think that a course devoted to crystallography would be too specialised and would include some other material as well.

I enjoyed reading a book at a reasonably low level containing new material.

ELMER REES

DOLLARD, J. D. and FRIEDMAN, C. N. *Product integration with applications to differential equations* (Encyclopedia of mathematics and its applications, Volume 10, Addison-Wesley, 1979) xxii+253 pp., U.S. $24.50.

The simplest case of the product integral

$$\prod_a^b e^{A(s)ds}$$

occurs when A is a square matrix-valued function whose elements are continuous on $[a, b]$. If $P(s_0, s_1, \ldots, s_n)$ is a partition of $[a, b]$ and A_P a step-function on P (i.e. one taking constant matrix values $A_1, A_2, \ldots, A_n$ on the subintervals of P), the product integral of A_P over $[a, b]$ is defined as

$$\exp\{A_n(s_n-s_{n-1})\}\exp\{A_{n-1}(s_{n-1}-s_{n-2})\}\dots\exp\{A_1(s_1-s_0)\},$$

a matrix product which is ordered since the matrices A_r may not commute. The product integral of a continuous A is defined as the limit of the integral of a step-function A_P which converges to A in an appropriate sense as the norm of P tends to zero. Such an integral is to the product what an ordinary integral is to the sum. For example, a product integral over $[a, b]$ is the *product* of integral over $[c, b]$ and $[a, c]$. One of the first results is that if

$$F(x, a)=\prod_a^x e^{A(s)ds}$$

where A is continuous, then F is a solution of the initial value problem

$$\frac{dF(x, a)}{dx}=A(x)F(x, a), \quad F(a, a)=I.$$

This has an immediate application to the solution of a system $Y'=AY$ of n linear ordinary differential equations in n unknowns.

The first chapter of the present book gives an extensive treatment of this simple case; it is quite elementary, and the required matrix algebra is covered in an appendix. Later chapters, which extend the notion of product integration to contour integrals, to measures and to the case when A is a more general operator-valued function, required some ideas from complex and functional analysis, and these are explained briefly. There are many examples throughout, with applications mainly to differential and integral equations but including the sufficiency part of the Hille-Yosida condition for an operator to be the infinitesimal generator of a contraction semigroup. The authors in a short final chapter, and P. R. Masani in an appendix, indicate several other areas of mathematics where product integration has been used, and some two hundred references are given.

Product integration, then, can be applied in many situations, and some results are obtained more easily than by conventional methods. Professors Dollard and Friedman have given the first modern survey, and have done so with admirable style and clarity. Their book can certainly be recommended to everyone whose research uses differential equations, and it may be of interest to other mathematicians also.

PHILIP HEYWOOD

SERRE, J.-P., *Trees* (Translated from the French by J. Stillwell) (Springer-Verlag, 1980), 142 pp., DM 48.

This is a translation of Serre's "Arbes, amalgames, SL_2", Astérisque no. 46, Soc. Math. France, 1977. It contains an exposition of the theory of groups acting on trees due to Serre and Bass. This important theory has had many applications and some of these are dealt with in this book. There are two chapters.

Chapter I consists mainly of an account of the basic theory. Given a group G acting on an (oriented) tree X the author describes (using methods from combinatorial group theory) how to obtain a presentation for G from a spanning tree in the quotient graph $G\backslash X$. The fundamental theorem gives this presentation for G in terms of the edges of $G\backslash X$ and the stabilizers (in G) of the vertices and edges of X. It is shown that free groups, tree products, HNN groups etc. can be realised in this way and the fundamental theorem is used to reprove the subgroup theorems of Schreier and Kurosh. Chapter I ends with the application of the basic theory to the case of groups which fix at least one point of every tree on which they act (the so-called "property (FA)"). In this way it is shown, for example, that $SL_3(\mathbb{Z})$ is not a proper free product with amalgamation.

In Chapter II the fundamental theorem is applied to subgroups of $GL(V)$, where $V=K^2$ and K is a local field. A *lattice* of V is an $\mathcal{O}$-submodule of V which generates the K-vector space V, where $\mathcal{O}$ is the valuation ring of K. $GL(V)$ acts on the set of lattices in a natural way and two