

Thereafter the book becomes slightly more abstract, with discussion of the properties of vector spaces over a field, up to direct sums, followed by a careful presentation of linear mappings with the barest mention of dual space, and then the matrices associated with linear mappings. The chapter dealing with eigenvalues and diagonalization of matrices considers geometric and algebraic multiplicities of eigenvalues, and gives criteria for diagonalizability, with a mention of the existence of a simple form of matrix for the non-diagonalizable case, but no statement of the form. The final chapter deals with euclidean spaces, orthonormal bases and orthogonal diagonalization, without mention of the extension to the complex field. A positive-definite matrix is defined, but there is no consideration of its place in the general inner product, and no discussion of quadratic forms.

Particularly in the early chapters the book proceeds gently, with ample explanation of what is being done. Even in the later chapters where the pace is faster a student should be well able to follow the arguments. The author includes helpful notes where needed, and points out very carefully which part of an "if and only if" result is being proved, a matter that many students seem to find difficult. To emphasise the parallelism between matrices and linear mappings many results which apply to both are separately stated. A summary of the main results in each chapter would be helpful, but perhaps it would be more profitable for a student to make his own summary. Each chapter ends with a set of exercises, answers being given where appropriate for numerical examples, with hints for other problems.

The printing of the book is disappointing in places, with irritating "breaking" of equations between lines, and some misalignment of matrices. But these are minor quibbles; the book is a useful text at this level, and can be confidently recommended.

H. G. ANDERSON

RICHTMYER, R. D., *Principles of advanced mathematical physics*, Volume II (Springer-Verlag, 1981), xii + 322 pp., DM 72.

Volume I under this title was published in 1978. It covers basic material on Hilbert spaces, Banach spaces, distributions and linear operators, with examples of ordinary and partial differential operators in physics. Other topics included are probability, initial value problems and fluid dynamics as an example of a non-linear problem. The three main areas of volume II are group and representation theory, Riemannian geometry and general relativity, and stability and dynamical systems.

The group theory is self contained, with examples of finite and continuous groups of physical interest. Group representations are discussed first in the context of the rotation group and spherical harmonics and subsequently the basic general theory of reduction and invariant integration is presented. Ray and in particular spin representations are motivated by discussing their role in quantum mechanics. Some elementary theory of manifolds is introduced prior to a discussion of Lie groups which ends with a sketch of the classification proof for simple complex Lie algebras.

The section on differential geometry begins with the definition of scalar, vector and tensor fields and introduces the important concepts in (pseudo-) Riemannian and affine geometry. A brief but clear presentation of the Einstein field equation is a starting point for a discussion of the Schwarzschild, Kerr and other solutions.

The presentation of stability and bifurcation is made in the context of fluid dynamics. Starting from a clear illustration of bifurcations in the flow between coaxial rotating cylinders, stability analysis by linearisation and simple examples of bifurcations are presented. A short chapter is devoted to some details of the rotating cylinder problem and the final chapter provides an introduction to the theory of the onset of turbulence, ending with a brief sketch of some period-doubling phenomena.

As its title indicates the book is at a higher level of mathematical sophistication than is conventional in a physics course but throughout the book the author bases his motivation and examples on physical problems. Much of the material could be used in specialised undergraduate courses. I would also recommend the book to postgraduate students and research workers in

theoretical physics who wished to pick up some more of the mathematics presently needed in their subject. Different parts of the book can largely be read independently and with its clarity of presentation it could very readily act as a reference volume.

Inevitably in a book which covers such a wide range of topics one may expect to have minor quibbles in areas where one feels particularly competent. Also, some may view the absence of bundles and forms as a notable omission. Nevertheless this seems to me a very valuable book, well written at a good mathematical level; I recommend it highly to anyone who needs a clear introduction to any of the topics it covers.

D. J. WALLACE

BEINEKE, L. W. and WILSON, R. J. (eds.), *Selected topics in graph theory* (Academic Press, 1979), pp. 451, £34.40.

Books with two editors and a title as vague as this tend to be collections of invited (or uninvited) conference lectures hastily gathered together in book form. The result is often disappointing for one of the following reasons: it is unreadable except for the person who knows it all already, each chapter uses different conventions of notation and terminology, and the topics are of such specialised interest that few will want to read about them anyway. Happily, this present volume is from a different stable. The editors have got together a group of well-known expositors who together survey some of the most important areas in modern "pure" graph theory. The result is a beautifully produced volume which deserves a place in every mathematics library, although most of the material can, of course, be found elsewhere. Topics covered include the four colour theorem, topological graph theory, hamiltonian graphs, tournaments, the reconstruction problem, minimax theorems, strongly regular graphs, enumeration and Ramsey theory, line graphs and edge colourings and contributors include P. J. Cameron, C. St. J. A. Nash-Williams, A. T. White and D. R. Woodall among others, as well as the two editors. A final chapter by Ronald Reid considers the impact computers have made and can make on graph theory research.

For those more interested in applications of graph theory, a companion volume entitled *Applications of Graph Theory* has also appeared. Finally, for those interested not so much in existence theorems but more in how to actually find a Hamiltonian cycle, for instance, the recent book *Graphs and Networks* by B. Carré (O.U.P. 1979) is recommended

I. ANDERSON

BILLINGSLEY, P., *Probability and measure* (Wiley, 1979), pp. 532, £28.95.

First and foremost the game-plan is excellent. Probabilistic intuition and measure-theoretic competence are developed side by side. For example, in the first chapter coin-tossing is used to show the need for events more complicated than finite unions of intervals and to lead rather quickly to Lebesgue measure. The calculus of infinite sequences of events (Borel-Cantelli lemmas, Zero-One Law etc.) is developed without recourse to integration. That was probability—but quite a deal of measure-theoretic intuition was picked up on the way. The second chapter takes up general measures in earnest, develops integration, Fubini's theorem, etc. That's measure theory—so we go on to random variables, expected values....

All of this is admirable and there's a healthy redundancy about it too. For example the treatment of differentiation lingers long enough on the line to make the power and efficiency of the abstract formulation of absolute continuity entirely convincing; then conditional probability, conditional expectation are introduced with great care and with many worked examples. In fact these topics and the most basic facts about martingales occupy a solid chapter of 176 pages.

That means I strongly approve the treatment and the book itself. Of course there are problems. There are many students for whom the main motivation of Lebesgue integration should be that it offers efficient no-nonsense convergence theorems which are simple to apply. The present treatment is not well-adapted to their needs, nor will they find the ideas surrounding the Riesz representation theorem and the links between measure and topology. (Rudin's "Real and Complex Analysis" and Hewitt and Stromberg's "Real and Abstract Analysis" are favourites of mine which cover some of that ground.)