The selected papers of E. S. Pearson, issued by the Biometrika Trustees to celebrate his 30 years as editor. University of California Press, Berkely and Los Angeles, 1966. $6.75.

This volume contains a selection of 21 of the 112 papers of Professor Pearson which are listed in the included bibliography. The selection illustrates the range of Professor Pearson's interests and contributions in both applied and theoretical statistics. 10 additional papers, written jointly with Professor Neyman, are to appear as a second volume by the same publisher. Selected early papers of Professor Neyman are to be published as a third volume.

Charles H. Kraft

Les fondements de la géométrie. Tome II: Géométrie projective par Béla Kerékjártó. Gauthier-Villars, Paris, et Académie des Sciences hongroise, 1966. 528 pages. $14.50.

The first volume of the voluminous work of the well-known Hungarian author deals with euclidean geometry; it was published in Hungarian in 1937. The publication of a French translation was planned, but delayed by the outbreak of the war. In the mean time Kerékjártó worked on the second volume which he completed in 1944. However, war time conditions as well as illness and death of the author (in 1946) caused delay in the publication. The French version of Vol. I "La construction élémentaire de la géométrie euclidienne" was published in 1955 by the Hungarian Academy in Budapest under the editorship of Frédéric Riesz. Publication of the second volume was further held up until finally in 1966 it appeared jointly in Budapest (Akadémiai kiado) and Paris, edited by György Hajós.

"Habent sua fata libelli!" The second volume of the "Fondements de géométrie" shows the same high level which secured world-wide recognition to the first: Careful exposition and complete proofs, unfortunately not always found in books on projective geometry, great precision in the language; but in spite of all this, in the present-day reader the book leaves the impression of something out of long by-gone days. The reason is the following: This book has been written about eight years before the appearance of R. Baer's "Linear Algebra and Projective Geometry" New York, 1952 (work on it must have started about 15 years earlier if we reckon that Kerékjártó started work on it immediately after the completion of the first volume) and it came out 14 years after Baer's book, i.e. after our views on projective geometry had considerably changed. In addition we may mention a certain decline in interest in the foundations of geometry in general and in Hilbert's axiomatics in particular (Kerékjártó's book is indeed much concerned with Hilbert's axiomatics which nowadays might be considered as a certain defect) as well as the fact that in the meantime we had some consistently axiomatic expositions of projective geometry from which I wish to mention in particular H. S. M. Coxeter's

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excellent book "The real projective plane" (first ed. 1949, 2nd ed. 1955). And I am afraid that, although the appearance of the first volume of Kerékjártó's book in its own time could be considered as an important event in the world of books on geometry, the second volume will pass by rather unnoticed by the average reader, let us say the younger generation of mathematicians.

To this result may contribute the fact that the new book appears explicitly as "second part" of a work which appeared a long time ago (and is probably out of print by now) and indeed it is difficult to consider Vol. II as an independent work. References to Vol. I are frequent and in certain parts quite important. In the first part of his book Kerékjártó deals with the foundations of euclidean geometry of the plane and the three-dimensional space on the basis of Hilbert's axiomatics (slightly modernised, mainly with regard to the axiom of continuity), a valuable careful exposition leading up to rather sophisticated geometrical theorems. At a time when the scientific and didactical value of Hilbert's "Foundations of geometry" was not in doubt by anybody, such a course on geometry could be considered as an important achievement of the author. Today, however, when everywhere less and less attention is given to Hilbert's axiomatical scheme, an exposition of projective geometry on this basis will hardly carry along the imagination of young geometers. This is tied up with the general "algebraization" of mathematics and in this connection we may refer to the lively introduction to the strongly polemical book by Jean Dieudonné "Algèbre linéaire et géométrie élémentaire", Paris, Hermann 1964, as well as to the more recently developed approaches to euclidean geometry (cf. e.g. F. Bachmann, "Aufbau der Geometrie aus dem Spiegelungsbegriff", Berlin, Springerverlag, 1959).

Construction of projective geometry on Hilbert's basis is carried through in Kerekjarto's Vol. II. But in spite of "Foundations of Geometry" on the title page with "Projective Geometry" added in smaller print, the work should be considered mainly as a textbook on projective geometry and not as one of the books on foundations. Everything necessary about Foundations of geometry has been analysed in Vol. I - here in Vol. II one reaps the benefit of earlier labour. In the beginning of this volume the author introduces the projective plane and space as completions of the euclidean plane and space by improper (ideal) elements assuming that this plane and space are known to the reader from the first volume. Thus he established the fundamental properties of incidence, cyclic order and continuïty of the projective space; in the completed space these properties are proved as <u>theorems</u> - and then they are taken as <u>axioms</u> of projective geometry. The question of independence of the axioms does not occur here; that they are non-contradictory and that the system is complete is established by the constructions of this large book. For this purpose the author refers mainly to the listing of the 33 axioms of space in the beginning of the book; sometimes, however, he turns immediately to Hilbert's axioms of euclidean geometry (as a rule every section of the book is preceded by a list of the axioms required in this section).

After this fundamental part - on rather traditional lines - there follow five chapters, musts for every course on projective geometry: "Projective geometry on the straight line", "Projective geometry in the plane", "Projective geometry in space", "Conic sections" (in the plane), "Quadrics". All this is well known; here it is accompanied by a detail discussion of projective transformations: collineations and correlations with particular attention to involutory collineations (involutions) and correlations (polarities), discussion of subgroups of the general projective group, commuting transformations, and generation of the full group by involutions. The analytic methods of projective geometry are not neglected, coordinates are introduced first on the line, then in the plane and space. Special sections are devoted to affine and euclidean geometry, now considered from the point of view of projective geometry. Chapter VII "Projective Metrics" deals as usual with Lobacevskii's hyperbolic and Riemann's elliptic geometry, in a rather old-fashioned way, including hyperbolic and elliptic trigonometry.

Less of traditional character is the last chapter of the book "On the axioms of projective geometry". Here we find a discussion of other methods of construction of projective geometry, e.g. the axiomatics of Veblen (cf. O. Veblen and J. W. Young, Projective Geometry, Vols. I - II, Boston 1910-1918) or that of Bieberbach ("Einleitung in die höhere Geometrie" Leipzig 1933). Questions concerning the independence of axioms are dealt with. Particular attention is given to the propositions (axioms or theorems, according to the point of view taken) of Desargues and Pappus and their connection with the algebraic properties of the basic field. Projective geometries over arbitrary, in particular finite, fields are touched in the beginning of the chapter; over hypercomplex algebras with a reference to L. Pontrjagin's well-known result on the topologization of such algebras at the end of the chapter. On the whole, however, the brief chapter VIII does not destroy the impression of a certain traditionality of Kerékjártó's book.

From other books on projective geometry the present work differs mainly in its initial set-up and in the careful manner in which all the constructions are carried out; thus it may well be a pleasure to read for all lovers of classical projective geometry, unfortunately a very small subset of the set of all mathematical readers.

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Translated by H. Schwerdtfeger