to be crucial here.

A brief description of the contents of the book (which is written for those who know real analysis but not necessarily potential theory) is as follows. Chapter 1 is concerned with preliminaries from real analysis (maximal functions, Fourier transform, etc.) and Sobolev spaces. Chapter 2 introduces the L^p-capacity and nonlinear potentials referred to above. Chapter 3 is a compendium of real variable estimates for Bessel and Riesz potentials which prove useful later. In Chapter 4 the Besov and Triebel–Lizorkin spaces are introduced and their role in potential theory established via a fundamental inequality of Tom Wolff. The next four chapters explore the consequences of what has been set up. Chapter 5 is a very useful guide to comparison of capacities with the (more familiar?) Hausdorff measures, while continuity properties of functions in Sobolev spaces – "continuous except on a 'small' set measured in terms of capacity" – are considered in Chapter 6. Chapters 7 and 8 treat trace and embedding theorems and Poincaré type inequalities respectively. Finally, Chapter 9 is concerned with a certain approximation property of Sobolev spaces and in Chapter 10 this property is re-examined in the light of Netrusov's ideas and a further theorem of Netrusov is presented. The last chapter deals with rational and harmonic approximation.

The book is carefully and thoroughly written and prepared with, in my opinion, just the right amount of detail included. The prose flows naturally. A couple of topics which might have been treated at least to some extent are irregular domains (of vast importance in potential theory) and trace and embedding theorems between function spaces when the target space also measures smoothness. But these are perhaps just the minor grumbles of a harmonic analyst – the book addresses a much wider audience than just the harmonic analysts – and indeed it is pleasing to note just how indispensable the Calderón-Zygmund theory is in the whole development of the subject.

Function spaces and potential theory will certainly be a primary source that I shall turn to when I need to consider potential-theoretic matters and, I suspect, it will be for many others too.

A. CARBERY

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Howie, J. M. *Fundamentals of semigroup theory* (London Mathematical Society Monographs No. 12, Clarendon Press, Oxford, 1995), x + 351 pp., 0 19 851194 9, (hardback) £45.00.

John Howie's latest book is a substantial updating of his 1976 book *An introduction to semigroup theory* (Academic Press). Like its predecessor the new book does not attempt to cover the whole field, but concentrates instead on the algebraic theory with a particular emphasis on the class of regular semigroups. Regular semigroups are easier to handle than arbitrary semigroups but, more importantly, they play a paradigmatic role in semigroup theory as a whole.

The book is divided into eight chapters. The first three might well have been subtitled "What every mathematician should know about semigroups". In the first chapter basic definitions from algebra are introduced with the minimum of fuss, including a useful treatment of equivalence relations and congruences. Semigroup theory proper begins with Chapter 2 and the definition of Green's equivalence relations. With the help of these relations the elements of a semigroup can be sorted according to their mutual divisibility properties and arranged in what is termed an "egg-box diagram". The properties of these diagrams are basic to many arguments in semigroup theory. The third chapter is an exposition of the Rees Theorem, one of the most influential results in semigroup theory. This result describes in a way reminiscent of the Wedderburn–Artin Theorem the class of completely 0-simple semigroups. With the help of this theorem all finite simple semigroups can be described. In the next three chapters regular semigroups take centre stage. A regular semigroup is one in which every element a possesses a "generalised inverse", that is, an element b such that $a = aba$ and $b = bab$. The semigroup of all transformations of a set to itself is regular, as are all completely 0-simple semigroups. Chapters 4 and 5 develop the theory of two classes of regular semigroups which are extremal in some sense: the completely regular semigroups and the inverse semigroups. The former are semigroups in which every element is required to belong to a group contained in the semigroup (hence their alternative name "unions of groups"), whereas the latter are regular semigroups in which every element has a unique generalised inverse. These two chapters provide thorough introductions to their respective classes of semigroups. Chapter 6, on the other hand, provides brief introductions to three different classes of regular semigroups: the locally inverse semigroups, the orthodox semigroups and the semibands. The first two are generalisations of inverse semigroups; the former are the regular semigroups S in which all local submonoids (that is, subsemigroups of the form eSe, where $e^2 = e$) are required to be inverse; the latter are the regular semigroups in which the idempotents form a subsemigroup. The semibands are the regular semigroups generated by their idempotents. The last two chapters of the book discuss two rather different topics. Chapter 7 is an introduction to a fascinating theory arising from free semigroups. Not every subset of a free semigroup need generate a free subsemigroup, but those that do, called "codes", have important applications in the efficient representation of information. Chapter 8, the most difficult in the book, is an introduction to semigroup amalgams.

Although the overall structure of the original has been preserved, there have been many changes in both detail and substance. The chapter on inverse semigroups has been extended to include results on E-unitary semigroups and free inverse monoids; the chapter on free semigroups is new, as are the sections on locally inverse semigroups and semibands. The chapter on semigroup amalgams has been substantially rewritten to incorporate techniques from category theory. The chapter on orthodox semigroups in the 1976 book has been considerably slimmed down and now forms just part of a chapter. In addition to the formal mathematical development in the book there are many new questions at the conclusion of every chapter and every chapter now ends with notes containing background remarks.

Twenty years is a long time in mathematics and in this time semigroup theory has come of age. Twenty years ago there was still a visible join between the algebraic and automata theoretic strands of the subject. Since then there has been a genuine rapprochement between these two traditions, in which regular semigroups have played an important role. With this well-written and well-organised book I think the author has ensured that "Howie" will continue to be a byword for semigroup books.

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