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EXTRACTION OF DENSITY-LAYERED FLUID FROM A POROUS MEDIUM

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Flow of fluids in porous media is very important across a range of applications. Oil recovery from underground, pumping fresh water from aquifers and mineral leaching in mining applications are obvious examples. A characteristic of the fluids in many of these applications is that they are stratified in density, either due to fluid properties (oil-water), salt content (fresh-water, salt-water) or temperature.

We consider axisymmetric flow towards a point sink from a stratified fluid in a vertically confined aquifer. We present two approaches to solve the equations of flow for the linear density gradient case. First, a series method results in an eigenfunction expansion in Whittaker functions. The second method is a finite difference method. Comparison of the two methods verifies the finite difference method is accurate, so that more complicated nonlinear, density stratification can be considered. Interesting results for the case where the density stratification changes from linear to almost two-layer are presented, showing that in the nonlinear case, there are certain values of flow rate for which a steady solution does not occur. A spectral method is then implemented to consider cases in which there is a stagnant region beneath a sharp interface between two layers of different, but constant density. In this situation, flows also exist only for flow rates beneath a critical flux value, consistent with the results for the continuous density stratification. Finally, we considered supercritical solutions, in which both layers flow out through the sink, using a modified spectral method. We found two exact solutions, one in which the sink is at the middle height of the interface and another in the limit as the flow rate becomes large. For all other flow rates, we found a numerical solution. As the sink moves away from the initial value of the interface

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height, the required critical flow rate that induces coning and results in supercritical flow increases significantly. This result is consistent with the results of the single-layer case.

Some of this research has been published in [1, 2].

References

[1] J. Jose, G. Hocking and D. Farrow, 'Extraction of density-layered fluid from a porous medium', *ANZIAM J.* **61** (2020), C137–C151.

[2] J. Jose, G. Hocking and D. Farrow, 'Extraction of density-layered fluid from a porous medium', *J. Engrg. Math.* **135** (2022), Article no. 3, 18 pages.

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