Index

Index of symbols. These are grouped according to the type of object to which they most closely relate.

Categories

$[n]$: $\{0, \ldots, n\}$ as a category, 37
$[\mathcal{C}, \mathcal{D}]$: functor category, 36
$\bigoplus \mathcal{C}_\lambda$: direct sum of categories, 108
$\prod_\Lambda \mathcal{C}_\lambda$: product category, 13
$C \backslash F$: right-fibre category, 24
$C \backslash \mathcal{C}$: slice category, 24
$\mathrm{Sk}(\mathcal{C})$: skeleton of $\mathcal{C}$, 39
$\mathcal{A}\mathcal{B}$: category of abelian groups, 3
$\mathcal{A}/\mathcal{C}$: quotient category, 281
$\mathcal{A}_{\mathcal{DD}}(X)$: additive category generated by X, 92
$\mathcal{A}_{\mathcal{RT}R}$: category of Artinian modules, 102
$\mathcal{ASS}_R$: Artinian semisimple modules, 90
$\mathcal{A}_{\mathcal{UT}}\,\mathcal{C}$: automorphism category, 14
$\mathcal{B}_{\mathcal{AL}R}$: category of balanced bimodules, 160
$\mathcal{B}_{\mathcal{ASES}R}$: based free modules of finite rank, 32
$\mathcal{B}_{\mathcal{ASES}R}^{\infty}$: based free modules, 44
$\mathcal{B}G$: group as a category, 14
${}_R\mathcal{B}_{\mathcal{IMOD}R}$: bimodule category, 160
$\mathcal{B}_{\mathcal{RANDT}}(\Sigma)$: Brandt groupoid, 219
$\mathcal{C} \times \mathcal{D}$: product category, 11
$\mathcal{C}, \mathcal{D}, \ldots$: categories, 2
$\mathcal{C}^\Lambda$: product of copies of $\mathcal{C}$, 13
$\mathcal{C}^{\odot}$: mirror category, 6
$\mathcal{C}^{\oplus}$: split exact category, 120
$\mathcal{C}^{\mathrm{op}}$: opposite category, 8
$\mathcal{C}^n$: product of copies of $\mathcal{C}$, 13
$\mathcal{C}_1 \times \cdots \times \mathcal{C}_n$: product, 13
$\mathcal{C}_{\mathcal{AT}}$: category of categories, 23
$\mathcal{C}_{\mathcal{OK}}(\gamma)$: in which $\mathrm{Cok}(\gamma)$ is initial , 60
$\mathcal{C}_{\mathcal{OPRD}}(C(\Lambda))$: in which the coproduct is initial, 58
$\mathcal{C}_{\mathcal{OPRD}}(C', C'')$: in which the coproduct is initial, 58
$\mathcal{E}G$: group as a category, 14
$\mathcal{E}_{\mathcal{ND}}\,\mathcal{C}$: endomorphism category, 14
$\mathcal{E}_{\mathcal{NS}}$: category of sets, 3
$\mathcal{F}_R$: free right modules, finite rank, 10
${}_R\mathcal{F}$: free left modules, finite rank, 10
$\mathcal{F}_n^+\mathcal{C}$: category of sequences, 132
$\mathcal{F}_n\mathcal{C}$: category of sequences, 132
$\mathcal{FA}_{\mathcal{B}}$: finite abelian groups, 90
$\mathcal{F}_{\mathcal{IELD}}$: category of fields, 48
$\mathcal{F}_{\mathcal{REE}R}$: free right modules, 10
${}_R\mathcal{F}_{\mathcal{REE}}$: free left modules, 10
${}^*\mathcal{F}_{\mathcal{REE}R}$: based free modules, 44
$\mathcal{G}_{\mathcal{P}}$: category of groups, 3
$\mathcal{HA}_{\mathcal{B}}$: Hausdorff topological abelian groups, 88
$\mathcal{I}_{\mathcal{NF}}\mathcal{A}_{\mathcal{B}}$: infinite abelian groups, 60
$\mathcal{I}_{\mathcal{NJ}R}$: category of injective modules, 80
$\mathcal{I}_{\mathcal{SO}}\,\mathcal{C}$: isomorphism category, 14
$\mathcal{K}_{\mathcal{ER}}(\gamma)$: in which $\mathrm{Ker}(\gamma)$ is terminal, 59
$\mathcal{M}_{R,\mathcal{O}}$: relative category, 161
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${}_R\mathcal{M}$: finitely generated left modules, 10
$\mathcal{P}_{R,\mathcal{O}}$: relative category, 161
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${}_R\mathcal{P}$: finitely generated projective left modules, 10
$\mathcal{M}_{\mathcal{OD}R}$: category of right R-modules, 3
${}_R\mathcal{M}_{\mathcal{OD}}$: category of left R-modules, 3
$\mathcal{M}_{\mathcal{OD}R,\mathfrak{a}}$: quotient mod nilmodules, 286
$\mathcal{M}_{\mathcal{OR}}(W; R, S)$: relative morphism category, 148
$\mathcal{M}_{\mathcal{OR}}\mathcal{C}$: morphism category, 13
$\mathcal{M}_{\mathcal{OR}^2}\mathcal{C}$: morphisms of morphisms, 16
$\mathcal{N}_{\mathcal{IL}R,\mathfrak{a}}$: nilmodules, 286
$\mathcal{N}_{\mathcal{OETH}R}$: Noetherian modules, 102
$\mathcal{O}_{\mathcal{RD}}$: category of ordered sets, 4

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$\mathcal{PRD}(C(\Lambda))$: in which the product is terminal, 58
$\mathcal{PRD}(C', C'')$: in which the product is terminal, 57
$\mathcal{PREADD}(X)$: category generated by X, 85
$\mathcal{PROJ}_R$: projective right modules, 10
${}_R\mathcal{PROJ}$: projective left modules, 10
$\mathcal{RING}$: category of rings, 3
$\mathcal{RNG}$: category of nonunital rings, 3
$\mathcal{SET}$: category of sets, 3
$\mathcal{STANBASES}_R^{\infty}$: free modules with standard bases, 44
$\mathcal{T}_{A(\mathcal{O}),\mathcal{O}}$: torsion adèle modules, 306
$\mathcal{T}_{A(\mathcal{O}),\mathfrak{p}}$: primary adèle modules, 306
$\mathcal{T}_{\mathcal{O},R}$: relative f. g. torsion modules, 130
$\mathcal{T}_{\mathcal{O}}$, $\mathcal{T}_{\mathcal{O},R}$, $\mathcal{T}_{\mathfrak{p},R}$: categories of f. g. torsion modules, 267
$\mathcal{T}_{\mathcal{O}}$: finitely generated torsion modules, 112
$\mathcal{T}_{\mathfrak{p},\mathcal{O}}$: primary modules, 112
$\mathcal{TF}_{\mathcal{O}}$: finitely generated torsion-free modules, 112
$\mathcal{TF}_{\mathcal{O},R}$: relative torsion-free modules, 130
$\mathcal{TF}_{\mathcal{O}}$, $\mathcal{TF}_{\mathcal{O},R}$,$\mathcal{TF}_{\mathfrak{p},R}$: categories of f. g. torsion-free modules, 267
$\mathcal{TOP}$: category of topological spaces, 4
$\mathcal{TOPAB}$: topological abelian groups, 88
$\mathcal{TOR}_{\mathcal{O},R}$: relative torsion modules, 130
$\mathcal{TOR}_{\mathcal{O}}$, $\mathcal{TOR}_{\mathfrak{p},R}$, $\mathcal{TOR}_{\mathcal{O},R}$: categories of torsion modules, 267

Exact sequences

$C(M, \mathbf{E})$: measures non-exactness of Hom sequence, 335
$\mathrm{Ex}(\mathcal{C})$: specified exact sequences, 119
$\mathrm{Ext}^1_R(M'', M')$: group of extensions, 81
$\phi_*\mathbf{E}$: push-out of sequence, 80
$\theta^*\mathbf{E}$: pull-back of sequence, 80

Functors

η: natural transformation, 26
id_F: identity natural transformation, 36
$\simeq$:
natural isomorphism of functors, 30
equivalence of categories, 38
$- \otimes_R -$: tensor bifunctor, 142
$- \otimes_R N$: tensor functor, 140
Ab: abelianization functor, 25
Cnst_D, $\mathrm{Cnst}_{\mathcal{D}}$: constant functors, 43
$\mathrm{Ext}^n_R(-,-)$: extension bifunctors, 77
$f^{\#}$: restriction of scalars, 165
$f_{\#}$: extension of scalars, 166
$F(C_1, \ldots, C_{h-1}, -, C_{h+1}, \ldots, C_k)$: restriction of multifunctor, 35
Fr: free object functor, 19
GF, $G \circ F$: composite or product of functors, 19, 20
$H^M(-)$, $H_X(-)$: Hom functors, 76, 77
$\mathrm{Id}_{\mathcal{C}}$: identity functor, 18
Inc: inclusion functor, 18
$\varprojlim^1$: derived functor, 250
Mir: mirror functor, 21
$M \otimes_R -$: tensor functor, 141
$\mathrm{Mor}_{\mathcal{C}}(-,-)$: morphism bifunctor, 33
$\mathrm{Mor}(-, X)$, $\mathrm{Mor}(L, -)$: morphism functors, 22
Op: opposite functor, 18
$P : R \mapsto \langle 1_R \rangle$: ring to prime subring, 28
$\mathrm{Tor}_n(-,-)$: Tor bifunctors, 153
Υ: forgetful or underlying functor, 18

Groups

$ghg^{-1}h^{-1}$: commutator, 25
$\langle T \rangle$: monoid generated by T, 254
$[C]$: element in Grothendieck group, 123
$\mathrm{Cl}(\mathcal{O})$: ideal class group, 217
$\mathrm{Fr}(\mathcal{C})$: free abelian group on isomorphism classes, 123
G_{ab}: commutator quotient group, 25
$[G, G]$: commutator subgroup, 25
$G' * G''$: group free product, 58
$\mathrm{GL}(R)$: stable general linear group, 229
$\mathrm{GL}_n(R)$: general linear group of degree n, 29
H^G: normal closure, 64
$K_0(\mathcal{C})$, $K_0(\mathcal{C}, \mathrm{Ex}(\mathcal{C}))$: Grothendieck group, 123
$\mathrm{Pic}(R)$: Picard group, 214
$\mathrm{Rel}(\mathcal{C})$: relation subgroup, 123
S_n: symmetric group, 16

Homomorphisms and morphisms

$\cong$: isomorphism, 14
$\alpha_{\mathfrak{p}}$: localized homomorphism, 265
$\alpha \oplus \beta$: direct sum, 117
$f^{\#}\alpha$: homomorphism arising from restriction, 165
$f_{\#}\alpha$: homomorphism arising from extension, 167
$\mathrm{Hom}(C, D)$: homomorphisms, 3
$\mathrm{Hom}(L_R, X_R)$: homomorphisms, 69
$\mathrm{Hom}_{A-C}(L \otimes_B M, N)$: bimodule homomorphisms, 146
id_C: identity morphism, 5
ι_C: morphism from initial object, 48
$\iota_R : R \to M_n(R)$: inclusion as scalars, 28
$\iota^{\lambda\mu}$: $\leq$ as a morphism, 4
λ^*, ξ_*: induced maps on morphisms, 23
$\mathrm{Mor}_{\mathcal{C}}(C, D)$: morphisms in a category, 2

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$\mathrm{Mor}(L, \xi)$, $\mathrm{Mor}(\lambda, X)$: induced maps on morphisms, 22

Ideals

$\{\mathfrak{a}\}$: class of $\mathfrak{a}$ in $\mathrm{Cl}(\mathcal{O})$, 109
$\mathfrak{ab}$: product, 109
$\mathrm{Ann}(M)$: annihilator, 111
$\mathrm{Cl}(\mathcal{O})$: ideal class group, 109
$\mathfrak{c}(S, R)$: conductor, 339
$\mathrm{Frac}(\mathcal{O})$: fractional ideals, 109
$\mathrm{In}(R)$: invertible fractional ideals, 220
$\mathfrak{n}(L, M)$: an invariant, 321
$\mathrm{Pr}(\mathcal{O})$: principal ideals, 109
$\mathrm{rad}(R)$: Jacobson radical, 103

Matrices

$A \oplus 1$: augmented matrix, 29
A^t: transposed matrix, 29
$M_{m,n}(R)$: $m \times n$ matrices, 145
$M_n(R)$: matrix ring, 28

Measuring functions

$\|\cdot\|$: norm, 310
$d(r, s)$: metric, 310
$\mathrm{pd}_R(L)$: projective dimension, 342
$v_{\mathfrak{p}}$: $\mathfrak{p}$-adic valuation, 292

Modules

$\{W\}$: class of W in $\mathrm{Pic}(R)$, 214
${}^{\alpha}N$: twisted module, 176
$A(M)$: adèle module, 328
$f^{\#}M$: module obtained by restriction, 165
$f_{\#}M$: module obtained by extension, 166
$\mathrm{Fr}_R(X)$: free right R-module on X, 19
$\mathrm{Ind}(R)$: indecomposable projectives, 317
$M \otimes_R N$: tensor product, 136
$M \times_{M''} L''$: pull-back, 67
$M' \oplus M''$: direct sum, 91
$M' \oplus_{L'} L$: push-out, 65
M^G: induced module, 178
$M_{\mathfrak{p}}$: localization at $\mathfrak{p}$, 264
$M\mathfrak{a}$: module times ideal, 169
M^{α}: twisted module, 175
M^{Λ}: direct sum of copies of M, 144
M^n: direct sum of copies of M, 144
MS: $M \otimes_R S$ when M_R is flat, 169
MR_{Σ}: M_{Σ} for R torsion-free, 269
M_{Σ}: localization of module, 264
N_H: restricted module, 178
$\mathrm{rad}(L)$: radical of a module, 208
R^n: standard free right module, 30
R^{Λ}: standard free right module, 44
${}^nR$, ${}^{\Lambda}R$: standard free left module, 144
SN: $S \otimes_R N$ when ${}_RN$ is flat, 169
$T(M)$: torsion submodule, 110
$T_{\Sigma}(M)$: torsion submodule, 264
$T_{\mathfrak{p}}(M)$: $\mathfrak{p}$-primary component, 111
$W^* = \mathrm{Hom}({}_RW, {}_RR)$: dual, 185
$W^{**} = \mathrm{Hom}(W^*_R, R_R)$: double dual, 186

Objects in categories

$C' \coprod C''$: coproduct, 58
$\coprod_{\Lambda} C_{\lambda}$: coproduct of objects, 59
$C' \prod C''$: product, 57
$\prod_{\Lambda} C_{\lambda}$: product, 58
$\prod'$: restricted direct product, 305
(C, D, χ): morphism from C to D, as object in morphism category, 13
$[c_{\lambda}]$: image in direct limit, 226
C_{Λ}:
object of direct sum of categories, 108
direct system of objects, 225
$(\mathrm{Cok}\,\gamma, \chi)$: cokernel, 60
colim_{Λ}: colimit of elements, 227
Colim: colimit of objects, 237
$\mathrm{D}(M, N)$: denominator set, 279
$\mathrm{dir\,lim}_{\Lambda}$: direct limit, 226
$\mathrm{ind\,lim}_{\Lambda}$: inductive limit, 227
$\varinjlim$: colimit, 227
$\varprojlim$: limit, 247
inv lim: inverse limit, 247
$\mathrm{Is}(\mathcal{C})$: isomorphism classes of objects, 123
$(\mathrm{Ker}\,\gamma, \kappa)$: kernel, 60
$\mathrm{M}(M, N)$: direct system of abelian groups, 280
$\mathrm{Ob}\,\mathcal{C}$: objects of a category, 2
$(R)^{\#}$: $\mathcal{M}_{OD_R}$, viewed as an object in $\mathcal{C}_{AT}$, 165
$X \backslash \Upsilon$: objects Υ-under X, 50

Rings

$\langle 1_R \rangle$: prime subring, 28
$\mathbb{F}_{p^m}$: finite field, 228
$\overline{\mathbb{F}}_p$: algebraic closure of $\mathbb{F}_p$, 228
$\widehat{\mathbb{Q}}_p$: p-adic rationals, 302
$\widehat{\mathbb{Z}}_p$: p-adic integers, 248, 302
$\mathcal{D}(T)$: rational functions, 271
$\mathcal{D}(T, \alpha)$: skew rational functions, 271
$\mathcal{F}[[T]]$: formal power series, 302
$\widehat{\mathcal{K}}$: complete field, 297
$\mathcal{O}_a$: powers of a inverted, 274
$\mathcal{O}_{\mathfrak{p}}$: localization, 254
$\mathcal{O} \setminus \mathfrak{p}$: difference set, 254
$\widehat{\mathcal{O}}$, $\widehat{\mathcal{O}}_{\mathfrak{p}}$: completion or complete ring, 296
$\mathcal{V}$: valuation ring, 291
$A[\epsilon]$: ring of dual numbers, 99

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AG: group ring, 178
$A(\mathcal{K})$: adèle ring, 305
$A(\mathcal{O})$: adèle ring, 305
$A[T, \alpha]$: skew polynomial ring, 180
$A[T, T^{-1}, \alpha]$: skew Laurent polynomial ring, 254
CR: cone of a ring, 288
mR: nonunital ring of all finite matrices, 229
$M_n(R)$: matrix ring, 28
$\overline{R}$: unitalization, 28
$R_{\mathfrak{p}}$: localization of an order, 254
R_Σ: ring of fractions, 256
${}_\Sigma R$: ring of left fractions, 272

Sets

$\bigsqcup_\Lambda X_\lambda$: disjoint union, 65
Λ: partially ordered set, 4
$\mathrm{Lat}(R)$: division lattice, 224
$\mathrm{Latid}(R)$: ideal lattice, 237
$\mathrm{Map}(X, Y)$: maps between sets, 3
ω: natural numbers as ordered set, 224
$\mathbf{P}(X)$: power set, 225
$\mathbf{P}_f(X)$: finite subsets of X, 225

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