The five Companies chosen for this comparison, have been taken indiscriminately, and from their rates being all identical, it is reasonable to suppose that they represent the average premiums charged by the French Companies.

Table, showing the Annual Premiums required by certain French and English Companies, for the Assurance of £100 on a single Life:—

Ages.	French Companies.		English Companies.					
			Royal Exchange.		London Assurance.		Alliance.	
	One Year	Whole of Life.	One Year.	Whole of Life.	One Year	Whole of Life.	One Year.	Whole of Life.
	£ s. d.	£ s. d.	£ s. d.	£ s. d.	£ s. .	£ s. d.	£ s. d.	£ s. d.
30	1 11 0	2 9 10	1 13 3	2 13 3	1 8 9	2 11 11	1 6 9	2 9 2
40	1 17 10	3 5 8	2 0 9	3 8 0	1 11 7	3 7 0	1 13 7	3 6 6
50	2 12 0	4 13 3	2 15 0	4 10 9	1 19 6	4 16 1	2 10 9	4 14 2
60	4 6 0	7 2 8	3 18 0	6 7 3	3 10 2	7 3 8	4 3 7	7 14 11

The French Companies alluded to above, are, La Compagnie d'Assurances Generales (Established 1818); L'Union, (1829); La Nationale, (1830); La France, (1843); Le Phénix, (1846); each of which charges the same premiums as the others, both for one year and the whole of life.

It will be seen that the rates in question both for one year, and for the whole of life agree very nearly with those charged by the Alliance Assurance Company of London, whose rates, founded as they are upon the Carlisle mortality*, which is supposed to give a correct view of human life, may be considered to be as nearly equitable as possible.

From these circumstances, and from the fact that our neighbours do not appear to be attempting any dangerous experiments in Life Assurance, there is every reason to consider that their Offices, if well conducted, and prudently managed, are, to say the least, very superior to many of the new English Companies, with the announcement of whose novel plans the advertising columns of our newspapers are daily burthened. I remain, &c.

Alliance Assurance Company, *H. W. PORTER.*
Nov. 2nd, 1850.

ON THE COMPARATIVE ADVANTAGES OF THE OLD AND THE NEW METHODS OF COMPUTATION.

To the Editor of the Assurance Magazine.

Sir,—I shall be glad if you can make room for the following remarks. The distinction between the old and the new methods of computation in Life Contingencies, consists in the different nature of the data which they respectively supply for the solution of practical questions. The principle on which all such questions are solved, is to found an equation between the present values of the benefits to be purchased and the payments to be given for it, using a symbol for the unknown amount of the one or the other, as the case may be. The solution of this equation, which is always of the simplest kind, then gives the value of the symbol employed for the unknown amount. Or if symbols be used for both amounts, the equation will then be of a more general kind, and

* The rates of premium given in the Table as those of the Royal Exchange, are the Northampton rates, still used by the Equitable Society, and many of the older Companies. These rates of the Alliance appear to be deduced from the Carlisle Tables at 4 per cent, with 40 per cent. added. The premiums charged by the French Companies approximate still more nearly for the whole continuance of life, to those obtained from Farr's English Life Table, (census 1841), at 3 per cent.. with 20 per cent. added to the pure premium, as may be noticed in the following comparison of the latter. Age 30, £2 9s. 4d.; Age 40, £3 5s. 6d.; Age 50, £4 13s. 0d.; and Age 60, £7 7s. 1d.—(Ed. *A. M*)

will give either amount when the other is known. Thus, if a benefit of a certain kind, amount m, is to be purchased by a premium, amount p, payable in a stipulated manner; then, if B denote the present value of a benefit of the same kind, amount £1, the present value of the benefit to be purchased will be denoted by mB. Also, if the present value of a payment, amount £1, to be made in the manner stipulated, be P, the present value of the payment p, will be p P., thence, equating these present values,

$$pP = mB;$$

whence we get,

$$p = \frac{mB}{P}, \text{ or } m = \frac{pP}{B},$$

according as m or p is given, and p or m required.

It is now assumed that B and P are known; unless they are so, we get no numerical result. And these are *present* values, that is, values *having reference to a single, and specified epoch,* and of a known amount, namely £1. It thus appears, therefore, that the data we require for practical purposes must be such as to afford us the means of forming with facility the present values of all conceivable benefits and payments. To tabulate the whole, with reference to each age, is obviously and altogether an impracticable task. It is, as I have stated, in the means they respectively afford for this formation, that the distinction between the two methods of computation consists.

The epoch to which, in the solution of a specified problem, the values made use of have to be referred, is the present age of the life or lives involved in the problem, and, as just intimated, the *amounts* must be the same. This is apparent, from the example just given. Now the data in the old method are one or more series of values,* of *the same* amount, (£1), but having reference, considered as present values, not to our epoch, bnt to as many epochs as there are values in each series. In using these data, consequently, when any other than the *present* age, is in question, a *reduction to the epoch* becomes necessary, for every additional age. These cases arise when either a benefit is to be enjoyed or a payment to be continued during only a portion of existence.

On the other hand, the data presented to us by the new method consist of two or more series of values, all of which, choose which epoch we may, *are present values with respect to that epoch;* and, with respect to each individual epoch, *are also of the same amount.* Thus, take any age, x; opposite this age in column D, we find a number, D_x; now take any other age $x+n$; opposite this age in D, we find D_{x+n}, which is the *present* value of an endowment on (x), of D_x pounds, payable in n years; in N we find N_{x+n}, the *present* value of an annuity on (x) of D_x pounds, to be entered upon in n years; and so on. The value, in short, consists of nothing but *present* values, having reference as such *to each and every age in the table.* In the use of the new method, therefore, the necessity for what I have called *reduction to the epoch,* is entirely superseded. But this is not all; the amounts of the benefits with respect to such individual age, are also the same. With reference to age x as we have just seen, the uniform amount of the endowments whose present values occupy column D, is D_x pounds; and that of the annuities, whose present values occupy column N, is also D_x pounds. And so of the remaining columns. Now, in practical questions, it is of no moment what the *amounts* of the benefits and payments that enter the solution are, provided only these amounts are the same. This appears, from the foregoing type of solution. Thus, the values of p and m being respectively,

$$m.\ \frac{B}{P}, \text{ and } p.\ \frac{P}{B},$$

they are obviously not affected by any vaiiation of B and P in the same ratio. Hence, except in the single case in which a present value is wanted as a final result (when division by D_x is necessary,) the data of the new method admit of being employed without any previons preparation.

The *reduction to the epoch,* of such frequent occurrence in the use of the old

* Usually only one,—the annuities; but I found nothing on this at present.

method, consists in a multiplication by $v^n \cdot p_{x\,.\,n}$ for every value of n that occurs in the problem; or, which is the same thing, for every age, other than the present age, at which either a benefit or a payment is to commence; or, other than the end of life at which either a benefit or a payment is to terminate. This operation must, almost of necessity, be performed by means of logarithms; and as four or five tabular entries are required, it is obvious that it is attended with no small amount of trouble and liability to error.

I now give a few examples of corresponding formulæ by the two methods, premising that a_x denotes the present value of an annuity of £1 on (x), and that $p_{x\,.\,n}$ written in full is $\frac{l_{x+n}}{l_x}$.

Annual premium, payable $n+1$ times, for annuity of m pounds on (x), deferred n years.

$$\frac{mv^n p_{x\,.\,n}\, a_{x+n}}{1+a_x-v^n p_{x\,.\,n}\, a_{x+n}}\,; \qquad \frac{m\mathrm{N}_{x+n}}{\mathrm{N}_{x-1}-\mathrm{N}_{x+n}}.$$

Annual premium, payable t times, for assurance of m pounds for n years on (x).

$$\frac{m\left[v\left(1-v^n p_{x\,.\,n}\right)-(1-v)\left(a_x-v^n p_{x\,.\,n} a_{x+n}\right)\right]}{1+a_x-v^t p_{x\,.\,t}\left(1+a_{x+t}\right)}\,; \qquad \frac{m\left(\mathrm{M}_x-\mathrm{M}_{x+n}\right)}{\mathrm{N}_{x-1}-\mathrm{N}_{x+t-1}}$$

Annual premium, payable t times, for assurance of m pounds on (x), payable in n years, or at death, if before.

$$\frac{m\left[v\left(1-v^n p_{x\,.\,n}\right)-(1-v)a_x-v^n p_{x\,.\,n}\left\{1-(1-v)a_{x+n}\right\}\right]}{1+a_x-v^t p_{x\,.\,t}\left(1+a_{x+t}\right)}\,; \qquad \frac{m\left[\mathrm{D}_{x+n}+\mathrm{M}_x-\mathrm{M}_{x+n}\right]}{\mathrm{N}_{x-1}-\mathrm{N}_{x+t-1}}.$$

The disadvantage of the old method here clearly appears, in the large number of quantities that have to be dealt with, and, in particular, in the constantly recurring multiplications by the discounted probabilities. It will be rendered yet more apparent if any of the corresponding formulæ be worked out at length. I subjoin an example so worked out. It is one which really occurred, and the consideration of which led to the preparation of the foregoing remarks.

PROBLEM.

A party now aged (x) is assured for m pounds, at an annual premium, P, payable till death. He desires to change his assurance into another of the same amount payable in n years, or at death, if before. Required, the new premium P′, payable till the risk is determined.

This problem admits of treatment in various ways, all, of course, giving the same result. I adopt the method which seems on the whole the best, as less likely than the others to lead astray.

Benefit Terms.

(x) receives, first, an assurance of m pounds, payable at $x+n$ or at death, the present value of which, multiplied by D_x, is - - - - - - -	$m\left(\mathrm{D}_{x+n}+\mathrm{M}_x-\mathrm{M}_{x+n}\right)$;
And, secondly, remission of the premium P, the present value of which, multiplied by D_x, is - - -	PN_{x-1}.

Payment Terms.

(x) gives up his present assurance, the present value of which, multiplied by D_x, is - - - -	$m\mathrm{M}_x$;
And he pays a premium P′ for n years, the present value of which, multiplied by D_x, is - - -	$\mathrm{P}'\left(\mathrm{N}_{x-1}-\mathrm{N}_{x+n-1}\right)$.

https://doi.org/10.1017/S2046164X00055332 Published online by Cambridge University Press

Hence, equating the sum of the benefit terms to that of the payment terms,

$$P'\left(N_{x-1}-N_{x+n-1}\right)+mM_x=m\left(D_{x+n}+M_x-M_{x+n}\right)+PN_{x-1};$$

Whence,

$$P'=\frac{m\left(D_{x+n}-M_{x+n}\right)+PN_{x-1}}{N_{x-1}-N_{x+n-1}}.$$

Let $x=40$; $n=20$; $\therefore$ $x+n=60$; also, $m=$ £1000; P = £20 10s. 6d. $=20\cdot525$; and the rates of mortality and interest, Carlisle, 3 per cent. The formula then becomes,

$$P'=\frac{1000\,(D_{60}-M_{60})+20\cdot525\,N_{39}}{N_{39}-N_{59}}$$

and the working is as follows:

N_{39} = ...		28225·536		
		52502		
		56451072		
		1411277		
		56451		
		14113		
		579329·13		
$1000D_{60}$ =	618337·6			
„ M_{60} =	411379·7	206957·9		
		786287·0	log.	5·8955811
N_{39} =	28225·536			
N_{59} =	7105·556	21119·98	„	4·3246935
P′ = ...		37·22953	„	1·5708876

The formula by the old method is,

$$P'=\frac{mv^n p_{x\,.\,n}\left(1-A_{x+n}\right)+P\left(1+a_x\right)}{1+a_x-v^n p_{x\,.\,n}\left(1+a_{x+n}\right)};$$

or, for the particular case before us,

$$P'=\frac{1000v^{20}p_{40\,.\,20}\left(1-A_{60}\right)+20\cdot525\left(1+a_{40}\right)}{1+a_{40}-v^{20}p_{40\,.\,20}\left(1+a_{60}\right)}$$

and the working is as follows:

v^{20}	(Jones, p. 103)	log.	$\bar{1}$·7432555
l_{60}	(„ p. 290)	„	3·5614592
l_{40}	(„ „)	colog.	$\bar{4}$·2945640
$v^{20}\,.\,p_{40\,.\,20}$		log.	$\bar{1}$.5992787
A_{60} = ·6652994	(Jones, p. 540)		
$1-A_{60}$ = ·3347006	. . . .	„	$\bar{1}$·5246565
1000	. . . .	„	3·
$1000v^{20}\,.\,p_{40\,.\,20}\;(1-A_{60})$ = . . .	133·0256	„	2·1239352

https://doi.org/10.1017/S2046164X00055332 Published online by Cambridge University Press

$1+A_{40}$ =	18·14242			
	52502			
	3628484			
	90712			
	3628			
	907			
	372·3731	372.3731		
Numerator =		505·3987	log.	2·7036341
$v^{20} . p_{40\,.\,20}$ as above.	. . . . . .		log.	$\bar{1}$·5992787
$1+a_{60}$ = 11·49139 . . .	(Jones, p. 312)		,,	1·0603725
		4·56721	,,	0·6596512
$1+a_{40}$ =	(Jones, p. 311)	18·14242		
Denominator =		13·57521	log.	1·1327465
			colog.	$\bar{2}$·8672535
Numerator, as above	. . . .		log.	2·7036341
P′ =		37·22953	,,	1·5708876

Both methods, of course, give for the required premium the same value, £37 4s. 7d.; but the second is attended with at least four times the amount of labour* required by the first.

Another most important advantage possessed by the new method, is the facility it affords for the extension of the data. Columns of *present* values, unlimited in number, can be found by mere addition, while, by the old method, the formation of every additional column is attended with as much labour as the formation of the first.

P. GRAY.

Baker Street, Lloyd Square,
Nov. 23, 1850.

ON THE VALUE OF ANNUITIES CERTAIN, OF WHICH THE SUCCESSIVE PAYMENTS ARE THE FIGURATE NUMBERS.

To the Editors of the Assurance Magazine.

Gentlemen,—The following remarks will probably be found useful to many of your readers, and I therefore place them at your disposal.

I call 1, 1, 1; 1, 2, 3; 1, 3, 6, &c., figurate numbers, of 1th, 2th, 3th orders; and denote the respective annuities by Σv^n $\Sigma^2 v^n$ $\Sigma^3 v^n$ . . $\Sigma^p v^n$.

The first few cases are deduced from each other by finite integration, from which cases the general form is found by induction.

$$\sum v^n = \frac{1}{i} - \frac{v^n}{i} \quad \sum\nolimits^2 v^n = \frac{n}{i} - \frac{1}{i}\left(\frac{1}{i} - \frac{v^n}{i}\right) = \frac{n}{i} - \frac{1}{i^2} + \frac{v^2}{i^2}$$

$$\sum\nolimits^3 v^n = \frac{n(n+1)}{1\,.\,2\,.\,i} - \frac{n}{i^2} + \frac{1}{i^2}\left(\frac{1}{i} - \frac{v^2}{i}\right) = \frac{n(n+1)}{1\,.\,2\,.\,i} - \frac{n}{i^2} + \frac{1}{i^3} - \frac{v^2}{i^3}$$

and generally,

$$\sum\nolimits^p v^n = \frac{n\,(n+1)\,.\,.\,.\,.\,(n+p-2)}{1\,.\,2\,.\,.\,(p-1)\quad i} - \frac{n\,(n+1)\,\,(n+p-3)}{1\,.\,2\,.\,(p-2)\,\,i^2} + \,.\,.\,.\,.\, \pm \frac{n}{i^{p-1}} \mp \frac{1}{i^p} \pm \frac{v^n}{i^n}$$

in which the first term is invariably plus, then — + — + &c.

* This would be considerably increased if the value of A_{60} were not assumed.