VERTICAL REFRACTION ANGLE DERIVED FROM THE VARIANCE OF THE ANGLE-OF-ARRIVAL FLUCTUATIONS

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ABSTRACT. A theory is developed for evaluating the vertical refraction angle from the variance of the angle-of-arrival fluctuations, assuming a horizontally homogeneous turbulent atmospheric surface layer. The vertical refraction angle is mainly a function of the vertical temperature gradient, and the variance of the angle-of-arrival is related to the temperature structure parameter C_T^2. However, surface layer similarity theory states that both the mean vertical temperature gradient and C_T^2 are functions of the same scaling temperature T_* and a thermal stability parameter. This therefore provides an indirect method of determining the vertical refraction angle from a measurement of the variance of the angle-of-arrival and an estimate of the thermal stability parameter. Advantages of this method over other techniques of evaluating vertical refraction are discussed.

1. INTRODUCTION

The precise determination of the atmospheric effect on vertical angle measurements must still be considered a primary research topic in geodesy. An improvement of precision in determining the vertical refraction angle would greatly benefit many geodetic operations.

Considering the present state-of-the-art in determination of the vertical refraction angle, two different approaches can be distinguished (Prilepin, 1974): the meteorological and the instrumental solution. The meteorological solution is based either on the selection of favourable observation times when refraction effect prediction is more reliable, or on design of a realistic atmospheric model for which meteorological parameters may be determined from measurements such as temperature gradients or heat fluxes (Brunner, 1978). Much of the present understanding of the nature of atmospheric refraction must be attributed to the meteorological approach. It seems unlikely, however, that it will generally yield an accuracy of 0.5" for the vertical refraction angle.

The underlying principle of the instrumental solution is the dispersion

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effect of light waves propagating through the atmosphere. Several technical solutions have been proposed (Prilepin, 1974), and a few have resulted in actual prototypes (Tengström, 1978; Glissmann, 1976; Williams, 1978). It is not unrealistic to predict that these instruments will yield an accuracy of 0.5" for the vertical refraction angle in the near future. However, test measurements have shown that atmospheric turbulence causes considerable problems in measuring the small dispersion angle, and precise measurements are only possible during favourable observation times.

The theory of a new approach to the determination of the vertical refraction will be presented here. In principle, it utilizes exactly that effect of the turbulent medium on light wave propagation, which has caused difficulty for the instrumental solution. A single vertical angle observation of a remote target through the telescope of a theodolite can be considered as the sum of the mean value and the momentary deviation from this mean value caused by turbulence in the atmosphere. For all derivations in this paper the ergodic hypothesis is invoked, replacing ensemble averages by time averages (denoted by overbars). The angle-of-arrival fluctuations are defined as the fluctuations of the normal on to the arriving wave front at the telescope (Lawrence and Strohbehn, 1970). The variance of the vertical component of the angle-of-arrival fluctuations is denoted by $\sigma_\alpha{}^2$.

It has been shown (e.g., Brunner, 1978) that the mean angle of refraction is related to the mean vertical refractive index gradient, and (Appendix) that the variance of the angle-of-arrival fluctuations is related to the refractive index structure parameter $C_n{}^2$, characterising the structure of the atmospheric turbulence along the line of sight. Atmospheric surface layer theory (Appendix) states that statistics of the mean and the turbulent flow fields are functions of scaling and a stability parameter. For dry air it follows that the mean gradient and the structure parameter of the refractive index can both be expressed as functions of the scaling temperature T_* and an atmospheric stability parameter. Eliminating T_* in these expressions, it is possible to calculate the mean angle of refraction from a measurement of the variance of the angle-of-arrival and an estimate of the atmospheric stability parameter.

The remainder of this paper explains the principal features of the theory, treating the simple case of a horizontal line of sight in a horizontally homogeneous turbulent medium. Humidity effects have also been neglected in the present derivations. The possible extension of this theory to more realistic cases (inclined line of sight, general topography, humidity effects for water crossings) is briefly discussed. In anticipation of this extension the theory is based on the height-independent scaling temperature T_*, rather than on the mean temperature gradient which is a function of height.

2. THEORY

2.1 Mean angle of refraction

The curvature of an optical ray is related to the vertical gradient of refractivity $(\partial N/\partial z)$. The time average of the refraction angle, $\overline{\delta}$, observed at A for a horizontal line of sight from A to B, can then be derived as (Brunner, 1978)

$$\overline{\delta} = -10^{-6} \int_{0}^{S} \frac{\partial \overline{N}}{\partial z} \left(1 - \frac{x}{S}\right) dx \tag{1}$$

where $(\partial \overline{N}/\partial z)$ is the mean vertical refractivity gradient, S is the path length, and x is an integration variable, see Figure 1. The second term of the integral of equation (1) represents a weighting function for the refractivity gradients.

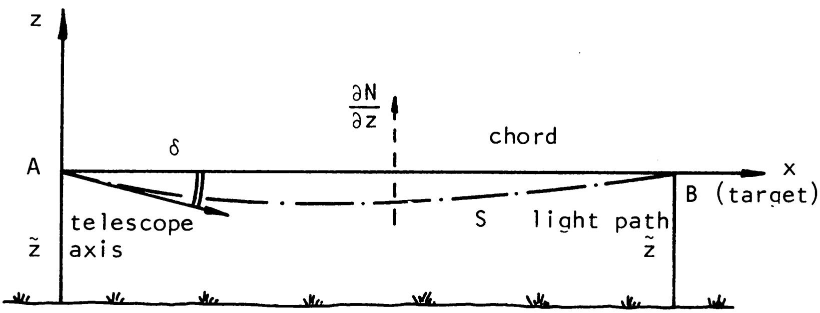

Figure 1. Geometry of the observation set-up.

Using the formula of Barrel and Sears, equation (A12), the gradient of refractivity can be derived as

$$\frac{\partial N}{\partial z} = \frac{N}{T} \left(\frac{T}{P}\frac{\partial P}{\partial z} - \frac{\partial T}{\partial z} \right) \tag{2}$$

where T is the temperature in degrees Kelvin, $\partial P/\partial z$ is the vertical pressure gradient, and $\partial T/\partial z$ is the vertical temperature gradient. The small effect of the water vapour pressure gradient has been neglected in equation (2). Substituting the hydrostatic equation

$$\frac{\partial P}{\partial z} = - \frac{g}{R}\frac{P}{T} \tag{3}$$

and the relationship (sufficiently accurate in the present context) for the potential temperature gradient

$$\frac{\partial \Theta}{\partial z} = \frac{\partial T}{\partial z} + \Gamma \qquad (4)$$

in equation (2) yields

$$\frac{\partial N}{\partial z} = -\frac{N}{T}\left(\frac{g}{R} - \Gamma + \frac{\partial \Theta}{\partial z}\right) \qquad (5)$$

where Γ is the adiabatic lapse rate (0.0098 K m^{-1}), and g/R is the ratio of gravity to the gas constant (0.0342 K m^{-1}). Transition of equation (5) to the average values of the involved parameters, and substitution of the flux-profile relationship (A5) for the mean potential temperature gradient ($\partial\overline{\Theta}/\partial z$) yields

$$\frac{\partial \overline{N}}{\partial z} = -\frac{\overline{N}}{\overline{T}}\left(0.0244 + \frac{T_*}{k\tilde{z}}\phi_h\right) \qquad (6)$$

where T_* is the scaling temperature, k is the von Kármán constant, $\tilde{z}$ is the path height above the ground (see Figure 1), and ϕ_h is the flux-profile function.

For a light ray parallel to the ground in a horizontally homogeneous surface layer, all the terms in equation (6) are constant. Therefore the integration of equation (6) according to equation (1) yields the final result

$$\overline{\delta} = 10^{-6}\frac{\overline{N}S}{2\overline{T}}\left(0.0244 + \frac{T_*}{k\tilde{z}}\phi_h\right) \qquad (7)$$

In this equation for $\overline{\delta}$, the first part represents the effect of the atmosphere for neutral conditions (Appendix), and the second part, through $T_*\phi_h$ accounts for deviations from neutral conditions, the diabatic effect. The sign of this second term is determined by the sign of the sensible heat flux in equation (A2) for the scaling temperature T_*, which will be negative for unstable (clear day) conditions, and positive for stable (clear night) conditions.

2.2 Variance of the angle-of-arrival

Much of the theory of wave propagation in a turbulent atmosphere has been given by Tatarskii (1971). The papers by Lawrence and Strohbehn (1970) and de Wolf (1974) also have been found very useful. When an electromagnetic wave propagates in a turbulent medium, it experiences random fluctuations of amplitude, intensity, phase and angle-of-arrival due to the refractive index fluctuations.

It can be shown that the variance of the angle-of-arrival, $\sigma_\alpha{}^2$, is a function of the phase structure function, $D_\phi(b)$, of the electromagnetic wave propagation

$$\sigma_\alpha^{\ 2} = \frac{D_\phi(b)}{\kappa^2 b^2} \tag{8}$$

where b is the interferometer separation and κ is the wave number $\kappa = 2\pi/\lambda$, with λ being the wave length. Equation (8) is derived under the assumption that the geometrical optics approximation is valid (Tatarskii, 1971).

The phase structure function has been derived by Tatarskii (1971) for a spherical wave propagating through a distance S in a homogeneous and locally isotropic turbulent medium with

$$D_\phi(b) = 1.09\ \kappa^2 C_n^{\ 2}\ S\ b^{5/3} \tag{9}$$

where C_n^2 is the refractive index structure parameter. Equation (9) is applicable when

$$l_o \ll (\lambda S)^{\frac{1}{2}} \ll L_o \tag{10}$$

where $(\lambda S)^{\frac{1}{2}}$ is the radius of the first Fresnel zone, l_o is the inner scale of the turbulence (in the order of a few millimetres), and L_o is the outer scale of the turbulence (about twice the path height above the ground).

The angle-of-arrival variance for a telescope can be obtained from the above equations, if the interferometer separation b is replaced by the diameter D of the receiving objective

$$\sigma_\alpha^{\ 2} = 1.09\ C_n^{\ 2}\ S\ D^{-1/3} \tag{11}$$

and is valid for $(\lambda S)^{\frac{1}{2}} \ll 2D$ (Lawrence and Strohbehn, 1970). In the Appendix the relationship between C_n^2 and the temperature structure parameter C_T^2 is given, and subsequently an expression is derived for C_T^2 as a function of the scaling temperature T_*, the stability parameter (z/L) and the height above the ground $\tilde{z}$. Substituting (A13) and (A17) into equation (11) yields

$$\sigma_\alpha^{\ 2} = 1.09 \cdot 10^{-12}\ S\ D^{-1/3} (N/T)^2\ \tilde{z}^{-2/3}\ T_*^{\ 2}\ f(z/L) \tag{12}$$

where N is the refractivity of air, T is the temperature in degrees Kelvin, $\tilde{z}$ is the height of the optical path above the ground, T_* is the scaling temperature, and $f(z/L)$ is a function given by equation (A18). Equation (12) represents an expression for the magnitude of T_* only.

2.3 Method

It has been shown in the previous derivations that the mean angle of refraction $\bar{\delta}$ and the variance of the angle-of-arrival $\sigma_\alpha{}^2$ are both functions of $|T_*|$ which can therefore be eliminated in equation (7) using (12). In order to retain the sign of T_* in equation (7), which equals also the sign of the stability parameter, sign (z/L) is incorporated in the final equation for $\bar{\delta}$

$$\bar{\delta} \;=\; 1.22\cdot 10^{-8}\, S\bar{N}/\bar{T} + \text{sign}(z/L)\; 0.479\; D^{1/6}\, \tilde{z}^{-2/3}\, S^{\frac{1}{2}} \sigma_\alpha\, p(z/L) \qquad (13)$$

where the stability function $p(z/L)$ is given as

$$\begin{aligned} p(z/L) &= \phi_h [k^2 f(z/L)]^{-\frac{1}{2}} \\ &= \phi_h{}^{\frac{1}{2}} (\phi_m - z/L)^{1/6} \end{aligned} \qquad (14)$$

In equation (13) the first term once again accounts for adiabatic conditions in the atmosphere, and the second term represents the diabatic correction. The discontinuity of this second term at $z/L = 0$ is caused by the change of the sign of z/L. This will not be very critical, however, as for neutral conditions the value of the whole second term in (13) tends towards zero. The stability function $p(z/L)$ should be evaluated for $\tilde{z}$, using the formulae for ϕ_h and ϕ_m given in the Appendix. The form of the stability function $p(z/L)$ versus z/L is shown in Figure 2.

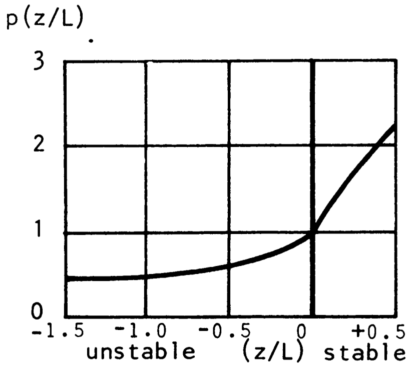

Figure 2. Stability function $p(z/L)$ versus z/L.

The theory developed above expresses the mean angle of refraction $\bar{\delta}$ as a function of the length of the line of sight, average value of refractivity and temperature, the telescope diameter, height above the ground of the line of sight, the standard deviation of the angle-of-arrival fluctuations σ_α, and a stability function $p(z/L)$. If the standard deviation of $\bar{\delta}$ should not exceed ±0.5", then an error analysis of equation (13) shows that the determination of those parameters is

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uncritical with the exception of σ_α and $p(z/L)$. σ_α should be determined with a relative precision of 10 to 20% which is certainly not an impossible task, even for visual observations through the theodolite telescope. Excellent reviews of the experimental determination of σ_α^2 have been given by Lee (1969), Lawrence and Strohbehn (1970), and Tatarskii (1971). For the practical evaluation of $p(z/L)$, the value of the Obukhov length L can be calculated using equation (A3) when wind speed and sensible heat flux are measured or estimated from empirical formula (Webb, 1964; Brunner and Fraser, 1977). Figure 2 indicates that for daylight observations, when z/L is negative, the determination of $p(z/L)$ is not too critical. For night observations, the stability function $p(z/L)$ is not extended in Figure 2 beyond $z/L = +0.5$, because the atmosphere shows generally insufficient thermal fluctuations during strong stability conditions (Okamoto and Webb, 1970), and consequently the proposed method will not be applicable for such conditions.

3. DISCUSSION

A new approach for determining the mean angle of refraction, $\overline{\delta}$, has been developed. For dry air $\overline{\delta}$ is mainly a function of the mean temperature gradient which is generated by the turbulent processes in the atmosphere. The mean temperature gradient can be obtained from a measurement of the variance of the angle-of-arrival, σ_α^2, using the temperature structure parameter C_T^2. The theory is developed for a horizontal line of sight parallel to the ground. Horizontally homogeneous turbulence is assumed for the derivations.

An obvious advantage of this method is that the effects of the turbulent medium on wave propagation which have been found adverse to other techniques are utilised here to advantage. Several pointings through the telescope of a theodolite are usually carried out to obtain a representative value for a vertical angle. During this time period the variance of the angle-of-arrival can be evaluated using the same telescope and along the same line of sight. It is beyond the scope of the present paper to give conclusive recommendations about the measuring technique of σ_α^2. However, without employing additional instruments, σ_α^2 could be inferred from the blurring of a target (Wesely and Derzko, 1975) or from the spread of the image dancing (Kukkamäki, 1950), estimated by visual observations through the telescope. Thus the method will not require new instrumental developments.

In the meteorological solution of the refraction problem point measurements generally are used for some atmospheric parameters. Representative values of these parameters require long averaging times. These averaging times are drastically reduced when path averaged values can be used, such as C_T^2 derived from σ_α^2 measurements (Wyngaard and Clifford, 1978), illustrating a further advantage of the method developed here.

The method presented here has not as yet been tested in field experiments. However, results of two independent experiments may be considered as preliminary verification of the method. Vertical refraction was successfully determined from measured sensible heat fluxes (Brunner, 1978), and sensible heat fluxes were determined from image blurring (Wesely, 1976). Further proof of the second step may be seen in the measurements by Coulman (1966).

The theory which has been intentionally derived for telescope observations, could easily be recast for laser beam propagation, using the appropriate beam equations. For laser beam propagation the variance of the log-amplitude, the phase-angle or the vertical displacement could be utilized for the determination of C_T^2. Appropriate corrections for the humidity effects of light wave propagation, significant for water crossings, can be incorporated in the present theory without great difficulties. Special attention must be given to the weighting functions in the integrals for the refraction angle and the variance of the angle-of-arrival, when the extension of this theory is considered for a more realistic line of sight with general topography. These additional considerations will be treated by the author in the future.

ACKNOWLEDGEMENTS

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APPENDIX: MICROMETEOROLOGICAL BACKGROUND

The surface layer of a horizontally homogeneous atmospheric boundary layer can be effectively described by a few ensemble-average statistical properties. For a comprehensive treatment, reference should be made to Priestley (1959), Lumley and Panofsky (1964), Webb (1964, 1965), Busch (1973), Businger (1973) and Wyngaard (1973).

The turbulent structure of the atmospheric surface layer may be expressed by simple similarity scaling. Neglecting the humidity effects in the atmosphere in the present context, basically three scaling parameters are adopted, defined as the friction velocity u_*, the scaling temperature T_*, and the Obukhov length L:

$$u_* = (\tau/\rho)^{\frac{1}{2}} \tag{A1}$$

$$T_* = - H(\rho c_p u_*)^{-1} \tag{A2}$$

$$L = u_*^2 T(k g T_*)^{-1} \tag{A3}$$

where τ is the shearing stress (downward flux of horizontal momentum), H is the sensible heat flux, ρ is the air density, c_p is the specific heat of air at constant pressure, T is the air temperature, g is the acceleration due to gravity, and k is the von Kármán constant. The Obukhov length L is used to form a dimensionless atmospheric stability parameter z/L, where z is the height above the ground. The atmospheric conditions characterised by negative values of z/L are called unstable. Positive values of z/L indicate stable conditions, and for z/L equal to zero or nearly zero, neutral conditions prevail.

In the atmospheric surface layer (a few tens of metres thick) fluxes of momentum and heat are considered essentially constant with height. Applying scaling to the vertical gradients of mean horizontal windspeed, $\overline{u}$, and mean potential temperature, $\overline{\Theta}$, the following flux-profile relationships are obtained:

$$\frac{\partial \overline{u}}{\partial z} = \frac{u_*}{kz} \phi_m \tag{A4}$$

$$\frac{\partial \overline{\Theta}}{\partial z} = \frac{T_*}{kz} \phi_h \tag{A5}$$

where the profile shape functions ϕ_m and ϕ_h account for the stability effect, and are functions of z/L. The forms of ϕ_m and ϕ_h have recently been reviewed by Dyer (1974), with the following results:

Unstable conditions ($z/L < 0$):

$$\phi_m = (1 - 16\ z/L)^{-\frac{1}{4}} \tag{A6}$$

$$\phi_h = \phi_m{}^2 \tag{A7}$$

Stable conditions ($z/L > 0$):

$$\phi_m = \phi_h = 1 + 5\ z/L \tag{A8}$$

For neutral conditions, where z/L approaches zero, both ϕ_m and ϕ_h tend to go to unity. For the numerical values in the above equations the value of the von Kármán constant has been assumed to be $k = 0.4$.

The determination of z/L, u_* and H from meteorological measurements has been discussed in great detail by Webb (1965) and the evaluation of these parameters in connection with refraction studies has been reported recently by the author (Brunner, 1978; Brunner and Fraser, 1977).

The structure function $D(r)$ is defined as the mean square difference of the values of a variable at distance r apart. If the Kolmogorov law is applicable, then

$$D(r) = C^2\ r^{2/3} \tag{A9}$$

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where C^2 is the structure parameter. If the variable is a scalar, the structure parameter C^2 is given (e.g. Panofsky, 1968) by

$$C^2 = a\, \varepsilon^{-1/3}\, \chi \tag{A10}$$

where a is a constant, ε is the rate of dissipation of turbulent energy, and χ is the rate of dissipation of the fluctuations of the scalar.

The refractive index of air, n, is often conveniently described by the refractivity, N,

$$N = (n - 1)\, 10^6 \tag{A11}$$

For the wavelength $\lambda = 0.56$ µm the refractivity of air is given with sufficient accuracy as (Barrel and Sears, 1939)

$$N = 79\, \frac{P}{T} - 11\, \frac{e}{T} \tag{A12}$$

where P is the total air pressure in mb, T is the temperature in degrees Kelvin, and e is the water vapour pressure in mb. Using (A12) the refractive index structure parameter C_n^2 of dry air can be related to the temperature structure parameter C_T^2 (Bouricius and Clifford, 1970)

$$C_n^2 = 10^{-12}\, (N/T)^2\, C_T^2 \tag{A13}$$

According to Panofsky (1968) the rate of dissipation of turbulent energy ε for dry air can be expressed as the sum of mechanical and thermal production rates for turbulent energy, and ε may then be expressed as (Busch, 1973)

$$\varepsilon = \frac{u_*^3}{kz}\, (\phi_m - z/L) \tag{A14}$$

where all parameters used here have been explained previously. The rate of dissipation of temperature fluctuations χ can be expressed as (Lumley and Panofsky, 1964)

$$\chi = K_h \left(\frac{\partial \overline{\Theta}}{\partial z} \right)^2 \tag{A15}$$

where K_h is the temperature exchange coefficient ($K_h = k\, u_*\, z/\phi_h$), and $(\partial\overline{\Theta}/\partial z)$ is the mean temperature gradient. Substitution of the flux-profile relationship yields

$$\chi = \frac{u_* T_*^2}{kz}\, \phi_h \tag{A16}$$

The numerical value for a still is the subject of conjecture (Panofsky, 1968; Wesely and Alcaraz, 1973; Wyngaard *et al.*, 1971), but the value

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3.2 is used here. Accordingly, substituting (A14) and (A16) into equation (A10) yields for C_T^2

$$C_T^2 = T_*^2 z^{-2/3} f(z/L) \tag{A17}$$

where

$$f(z/L) = 3.2 k^{-2/3} \phi_h (\phi_m - z/L)^{-1/3} \tag{A18}$$

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DISCUSSION

J. Milewski: I think it is a very interesting paper and promising idea, but practically I suppose that we can use this method only in that case if the ratio between the systematic influence, dependent on the atmospheric parameters and the random error of observation is rather large. Such conditions are typical for big turbulence of unstable status of atmosphere. If, however, we had a very stable condition, we have usually a very great value of absolute refraction index, but its variation is then very small. Under such a status of atmosphere the ratio between the variations of refraction and random errors of observation will be rather small, creating unconvenient situations for the use of Brunner's method.

P.V. Angus-Leppan: Dr Brunner is proposing the method for unstable conditions, where there is visible shimmer, and it is easy to estimate the variations of δ with some precision from simple telescope observations. Under stable conditions you may have to use some method other than estimation, because there are large but slow movements of the image.

D.G. Currie: One question I might have on that, is in some work of propagation of laser beams over horizontal paths. There seem to be other parameters that come in to make a significant variation, e.g. the way in which the turbulence varies above the ground over very smooth fields depends quite a bit on the wind. And I think the scaling height, or the height at which you get significant changes in the structure, does depend on the wind velocity, and therefore I suspect that the wind velocity will have a strong influence on the magnitude as well as the frequency of the variation.

P.V. Angus-Leppan: In region II under unstable conditions it is independent of wind. But under other conditions wind certainly is a factor. I think that is taken into account in the parameter T_*.

K. Poder: May I add that the people at the University of Hannover have made some experiments with laser propagation under wind and turbulence.

As far as I am recalling they are published in the proceedings from the Wageningen symposium.

D.G. Currie: Is that true? Thank you.

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