SOLAR AND SOLAR-LIKE OSCILLATIONS: THEORY

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ABSTRACT. Solar five-minute oscillations provide a means of testing theoretical models of the sun. By judiciously combining data from low-degree modes, properties of the central and surface regions of the sun can be inferred separately. In principle, it should be possible to draw similar inferences from other stars, once adequate data are available. Recent solar rotational splitting data imply that in the equatorial regions much of the radiative envelope of the sun is rotating more slowly than the photosphere.

1. INTRODUCTION

Seismological techniques are becoming very important for diagnosing the internal structure of the sun. In this discussion attention is restricted to just the frequencies of oscillation. These contain information about both the spherically symmetrical component of the stratification of the sun and the asymmetric deviations. The latter can be produced by material motion, in the form of rotation, large-scale meridional circulation and convection, and by magnetic fields.

At present the only unambiguous observations are of high-frequency acoustic modes: the so-called five-minute oscillations whose cyclic frequencies $\nu = \omega/2\pi$ lie between about 2 and 4 mHz. These provide information principally about the sound speed $c(\underset{\sim}{r})$, and it is therefore about this quantity that my discussion will be mainly devoted. However, there is an important small but identifiable contribution arising from the angular velocity $\Omega(\underset{\sim}{r})$ of the sun, which has permitted very interesting, albeit somewhat uncertain inferences concerning the solar rotation to be made. Other large-scale motion has not yet been convincingly resolved, though there are hints in the data of giant convective cells. There has also been some discussion of intense magnetic fields in the core of the sun, but that too is yet quite uncertain.

The high-frequency acoustic modes, which are normally called p modes, can be described by asymptotic techniques. Although in the long run delicate comparisons between theory and observation must undoubtedly be carried out using numerically computed eigenfrequencies of theoretical

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solar models, asymptotic descriptions of the oscillations have proved to be extremely useful in several respects for guiding investigations. First, the approximate formula can be used directly to obtain a first approximation to the internal stratification of the sun. In particular, an approximation to Equation (4) is analytically invertible, and has been used to obtain a first estimate of c(r) (Christensen-Dalsgaard *et al.*, 1985). A hybrid procedure that stems from this, whereby the asymptotic formula is used to estimate the hopefully small difference between numerically computed eigenfrequencies and observed frequencies, appears to provide a quick and quite accurate improvement to this approach. Second, the asymptotic formulae provide insight into how the frequencies are determined by the star, and hence what information the frequencies can give us and how it can be extracted.

2. ASYMPTOTIC ACOUSTIC MODES

If magnetic fields, rotation and other forms of large-scale motion are ignored, so that the equilibrium state of the star is spherically symmetric, perhaps the most natural way to analyse the oscillations is to separate the eigenfunctions into products of spherical harmonics and functions of radius r and solve the resulting one-dimensional eigenvalue problem. The most thorough asymptotic analysis using this approach has been carried out by Tassoul (1980), for modes of low degree ℓ and large order n. She found

$$\nu \sim (n + \ell/2 + \varepsilon)\nu_o - A(L^2 - \delta)\nu_o^2/\nu + \dots , \tag{1}$$

where the coefficients ε, ν_o, A and δ are constants of the equilibrium state and $L^2 = \ell(\ell + 1)$. In particular,

$$\nu_o = \left(2 \int_o^R c^{-1} dr \right)^{-1} , \tag{2}$$

and

$$A = \frac{1}{4\pi^2\nu_o} \left[\frac{c(R)}{R} - \int_o^R \frac{1}{r}\frac{dc}{dr}\, dr \right], \tag{3}$$

where R is the radius of the sun.

The most natural way of looking at this result is to think in terms of the independent variable $\tau(r) = \int c^{-1} dr$, which one might call acoustical radius. The characteristic frequency ν_o is thus simply the reciprocal of twice the acoustical radius T of the star, and corresponds to the fundamental frequency of an acoustic resonator of acoustical length T with fixed ends. The quantity A depends principally on conditions near $\tau = 0$, the inner 'boundary' of the resonator. It follows from Tassoul's analysis that both ε and $A\delta$ depend on the density stratification and are closely related, though that is perhaps more readily apparent from the expansion of Equation (4) discussed below: $\varepsilon = n_e/2$, where n_e is an effective polytropic index in the vicinity of $r = r_2$, and $A\delta$ is an integral over the star that is most strongly dependent on the stratification near the surface. It is evident, therefore, that a

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comparison of the quantities ν_0, δ, A and ε extracted from observed and computed frequencies can provide a degree of localization to the information concerning the differences between a theoretical solar model and the real sun.

Unfortunately, acoustical radius τ is not always the most natural independent variable for discussing other aspects of the sun. Studies of internal dynamics are most conveniently carried out with respect to r, and most workers in stellar evolution think in terms of a mass variable. With respect to r, the variation of τ is concentrated near the surface of the sun, as can be seen in Figure 1, because the sound speed near the surface is much less than it is near the centre. With respect to mass, the concentration is generally more severe. Thus the central concentration of the integrand in Equation (3), when expressed as a function of r as it is in Figure 2, is substantially weakened by the sound-speed variation. A histogram using mass fraction as independent variable is presented by Bahcall *et al*. (1982).

An asymptotic analysis of high-frequency p modes when ℓ/n is not necessarily small is outlined by Deubner and Gough (1984). They obtain

$$\frac{\pi(n+\alpha)}{\omega} = \int_{r_1}^{r_2} \left[1 - \frac{\omega_c^2}{\omega^2} + \frac{c^2L^2}{r^2\omega^2}\left(\frac{N^2}{\omega^2} - 1\right)\right]^{\frac{1}{2}} \frac{dr}{c}\,, \tag{4}$$

where

$$\omega_c^2 = \frac{c^2}{4H^2}\left(1 - 2\frac{dH}{dr}\right), \quad N^2 = g\left(\frac{1}{H} - \frac{g}{c^2}\right), \tag{5}$$

g being the local gravitational acceleration and H the density scale height. The limits of integration, r_1 and r_2, are turning points, where the integrand vanishes, and α is a constant that should be determined by matching the oscillatory eigenfunctions in the region of propagation (r_1, r_2) to the evanescent solutions outside; if the equilibrium state of the star varied much more slowly than the eigenfunctions, which is not actually the case near the upper turning point $r = r_2$, then α would take the value $-\frac{1}{2}$.

Expansion of Equation (4) in the limit $n/\ell \to \infty$ yields Tassoul's result, except that ℓ is replaced by $L-\frac{1}{2}$ in the first term of the right-hand side of Equation (1). The expansion also yields a simple expression for the ℓ-independent contribution $\varepsilon + A\delta\nu_0/\nu$ to Equation (1), namely

$$\varepsilon + A\delta\nu_0/\nu \simeq 2\nu \int_0^R (1 - f)c^{-1}dr - \tfrac{1}{2}, \tag{6}$$

where

$$f(r) = \begin{cases} (1 - \omega_c^2/\omega^2)^{\frac{1}{2}} & \text{if } \omega > \omega_c(r) \\ 0 & \text{otherwise}. \end{cases} \tag{7}$$

The integrand $(1 - f)/c$ is plotted in Figure 3.

Other properties of the asymptotic expansions are discussed by Christensen-Dalsgaard (1984a, b; 1986) and Provost (1984).

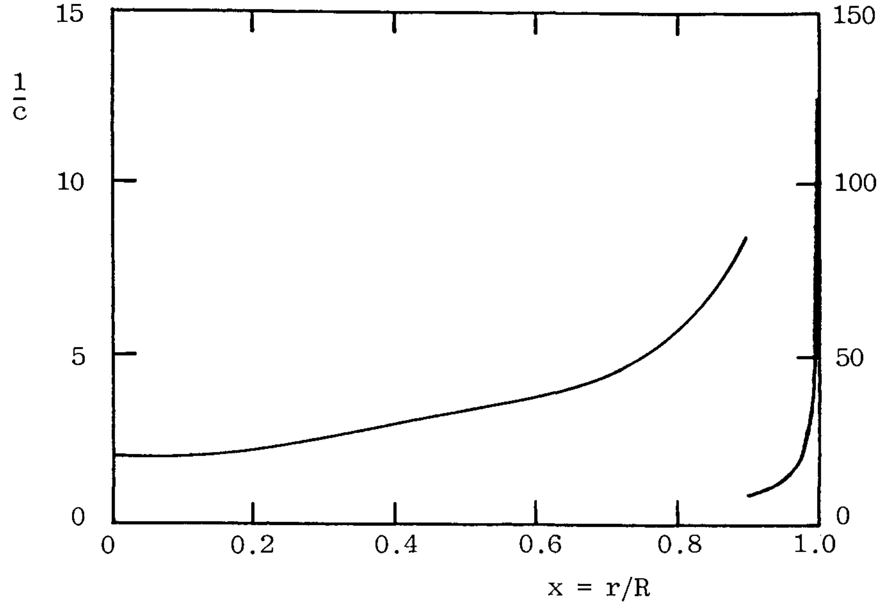

Figure 1. The integrand in Equation (2) computed from Model 1 of Christensen-Dalsgaard (1982). Units of c are Mm s^{-1}. The left ordinate scale refers to x = r/c < 0.9, the right scale to x > 0.9.

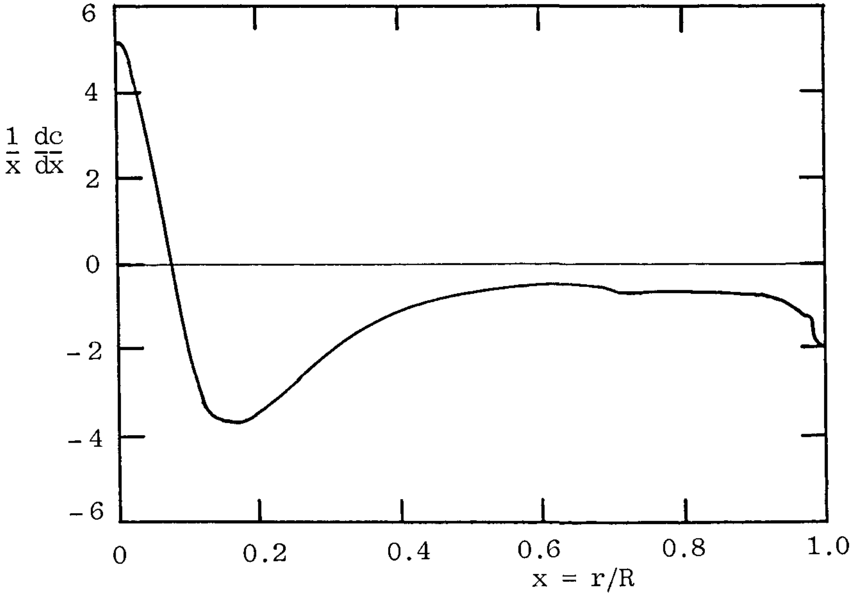

Figure 2. The integrand in Equation (3) computed from Model 1 of Christensen-Dalsgaard (1982). Units of c are Mm s^{-1}.

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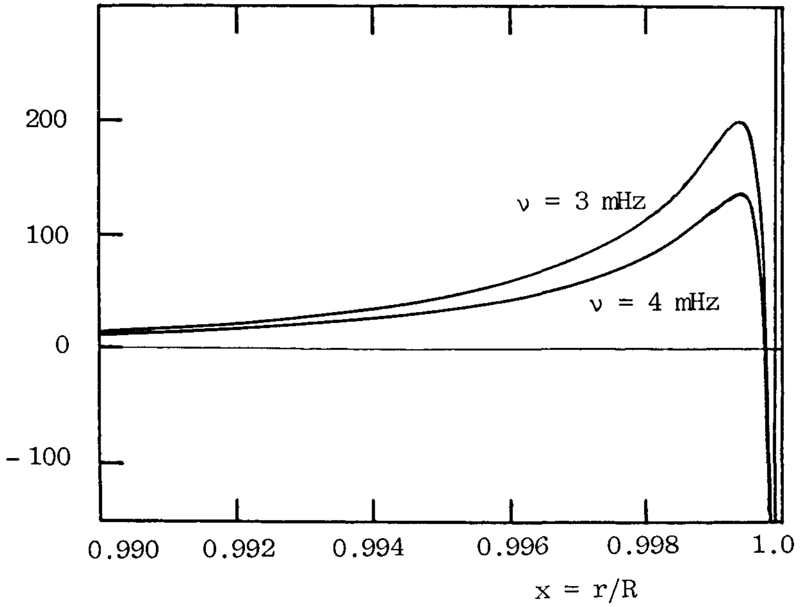

Figure 3. The quantity $2\nu R(1-f)/c$, which determines $\varepsilon + A\delta\nu_o/\nu_s$, for modes with cyclic frequencies 3 and 4 mHz, computed from Model 1 of Christensen-Dalsgaard (1982).

3. PROPERTIES OF SOLAR MODELS

In Table 1 are listed the parameters ν_o, A, ε and δ for the sun and for various theoretical solar models. The parameters were obtained by minimizing

$$E^2 = \Sigma_{n,\ell}[\nu^2_{n,\ell} - (n + \ell/2 + \varepsilon)\nu_o\nu_{n,\ell} + A(L^2 - \delta)\nu_o^2]^2/\Sigma_{n,\ell}\nu^2_{n,\ell} \qquad (8)$$

for all available frequencies $\nu_{n,\ell}$ between 2.0 and 4.0 mHz of modes with $\ell \leqslant 3$. The solar data used were equally weighted averages of the frequencies observed by Claverie *et al.* (1981), Grec, Fossat and Pomerantz (1983), Woodard and Hudson (1983) and Harvey and Duvall (1984).

Several features of the models are immediately noticeable. Those with 'normal' helium abundances ($0.23 \lesssim Y_o \lesssim 0.27$) have high values of ν_o and low values of ε compared with the sun. Thus the sound travel time from centre to surface is too short in the models, as is the effective polytropic index in the surface layers. The discrepancy cannot be reduced by varying the composition, for, as can be seen by comparing CDGMA and CDGMB, decreasing Y_o to bring ν_o in line with the observations is associated with a decrease in ε too. Thus there appears to be an error in the physics in either the radiative envelope or the convection zone, or both. The ℓ-independent contribution $\varepsilon + A\delta\nu_o/\nu$ to expression (1) for the frequency is sensitive principally to the structure of the outer layers of the star (Figure 3) and is consistently too low in the theoretical models, suggesting that the convection zone has been incorrectly modelled (though we do not yet know

Source	Y_o	ν_o(μHz)	A	ε	δ	$\varepsilon + A\delta\nu_o/\nu_s$	E_{min}
Observations		137.9	.247	.554	40	1.011	1.5×10^{-3}
CDGMA	.251	139.5	.323	.076	36	.617	8.6×10^{-4}
JCD1	.247	139.4	.265	.223	43	.749	6.4×10^{-4}
SNG1	.235	139.0	.235	.216	55	.811	8.2×10^{-4}
SNG2	.229	139.2	.194	.204	62	.763	1.3×10^{-3}
UCLA22		139.4	.255	.175	46	.723	1.1×10^{-3}
UR84		139.0	.273	.318	36	.777	9.0×10^{-5}
CDGMB	.187	138.4	.309	-.288	46	.375	4.1×10^{-4}
CDGMC	.160	131.5	.338	-.156	26	.229	4.7×10^{-4}

TABLE 1. All theoretical models are so-called standard solar models, except models CDGMB and CDGMC, which are deficient in helium and heavy elements in their radiative interiors and have normal heavy-element abundances in their convection zones. Y_o is the zero-age helium abundance and ν_s = 3.0 mHz. CDGMA, CDGMB, CDGMC refer to Models A, B, C of Christensen-Dalsgaard, Gough and Morgan (1979), JCD1 to Model 1 of Christensen-Dalsgaard (1982), SNG1 and SNG2 to Models 1 and 2 of Shibahashi, Noels and Gabriel (1983); the entries UCLA22 were computed from the frequencies of UCLA Standard Model 22 reported by Ulrich and Rhodes (1983), and UR84 were computed from the frequencies of that model reported by Ulrich and Rhodes (1984). The value of Y_o of the UCLA model is not given, but the initial hydrogen abundance X_o is quoted to be 0.716 (Ulrich, 1982); if the heavy-element abundance is assumed to be 0.0228 X_o, in agreement with the determination by Ross and Aller (1976), this yields Y_o = 0.267.

whether it is the equilibrium model or the computation of the eigenfrequencies that is predominantly at fault). This can be seen in Table 1, where the contribution is tabulated at the typical frequency $\nu = \nu_s$ = 3.0 mHz of the observed modes.

On the other hand, the structure of the energy-generating core appears to have been modelled more-or-less correctly by the standard models, at least to the extent that core conditions can be measured by the high-frequency p modes. The observed value of A is slightly lower than the mean theoretical value, but it is well within the scatter amongst the theoretical models. This is a particularly interesting result because it sets an important constraint on any attempt to resolve the solar neutrino problem. As is evident from Figure 2, much of the contribution to the integral in Equation (3) comes from almost cancelling components in the inner 20 per cent or so of the radius of the sun (within which about 90 per cent of the energy is generated), the positive contribution near the very centre arising because c(r) is strongly influenced by the variation of the mean molecular weight μ, and increases outwards. A decrease in Y_o, for example, which diminishes the theoretical neutrino flux, leads to a smaller relative change in μ (since the absolute increase in the helium abundance resulting from nuclear transmutations is insensitive to changes in Y_o) and a smaller positive contribution to the integral, thus augmenting A further from

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the observed value. In this case the structure of the outer layers is also moved further from that of the sun; this was first realised from a direct comparison of high-degree eigenfrequencies with solar observations (e.g. Gough, 1983a), and is evident from the variation in $\varepsilon + A\delta\nu_0/\nu_s$ tabulated in Table 1.

It is evident now that extensive smoothing of μ, such as would be produced by the turbulent diffusive mixing of the core with its environment that Schatzmann *et al.* (1981) postulated in an attempt to resolve the solar neutrino problem, is ruled out by the oscillation data. Diffusion of μ decreases the magnitude of the gradient near $r = 0$ and increases it further out. This decreases dc/dr in the innermost regions, where r^{-1} is large; indeed when the central regions are almost homogenized, as is necessary (within this scenario) if the theoretical neutrino flux is to be compatible with observation, c decreases with r throughout the star. The increase of dc/dr in the region beyond the core, where the r^{-1} weighting is smaller, has a lesser effect on the integral in Equation (3). Consequently A is increased. Computations by Ulrich and Rhodes (1983), Berthomieu *et al.* (1984), Cox and Kidman (1984) and Christensen-Dalsgaard (1986) have yielded a mean increase in the value of A by a factor of 1.4. Thus $A \simeq 0.37$, which is substantially greater than the value observed. On the other hand, enhanced energy transport by weakly ineracting massive particles, for example, could lower the central temperature, and the neutrino flux, without destroying the μ gradient (Faulkner *et al.*, 1986, Däppen *et al.*, 1986).

Further discussion of the seismological calibration of solar models can be found in the reviews by Christensen-Dalsgaard (1986), Gough (1983b) and Provost (1984).

4. THE SOLAR CORE

A recent analysis by Henning and Scherrer (1986) of p modes of degrees 2 - 5 with frequencies almost as low as 1 mHz raises an interesting question about the structure of the central regions of the sun. In Figure 4 are plotted the differences between the frequencies observed and those of a standard solar model. Points corresponding to modes of like degree are connected by straight lines. A startling feature of the plot is the behaviour of the lowest-frequency modes with $\ell = 5$. Whereas all the other modes differ from the theory by an ℓ-independent amount, the frequencies of these modes are anomalously high.

It is important to realise that the low-frequency modes have low amplitudes, and are difficult to detect. Therefore one should bear in mind that their frequencies may not have been well determined. Indeed, the lowest-frequency $\ell = 5$ modes reported by Duvall and Harvey (1984) and Libbrecht and Zirin (1986) show no significant sign of departing from the lower-degree modes (see Figure 4). However, these observations hardly extend beyond the point of departure of the $\ell = 5$ sequence, so one is left to conjecture the implications of accepting that Henning and Scherrer are correct.

A partial appreciation of the result can be obtained by considering the asymptotic formula (4), which even at such low frequencies should at

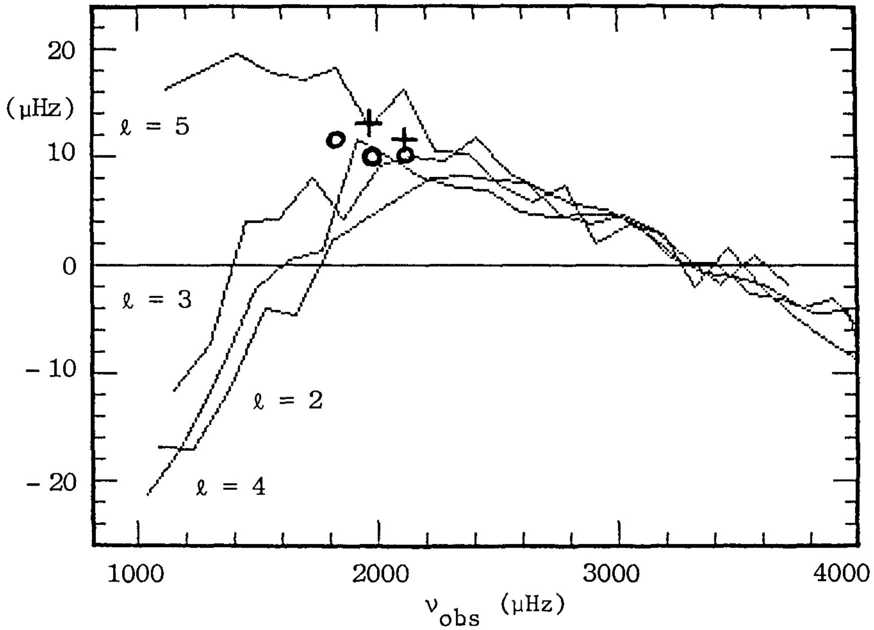

Figure 4. Differences between solar oscillation frequencies measured by Henning and Scherrer (1986) and the theoretical adiabatic eigenfrequencies of Christensen-Dalsgaard's (1982) solar model 1, plotted against the observed frequencies. The crosses and circles denote the lowest-frequency ℓ = 5 modes reported by Harvey and Duvall (1984) and Libbrecht (1986) respectively.

least be a guide to the behaviour of the oscillations. Except near the surface of the sun, ω is rather greater than both ω_c and N. Moreover, c/r is a monotonic decreasing function of r. Consequently the contribution to the ℓ dependence of ω comes predominantly from near $r = r_1$, which is roughly where $c/r = \omega/L$, a property that has been used of modes of intermediate degree to infer an error in a theoretical solar model in the outer layers of the radiative envelope (Christensen-Dalsgaard and Gough, 1984). It is interesting to note that in Figure 4 the deviant ℓ = 5 modes, with ν < 2 mHz, have r_1 > 0.27R and do not penetrate the energy-generating core, whereas those with ν > 2 mHz do. At first sight one might therefore wonder whether there is a sudden departure of the structure of the solar model from that of the sun near r = 0.3R.

Such an hypothesis is not obviously consistent with the whole of Figure 4, however, because there are other modes that do not penetrate beneath r = 0.3R, namely the ℓ = 4 modes with ν < 1.6 mHz and the ℓ = 3 modes with ν < 1.3 mHz. One should bear in mind, however, that the decay of the amplitude of the oscillations in the evanescent region $r < r_1$ is less rapid for the lower-degree modes, and consequently they might sense the structure of the core even though their turning points are outside. It will require more careful caclulations to decide.

5. INTERIOR ROTATION

Rotational splitting has been measured by Duvall and Harvey (1984), Brown (1985, 1986) and Libbrecht (1986). The principal result is that, after a possible slight rise immediately beneath the photosphere, the angular velocity Ω gradually declines with depth near the equatorial plane, at least down to $r \simeq 0.4R$ (Duvall *et al.* 1984). It is also evident from Brown's and Libbrecht's measurements that the latitudinal variation of Ω declines too, though an analysis of its dependence on depth has not yet been published.

At radii less than 0.4R a clear picture has not yet emerged. Both Duvall and Harvey (1984) and Brown (1985, 1986) find a peak in the linear splitting at $\ell = 11$, suggesting a low-latitude region of rapid rotation near $r = 0.4R$ and a region of slow rotation near $r = 0.25R$, just exterior to the energy generating core (Duvall *et al.* 1984). However, this feature is not seen in Libbrecht's (1986) data. There is, moreover, evidence of a region of strong latitudinal variation of Ω near $r = 0.4R$ in both Brown's and Libbrecht's data which reverses at the edge of the core, suggesting the presence of a central vortex. Furthermore, Duvall and Harvey, who are the sole observers to have splitting data for the lowest-degree modes which penetrate the core, report high rotational splitting for modes with $\ell = 1$ and 2, indicating rapid rotation of the central regions.

In view of the discrepancies between the different observations of particularly the low-degree modes, one should treat the evidence for rapid variation of Ω in and immediately outside the core with some caution. But the slow rotation of the outer part of the radiative envelope is found by all observers, and therefore needs explaining (cf. Gough, 1985; Rosner and Weiss, 1985). So far as the innermost regions are concerned, however, if the rapid variations suggested by the observations are real, they must be unsteady, because any steady angular velocity distribution that one would infer by taking the data at face value would be unstable. Since the different observations were taken at different times, the differences between them should perhaps not be surprising.

6. SOLAR-LIKE STELLAR OSCILLATIONS

Seismological diagnosis of the kind discussed here is likely to become a powerful tool for measuring the properties of other stars. Multiple periods have been measured in Ap stars (e.g. Kurtz, 1986), and evidence has been found for solar-like oscillations in α Centauri A (Fossat *et al.* 1984), ε Eridani (Noyes *et al.* 1984) and Procyon (Fossat *et al.* 1986). This success will no doubt encourage further work that seems bound to make asteroseismology an exciting reality. Only low-degree modes can be measured, but these contain a wealth of information.

The first basic quantities that can be measured are the asymptotic parameters ν_0, ε, A and δ. Christensen-Dalsgaard and Frandsen (1983) and Christensen-Dalsgaard (1984b) have predicted how ν_0 and A should vary on the main sequence, which provides an important standard against

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which observers can compare their results. Some caution must be exercised for stars that differ substantially from the sun, however, because phenomena not encountered in the sun might make significant differences to the oscillation parameters. For example, the intense large-scale magnetic fields in rapidly oscillating Ap stars could cause substantial deviations from spherical symmetry in the outer layers of the star that might have as great an influence on the parameter A as does the structure of the core. This adds to the riches that await discovery.

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