A Giant Step: from Milli- to Micro-arcsecond Astrometry
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Asteroid mass determination with the Gaia mission

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Abstract. The ESA astrometric mission Gaia, due for launch in late 2011, will observe a very large number of asteroids ($\sim$ 350,000 down to the magnitude 20), most from the main belt, with an unprecedented positional precision (at the sub-milliarcsecond level). Such high-precision astrometry will enable to considerably improve the orbits of a large number of objects, and also to determine the masses of the largest asteroids by analyzing their gravitational pull during close encounters with smaller ones. A global solution involving simultaneously all the perturbers and the smaller targets should yield about hundred masses with a precision better than 30 percent. The knowledge of these masses will be a rich source of information on the physics of main-belt asteroids and will increase the accuracy of modern solar system ephemerides. We outline the principle of the mathematical method based on variational equations developed to solve for the orbital parameters and the masses of the largest bodies. Calculations have been performed by taking into account realistic simulation of the Gaia observations such as geometry, time sequence, magnitude and by considering possible close approaches among 350,000 asteroids. We give a list of asteroids whose mass can be estimated along with its formal precision.

Keywords. astrometry, asteroids, solar system: general

1. Introduction

Gaia is an astrometric cornerstone mission of the European Space Agency. With a launch due in late 2011, Gaia will have much more ambitious goals than its precursor Hipparcos: to obtain a “3D census” of our galaxy with astrometric, photometric, and spectroscopic observations. It will pinpoint the objects with an unprecedented positional precision (at a sub-milliarcsecond level for a single observation) allowing to observe about 350,000 asteroids (mainly main-belt asteroids) down to magnitude 20. The high precision astrometry will enable to considerably improve the orbits of almost all observed asteroids, yielding the masses of the largest ones from mutual perturbations during close approaches with smaller asteroids. The objective of this work is determination of the masses of a small subset of minor planets from the perturbations on smaller objects during close encounters. Not only does the method consider multiple close encounters with different targets, but it also simultaneously includes all perturbing asteroids (*perturbers*) together with their target asteroids. Here, we briefly present the different steps of this method, and give a list of asteroids for which the mass can be derived. The expected precision of masses by the time of mission’s completion is estimated using realistic simulations of the Gaia observations (geometry, time sequence, magnitude).

The analysis of the gravitational perturbations during a close encounter between two asteroids, has been the most productive method to derive the masses. However, its application to ground-based observations has been limited in most cases to rare, particularly favorable encounters between a perturber and a small and faint target. This can be explained in part by the accuracy of observations allowing to analyse solely the very significant gravitational signature. As a consequence, only 21 asteroids have their mass estimates with a precision as good as 10% and the other 41 – at a 50% mark. Moreover, these estimates are often corrupted by systematic errors, which do not appear in the post-fit standard deviations. Thus, our knowledge of asteroid masses is still very poor, while their masses are of great interest for planetary ephemerides and the physics of asteroids.

Currently, the factor limiting the precision of the ephemerides for Mars is the uncertainty on the masses of the largest asteroids (Standish *et al.* 2002). The accuracy of their derivation will play a major role in the precision of ephemerides for the other inner planets when space missions will provide more precise observations of these asteroids. Regarding the physics of asteroids, the knowledge of mass and size (the latter being given by IRAS and many other techniques such as occultations, high angular resolution, but also from Gaia) of an asteroid leads to their bulk density, or equivalently to the amount of matter making up the body and the space occupied by its pores and fractures. The comparison with the grain density of meteorites with analogous composition, allows to estimate the porosity, a parameter related to the collisional history of asteroids and to their internal structure (Britt *et al.* 2002). In addition, a measurement of the bulk density will shed light on the relationship, if any, between the spectroscopic taxonomic class and the density, or possibly the porosity. Ultimately, this is related to the origin of asteroids and the formation process of the Solar System (Zappalà *et al.* 2002). If a well-defined relationship can be found, this could be used to derive density estimate for objects with a known taxonomic class. Eventually a reassessment of this relationship, used in planetary theories to estimate the masses of asteroids from their volume, will benefit to the accuracy of the ephemeris for the inner planets.

2. The mass determination method

The observed minus calculated positions (**O-C**) for each observation of minor planets expressed in Gaia longitude λ projected over a given Great Circle, can be linearized and solved by the least-squares method †,

$$\mathbf{O}-\mathbf{C}=\mathbf{A}\begin{pmatrix}\delta\mathbf{u_0}\\ \delta\mathbf{m_p}\end{pmatrix}\Rightarrow\begin{pmatrix}\delta\mathbf{u_0}\\ \delta\mathbf{m_p}\end{pmatrix}=(\mathbf{A}^t\mathbf{A})^{-1}\mathbf{A}^t(\mathbf{O}-\mathbf{C}) \tag{2.1}$$

where, $\delta\mathbf{u^0}=(\delta\mathbf{u^0_1},..,\delta\mathbf{u^0_n})^t$ are the corrections to the initial conditions of the n asteroids with $\delta\mathbf{u^0_k}=(\delta x^0,\delta y^0,\delta z^0,\delta\dot{x}^0,\delta\dot{y}^0,\delta\dot{z}^0)$ standing for the corrections to the position and velocity of the asteroid k at the reference time, and $\delta\mathbf{m_p}=(\delta m_1,..,\delta m_p)^t$ are the corrections to the masses of the p perturbers.

The matrix **A** in Eq.(2.1) represents the partial derivatives of the longitudes λ of the minor planets with respect to their initial parameters,

$$[\mathbf{A}]_{i,j}=\sum_{q=1}^{3}\frac{\partial\lambda_i}{\partial x_q}\frac{\partial x_q}{\partial C^0_j}\quad\text{where}\quad\begin{cases}\mathbf{x}_{q=1,..,3}=(x,y,z)\\ \mathbf{C}=(\mathbf{u}^0_1,..,\mathbf{u}^0_n,m_1,..,m_p)\\ \mathbf{u^0_k}=(x^0,y^0,z^0,\dot{x}^0,\dot{y}^0,\dot{z}^0)\text{ for the }k^{th}\text{ asteroid}\\ \lambda\ \text{ the observed longitudes for all asteroids.}\end{cases} \tag{2.2}$$

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† Weighting of the equations has been omitted for the sake of brevity.

The expression $\partial\lambda_i/\partial x_q$ is determined analytically, while the $\partial x_q/\partial C_j$ are evaluated numerically. This is done by expressing the variations of the rectangular heliocentric coordinates $x_{q=1,..,3}$ (in an inertial frame) of each asteroid with respect to the unknowns by integrating the variational equations simultaneously with the equations of motion (Herget 1968).

From Eq.(2.1), it is possible to obtain the precision $\sigma(\delta\mathbf{m_p})$ attainable for the masses as a function of the observational accuracy. Each line of the matrix $\mathbf{A}$ corresponding to an asteroid k for an observation i is weighed according to the error on the position $\sigma_{k,i}$ (depending on the magnitude) by a constant σ_0, and so, the measurements of the weighted **(O-C)** have the same variance σ_0^2. In addition, we make the assumption that the measurements of positions are independent, and, consequently, $\mathrm{cov}(\mathbf{O}-\mathbf{C}) = \sigma_0^2 Id$ and we obtain,

$$\mathrm{cov}\left(\delta\mathbf{u^0}, \delta\mathbf{m_p}\right)^t = \sigma_0^2(\mathbf{A}^t\mathbf{A})^{-1} \tag{2.3}$$

More details are given in Mouret *et al.* (2007).

3. The selection of the target asteroids

An important step prior to this solution is the indentification of suitable targets significantly perturbed during close approaches by larger asteroids. In the simulation for Gaia, the target asteroids are selected among the first 350,000 numbered asteroids from a systematic search of all close approaches with a set of bodies over a prescribed duration. The perturbers are chosen among the first 20,000 numbered asteroids as the bodies having a mass larger than a given threshold (here set at $10^{-13}M_\odot$). A close approach is considered meaningful if the impact parameter b (the minimum distance between the two asteroid trajectories in the case where we do not take into account their mutual perturbations) is smaller than 0.5 AU and the deflection angle θ greater than 1 *mas* which is estimated from the impulse approach by:

$$\tan\frac{\theta}{2} = \frac{G(m+M)}{v^2 b}, \tag{3.1}$$

where, G is the gravitational constant, M is the mass of the perturber, $m$is the mass of the target asteroid, and v is the relative velocity of the encounter.

For the period of time covering the Gaia mission—from year 2010.5 to 2016—the overall statistics are given in Table 1, where the second column indicates the number of bodies which are in the perturber list, but appears occasionally as target of a larger body.

Table 1. Results of the close approach simulations.

Number of			
perturbers	602	perturbers and targets	
target asteroids	43513	simultaneously	434

4. Results

We have produced realistic simulations of the Gaia observations (geometry, time sequence, magnitude) and the set of close approaches (Table 1) to evaluate the final precision achievable on the masses of the perturbers (Eq. 2.3). The main result is shown in

Table 2 (*left*) with the statistical distribution of the relative precision on the masses. The right part of the Table 2 lists individual objects for which we obtained the best relative precision. The number of target asteroids for each solved mass is also listed together with the formal precision $\sigma(m)$, the reference mass m of the perturber, and the relative precision $\sigma(m)/m$.

Table 2. Number of masses determined in each class of the relative precision (*left*). The formal precision on the perturbers masses (*right*). The given masses m are the ones used in our simulation. The new masses for which no direct measurement is known are marked by $\star\star$.

Number of perturbers	
Total	602
σ(m)/m < 0.1%	2
σ(m)/m < 1%	3
σ(m)/m < 10%	36
σ(m)/m < 15%	59
σ(m)/m < 20%	75
σ(m)/m < 30%	106
σ(m)/m < 40%	135
σ(m)/m < 50%	149

n^o	Asteroid name	number of target asteroids	$\sigma(m)$ [$M_\odot$]	mass m [$M_\odot$]	$\sigma(m)/m$ [%]
1	Ceres	6007	3.19×10^{-13}	4.75×10^{-10}	0.067
4	Vesta	6685	1.23×10^{-13}	1.36×10^{-10}	0.091
10	Hygiea	3180	3.42×10^{-13}	4.54×10^{-11}	0.753
14	Irene**	1973	1.53×10^{-13}	1.41×10^{-11}	1.090
16	Psyche	4699	3.91×10^{-13}	3.38×10^{-11}	1.160
27	Euterpe**	1053	6.30×10^{-14}	5.38×10^{-12}	1.170
7	Iris	1821	1.80×10^{-13}	1.41×10^{-11}	1.270
2	Pallas	1194	1.35×10^{-12}	1.00×10^{-10}	1.350
9	Metis	3046	2.10×10^{-13}	1.45×10^{-11}	1.450
15	Eunomia	1859	2.82×10^{-13}	1.64×10^{-11}	1.720

5. Conclusions

The results of our simulations are very encouraging. Analyzing the close approaches between 602 perturbers and 43,513 target asteroids between 2010.5 and 2016, we find 36 asteroids for which the mass can be estimated to better than 10% and this number rises to 149 for a 50% precision. Gaia will improve most of current the measurements of masses thanks to a very complete dynamical modelling which limit the effects of systematic errors. Gaia will also yield several new masses, among which we expect a very precise mass estimate for (14) Irene and (27) Euterpe. As shown in Mouret (2007), the well-planned ground-based observations before and after the Gaia mission should improve these results. In the end, the number of well-determined masses after the mission's completion could be larger than the above estimate.

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