2. R. Michel, Sur quelques problèmes de géométrie globale des géodésiques, *Bol. Soc. Bras. Mat.* **9** (1978), 19–38.

T. N. BAILEY

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Magurn, B. A. *An algebraic introduction to K-theory* (Cambridge University Press, 2002), 0 521 80078 1 (hardback), £80.

Algebraic and topological K-theories originate in certain generalizations of the category, Vect, of vector spaces over a field. It is well known that any finite-dimensional vector space V admits a basis, and any two bases are equivalent. Sending the isomorphism class $[V]$ of V into $\dim V$ establishes a one-to-one correspondence between isomorphism classes, Iso(Vect), of objects in Vect and the set $\mathbb{N}$ of non-negative integers. As $\dim(V \oplus W) = \dim V + \dim W$, this is in fact a one-to-one correspondence of Abelian monoids which can be made via an obvious enlargement of both sides (i.e. adding formal differences $[V] - [W]$ to Iso(Vect) and negative integers to $\mathbb{N}$) into an equivalence of Abelian groups, $K_0(\mathrm{Vect}) \simeq \mathbb{Z}$.

If one generalizes Vect to the category of finitely generated projective modules, Mod_R, over a ring R, then bases no longer exist (and even if they exist they may not be equivalent) so that dimension disappears. However, an analogue, $K_0(\mathrm{Mod}_R)$, of the group $K_0(\mathrm{Vect})$ survives! It is no longer isomorphic to $\mathbb{Z}$, in general, and measures the obstruction to existence of bases; another group, $K_1(\mathrm{Mod}_R)$, describes their non-uniqueness. These are the first two floors of the tower of groups K_n which are the main subject of study in algebraic K-theory.

If one generalizes Vect to the category of vector bundles over compact topological spaces, then one arrives at topological K-theory.

The book under review is an excellent introduction to the algebraic K-theory. It gives a nice overview of several deep problems solved by means of algebraic K-theory (such as the normal basis problem in number fields, the classification of normal subgroups of linear groups) and, rather surprisingly, assumes no prerequisite beyond standard undergraduate algebra. The book is very self-contained and can be recommended to graduate students.

Here is a list of contents:

Part III. Groups of matrices: K_1

Chapter 9. Definition of K_1 (elementary matrices; commutators and $K_1(R)$; determinants; the Bass K_1 of a category)

Chapter 10. Stability for $K_1(R)$ (surjective stability; injective stability)

Chapter 11. Relative K_1 (congruence subgroups of $GL_n(R)$; congruence subgroups of $SL_n(R)$; Mennicke symbols)

Part IV. Relations among matrices: K_2

Chapter 12. $K_2(R)$ and Steinberg Symbols (elementary matrices; commutators and $K_1(R)$; determinants; the Bass K_1 of a category)

Chapter 13. Exact sequences (the relative sequence; excision and the Mayer-Vietoris sequence; the localization sequence)

Chapter 14. Universal algebras (presentations of algebras; graded rings; the tensor algebra; symmetric and exterior algebras; the Milnor ring; tame symbols; norms on Milnor K-theory; Matsumoto's theorem)

Part V. Sources of K_2

Chapter 15. Symbols in arithmetic (Hilbert symbols; metric completion of fields; the p-adic numbers and quadratic reciprocity; local fields and norm residue symbols)

Chapter 16. Brauer groups (the Brauer group of a field; splitting fields; twisted group rings; the K_2 connection)

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Maclachlan, C. and Reid, A. W. *The arithmetic of hyperbolic* 3*-manifolds* (Springer, 2003), 0 387 98386 4 (hardback), £45.50.

The study of Kleinian groups and hyperbolic 3-manifolds involves the interplay of many different mathematical ideas. Much has been written on the subject from the points of view of complex analysis, topology, geometry and group theory. In this book Maclachlan and Reid give a comprehensive treatment of hyperbolic 3-manifolds and Kleinian groups from the viewpoint of algebraic number theory. Both authors have very successfully exploited arithmetic techniques in Kleinian groups and it is extremely useful to have a definitive account of the techniques and ideas they use and have developed.

As is well known, a hyperbolic 3-manifold M may be written as the quotient of hyperbolic 3-space $\boldsymbol{H}^3$ by a discrete, torsion-free subgroup Γ of $\mathrm{PSL}(2,\mathbb{C})$. If we drop the hypothesis that Γ is torsion-free, then the quotient $\boldsymbol{H}^3/\Gamma$ becomes an orbifold. In both cases the group Γ is said to be *Kleinian*. The hyperbolic manifolds and orbifolds discussed in this book are generally of finite volume (though they may have cusps). In this case the associated Kleinian group is said to have finite covolume. It is completely standard to switch between thinking of Γ as a group of Möbius transformations in $\mathrm{PSL}(2,\mathbb{C})$ and a group of matrices in $\mathrm{SL}(2,\mathbb{C})$.

If we are given Γ, a possibly torsion-free subgroup of $\mathrm{PSL}(2,\mathbb{C})$, then it is not an easy matter to decide whether or not it is discrete. Roughly speaking, there are three main techniques for showing the discreteness of Γ. Firstly, following Klein and Maskit, one may be able to show that Γ can be assembled from smaller groups by the operations of free product (possibly with amalgamation) and HNN extension. Secondly, we may use the geometry of Γ acting on $\boldsymbol{H}^3$