NEW OBSERVATIONAL TECHNIQUES AND PRECISE ORBIT DETERMINATION OF ARTIFICIAL SATELLITES

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Abstract. Modern observational techniques using ground-based and space-based instrumentation have enabled the measurement of the distance between the instrument and satellite to better than one centimeter. Such high precision instrumentation has fostered applications with centimeter-level requirements for satellite position knowledge. The determination of the satellite position to such accuracy requires a comparable modeling of the forces experienced by the satellite, especially when classical orbit determination methods are used. Geodetic satellites, such as Lageos, in conjunction with high precision ground-based laser ranging, have been used to improve for modeling of forces experienced by the satellite. Space-based techniques, such as Global Positioning System (GPS), offer alternatives, including kinematic techniques which require no modeling of the satellite forces, or only rudimentary models. This paper will describe the various techniques and illustrate the accuracies achieved with current satellites, such as TOPEX/POSEIDON, GPS/MET and the expectations for some future satellites.

1. Introduction

The motion of an artificial Earth satellite can be described by the differential equations of motion, based on Newton's Second Law, which describe the forces experienced by the satellite. The forces can be separated into gravitational and nongravitational categories defined by:

- Gravitational
  - Point mass two-body term

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- Static gravitational field (usually described with spherical harmonics)
- Temporal gravity field (tides)
- Gravitational fields of the Sun, Moon and planets
- General relativity

• Nongravitational

- Atmospheric resistance
- Solar radiation pressure
- Earth radiation pressure
- Satellite thermal radiation
- Other (charged particle drag, etc.)

These equations of motion are usually expressed in the J2000 system; thus, appropriate coordinate transformations are required for the forces. For example, the static gravitational field is commonly described in an Earth-fixed system, so the transformation of this force into J2000 is dependent on precession, nutation, polar motion, and UT1. A summary of the models for the gravitational and nongravitational forces, as well as the models for Earth orientation, are given by McCarthy (1996) for geodetic satellites with high accuracy orbit requirements.

With specified initial conditions for position and velocity at a specified epoch, numerical techniques may be invoked to solve the differential equations of motion. The resulting solution, or satellite ephemeris, is used for the following:

• Estimation (data fitting): The ephemeris will be used to form a computed "observation" (C), such as a range, range-rate or angle observations, and compared with the actual observation (O). The measurement residual, O-C, can be used in an appropriate estimation procedure to determine the initial conditions (orbit determination problem) as well as force and kinematic parameters that appear in the equations of motion or in the measurement model. In this application, there is a corresponding "data fit interval" or "arc length", which is the time interval during which observations are available. While this term is well defined in a batch estimation process, it may be less relevant in a sequential estimator, depending on the weights given to past observations.

• Prediction: The ephemeris may be used as a state-predictor. In this mode, the future (or past) satellite state is required outside the data-fit interval used to determine the initial conditions. Such predictions may be used for developing future observation scheduling or to study potential satellite collisions, for example.

The estimation problem and some aspects of the prediction problem (e.g., propagation of the error covariance matrix) require the state transition matrix (STM) to enable mapping state "corrections" at one time to another time. The STM is the solution to another set of differential equations, which are usually solved simultaneously with the equations of motion. For the basic orbit determination problem in which epoch position and velocity are estimated, the STM is a 6×6 matrix represented by 36 first order, ordinary differential equations. The inclusion of other estimated parameters will increase the dimensions of the STM, but with careful evaluation of the differential equations, the number of equations is usually less than n^2, where n represents the number of estimated parameters.

A variety of general purpose techniques for the solution of ordinary differential equations have been developed, as well as special purpose techniques for the solution of problems in orbital mechanics. These techniques are described and summarized by Shampine and Gordon (1975), Lundberg (1985) and Montenbruck (1992), for example. The selection of a numerical technique is based on several considerations, including the required accuracy (usually commensurate with observational accuracy requirements), the integration span, the available computer resources and the "turn-around time" (is the solution required in real-time?). Numerical methods exhibit well-known errors: truncation error and round-off error. However, with properly chosen techniques, these error sources do not dominate. In fact, much of the numerical integration error can be absorbed into estimated parameters in the observation-fitting process, especially drag-like forces (Schutz and Tapley, 1980).

The numerical solution of the equations of motion may proceed with the direct solution of the equations of motion and STM, referred to as Cowell's method. Alternatively, a reference solution may be adopted and the differential equations reformulated to represent displacements from the reference (Encke's method). The latter technique has been shown to effectively extend the numerical precision associated with the available computer (Lundberg, *et al.*, 1991).

2. Observational and Orbit Errors

The errors associated with observational techniques have been steadily declining in the last two decades. At the present time, the global network of ground-based satellite laser ranging (SLR) stations can measure the distance from the ranging observatory to the satellite with a precision of better than one centimeter. Coupled with the instrument bias at a comparable level, this one centimeter accuracy has enable the extraction of a wide range of small, but important, forces to be observed.

The analysis of satellite laser range (SLR) measurements to the Lageos satellite provides an example of high accuracy ephemerides for both data fitting and prediction. The Lageos satellite is spherical with a 30 cm diameter and covered with 426 corner cubes. The mass of the satellite is 407 kg and the resulting low area to mass ratio reduces the effect of nongravitational forces. The satellite was launched in May, 1976, and has been regularly tracked by a network of SLR stations since that time. The altitude is approximately 5900 km and the inclination is 110 degrees. Long arc analyses have been used for the analysis of the SLR data; however, these analyses also tests the fidelity of the physical models and the quality of numerical integration. In these analyses, the estimation interval spans the period from the 1976 Lageos launch to, for example, 1996 or an interval of about 20 years. The estimation parameters consist of the epoch (near launch) position and velocity and sub-arc drag-like force terms, as well as a solar radiation pressure parameter. In some cases, other force model parameters may be included for estimation, such as GM or gravity coefficients. Considerable force model improvement has occurred during this time with the simultaneous improvement achieved with SLR observations. If the 20-year record of SLR data are fit by estimating position and velocity in May 1976, as well as radiation pressure and along-track forces, the RMS of the SLR residuals is 2 meters (R. Eanes, personal communication, 1996). Since the radiation pressure and along-track forces change with time, the variation has been accommodated by estimating new values every 15 days over the 20-year span. Nevertheless, when this 20-year result is compared with the 5 meter RMS obtained in 1980 using just a 4-year span of data, the dramatic force model improvements attained during the last 20-years are clearly evident. It should be noted that the 2 meter RMS in 20-years can be further reduced through additional sub-arc analysis to the noise level of the observations, as described by Tapley *et al.* (1993).

The TOPEX/POSEIDON (T/P) oceanographic satellite is an excellent example of modern observational systems required for high accuracy orbit determination. Prior to the launch of the satellite in 1992, it was required that the radial component of the orbit be known with an accuracy of 13 cm. However, this accuracy has been significantly exceeded by the achieved level of about 3 cm, thereby opening new vistas for oceanographic applications of the altimeter measurements conducted on-board T/P. The satellite carried SLR and the doppler system, DORIS, as the primary systems for providing precise orbit determination at the 1335 km altitude. However, recognizing the future importance of the Global Positioning System (GPS), a GPS receiver was carried as a demonstration experiment. The T/P orbit has been determined independently by SLR, DORIS and GPS and the radial component of the resulting ephemeris agrees to 3 cm or better as described by Schutz *et al.* (1994), Yunck *et al.* (1994) and Melbourne *et al.* (1994).

A wide range of orbit accuracies can be achieved with GPS. With a GPS receiver carried on a low altitude satellite, single frequency pseudorange measurements will provide position determinations of 100 meters in the presence of "Selective Availability" or SA. The dominant effect of SA is a dithering the GPS frequency oscillators so there is an intentional error in the satellite clock. Without knowledge of the characteristics of this error, a receiver capable of making a minimum of four pseudorange measurements to the GPS satellites will be limited to a positioning accuracy of 100 meters. However, for many applications, 100 meters is quite adequate and this requirement can be met quite inexpensively. On the other hand, if the orbit ephemeris must be known with an accuracy of a few centimeters or a few meters, this requirement can be met using a receiver carried on the low altitude satellite, but additional processing of the measurements will be required.

Moderate to high accuracy (few meters to a few centimeters) can be achieved after removal of the SA effects. This removal is accomplished by differencing measurements, known in a broad sense as "differential GPS." For example, given measurements collected by a GPS receiver carried on a low Earth orbiting satellite (LEO), these measurements can be differenced with those obtained by a ground-based GPS receiver located at a site with well-determined coordinates. A double difference measurement can be formed using, for example, pseudorange measurements: 1) difference the pseudoranges obtained by the LEO and the ground receiver for a specified GPS satellite, which removes the effects of SA, known as a single difference, 2) form another single difference using another GPS satellite, then 3) form a double difference (DD) by differencing the two single differences. The resulting DD has removed the effects of SA as well as the clock errors form the two receivers. Using the DD, a new observable can be formed. Although this example used pseudoranges, which have a precision of about 0.5 meters in the best receivers, the carrier phase measurement of the GPS signal can be measured with a precision of a few millimeters. The DD formation can be formed for the carrier phase as well, but the result has an unknown cycle ambiguity that must be accommodated in the analysis. The previously described T/P results using GPS were obtained using carrier phase. The differencing method is described by Hoffmann-Wellenhof (1993). In addition to the use of classical dynamic orbit determination which requires the solution of the differential equations of motion, the measurements can be used in a "kinematic" approach in which the position of the LEO receiver is solved from an appropriate set of difference equations at a specific time. In either way, or combination of ways (e.g., "reduced dynamic"), the ephemeris of the LEO can be determined. Both the dynamic and kinematic approaches have advantages and disadvantages, which will not be elaborated here.

Applications of GPS require knowledge of the GPS ephemerides. The broadcast ephemerides suffice for applications which do not remove SA; however, highest accuracy require improved ephemerides. The International GPS Service for Geodynamics (IGS) has organized a network with approximately 100 globally distributed GPS receivers. These receivers make GPS measurements on both GPS frequencies (1227.6 MHz and 1575.4 MHz) to remove the ionosphere effects. The orbits are determined by seven different analysis centers and a combined orbit, based on the results from the analysis centers, is produced. This combined orbit is available within about two weeks, but predicted orbits are also available for real-time applications. The accuracy of one day arcs for each of the 24 GPS satellites is about 10–20 centimeters. The limiting factor for the generation of longer arcs is the nongravitational force modeling. For more information on the IGS, consult the World Wide Web, http://igscb.jpl.nasa.gov.

One other example with GPS, a low altitude satellite was deployed from the Space Shuttle into a 400 km altitude orbit carrying a GPS receiver. This GPS receiver was a TurboRogue, similar to those used by the IGS ground network. One of the purposes of this experiment was to study the low altitude environment, especially drag and the Earth gravity field, to assess the accuracy of a low altitude orbit determination process. The deployed satellite was the Wake Shield and the experiment is described by Schutz *et al.* (1995). The experiment is scheduled for a second flight in November 1996.

3. Conclusions

Using modern high precision measurements, such as satellite laser ranging and GPS pseudorange and carrier phase, the orbits of low altitude satellites have been determined with accuracies ranging from a few centimeters to a few decimeters. In those cases where the force models are not precisely known, such as nongravitational forces, the arc lengths (span in which the observational data are fit) are one day to a few days, but on some satellites (such as Lageos), the arc length is 20 years. These latter cases with long arcs have enabled considerable improvement to force model improvement and understanding of the nature of the forces experienced by a satellite. In the future, GPS is expected to be used for determining the orbit of low altitude satellites with accuracies of a few centimeters to 100 meters. Nevertheless, continued laser ranging is important for the scientific information available from long term data sets obtained with high precision measurements, especially to elucidate the nature of temporal changes in the Earth's gravity field.

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