# NUMERICAL SIMULATIONS OF TURBULENT COMPRESSIBLE FLOWS

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ABSTRACT. We give an overview of the use of numerical simulations in the modeling of turbulence in molecular clouds.

## 1. Introduction

Observations of spectral lines in molecular clouds reveal the existence of supersonic motions whose origin has not been clarified. Scaling laws relating velocity dispersion and cloud size can be attributed to turbulent motions. The physical properties of this turbulence and its feeding mechanisms are still unknown, but independent observations seem to confirm the existence of very irregular and hierarchical structures. Magnetic fields are probably dynamically important in most of these objects. They can help support the cloud against gravity and, through Alfvén wave turbulence, give an alternative explanation for the observed molecular linewidths.

Numerical simulations have become an important tool for studying nonlinear dynamics and are helpful in deciding between competing physical models of molecular clouds. It is generally admitted that nonlinearities are at play in these media and that they are in part the source of our lack of success in analytical modeling. In the absence of an adequate theory of turbulent flows, our understanding of nonlinear complexity necessitates high resolution numerical simulations.

In this paper, we concentrate on homogeneous compressible flows, leaving aside problems which lead to more physically elaborate modeling, including for example radiative transfer.

We begin with a brief account of several basic concepts in turbulence. The next Section of the review is devoted to the case of neutral two and three–dimensional flows without self–gravity, discussing the general properties of supersonic turbulence. In particular we will describe the structures observed in physical space, the temporal evolution of large scale variables, and the scaling laws for the velocity correlations. One of the most striking results that seems to persist in three dimensions is the distribution of density in patches, within which it is filamentary and with small fluctuations.



*E. Falgarone et al. (eds.), Fragmentation of Molecular Clouds and Star Formation*, 101–118.

The fourth Section is devoted to a brief account of recent numerical calculations of compressible two-dimensional MHD flows, focusing on overall aspects for different magnetic over kinetic energy ratios. A description will be given of the structures that develop, namely current sheets and bubbles of density.

In the following Section we show, on the basis of phenomenological arguments supported by two-dimensional numerical simulations, that supersonic turbulence can slow down and even stop the gravitational collapse.

Before concluding, we discuss the limit of small Mach numbers, relevant for both the large-scale interstellar medium and the sub-regions that develop within a supersonic flow.

## 2. Basic Concepts and Tools in Turbulence

Turbulence, as a strongly nonlinear phenomenon, is still lacking a definitive theoretical description, and a resort to a combination of theory, phenomenology, modeling, experiments and observations is needed to progress. In that light, it may appear bold to extend the analysis to more complex problems, involving coupling to rotation, magnetic fields, compressibility, convection and self-gravity, to name a few. But observational facts for one thing compel us in that direction. Also, and somewhat paradoxically, the problems at hand may become simpler, in that small parameters are introduced and at least some subsets become amenable to analytical treatment, *e.g.* through multiple-scale analysis : low Mach number, fast rotation, or strong magnetic fields.

The robustness of such regimes for the general case remains to be shown, and numerical experimentation has certainly become a primary way of investigation. Indeed some flows, for example at high magnetic Reynolds number $R^M = u_0 L_0/\eta$ where $u_0$ and $L_0$ are characteristic velocity and length and $\eta$ the magnetic diffusivity, or at high *rms* Mach number $M_a = u_0/c_s$ where $c_s$ is the sound speed of the medium, may not be feasible in the laboratory. Numerical experiments, on the other hand, do not allow to reach very high Reynolds numbers because of limitations in both memory and CPU time. Indeed, for a flow to be well resolved down to the dissipation length $\ell_D = (\nu^3/\epsilon)^{1/4}$ where $\nu$ is the kinematic viscosity, $\epsilon$ the rate of energy transfer and dissipation, and where a Kolmogorov energy spectrum has been assumed, *viz* $E(k) = \epsilon^{2/3} k^{-5/3}$, the dynamics of the numerical simulation *ie* the ratio of the large-scale $L_0$ to the smallest resolved scale $\Delta x$ must be of the order of the Reynolds number itself ($R^{3/4}$ in the incompressible case). In three dimensions, $10^6$ modes will thus be needed to experiment on a flow with a Reynolds number of $\sim 100$. Herein lies the fundamental limitation of numerical simulations. By-passes exist. One can reduce the space dimensionality to two (cylindrical) or one (spherical). Or one can decide that the precise way by which the flow dissipates the energy is not fundamental and thus resort to a model of dissipation. Among such methods, the more popular ones use the Euler equations (viscosity identically zero) and add some *ad-hoc* dissipation in steep gradients and shocks in the compressible regime (Woodward and Collela, 1984; Moretti, 1987). However, when dealing with small-scale phenomena, such as the reconnection processes in current sheets that may be at the origin of the heating of the solar corona, care must be taken in the precise treatment of the internal structure of dissipative layers. In that case the spectral methods retain all their advantages (Gottlieb and Orszag, 1977) because of their exponential precision for smooth flows.

Finally, one should expect only very slow progress in this experimental–numerical approach to turbulence : a factor two in resolution represents an eightfold increase in memory and twice that in CPU time, to follow explicitly all time scales involved. Some speed–up will come from hierarchical grids (Dorfi, 1982), from dynamical grid–tightening (Landman et al., 1990), and from heavy parallelisation of codes on computers such as the successor to the Connection Machine (Boghosian, 1990).

The concept of a cascade of energy from the large–scale containing eddies to the small–scale dissipative ones, in an energy–conserving way through the inertial range, is well–known (Rose and Sulem, 1978; Leslie, 1973; Monin and Yaglom, 1971). Modifications to the Kolmogorov spectral index of this range to take into account either magnetic fields (Iroshnikov, 1963; Kraichnan, 1965; Grappin et al., 1983; Matthaeus and Zhou, 1989), or compressible effects (Moiseev et al., 1983) as well as intermittency have been proposed.

Possibly less familiar is the concept of **inverse** cascade, from $L_0$ to scales larger than $L_0$. In incompressible MHD, an inverse cascade of magnetic helicity $H^M = \int \mathbf{a} \cdot \mathbf{b} \; d^3\mathbf{x}$ where $\mathbf{b} = \nabla \times \mathbf{a}$ with $\mathbf{a}$ the magnetic potential, leads to large–scale helical magnetic fields (Horiuchi and Sato, 1989; Pouquet, 1990). Such magnetic helical structures may have been observed in the Sun (Berger, 1988) and in molecular clouds (Heiles, 1987; see also the discussion in Scalo, 1990), but more data analysis is needed (Heiles, private communication). Whether such large scales will persist in a sustained supersonic flow is an open question. The precise mechanism by which these instabilities grow and saturate can be recast in the framework of multiple–scale analysis (Gilbert and Sulem, 1990).

Coherent structures in flows are pre–eminent, and their origin through an inverse cascade or more esoteric mechanisms (Nicolaenko and She, 1989) is unclear. The topological approach to turbulent flows (Moffatt, 1989) may be a helpful way to encompass the three–dimensionality of structures and also allow for a substantial reduction in data storage and analysis (Perry and Chong, 1987), such as for separated flows.

A consequence of the **direct** cascade of energy to the small scales is the added dissipation that takes place in a turbulent flow. These effects are modelized through turbulent transport coefficients, the precise computation of which still remains a problem of current research (Dubrulle and Frisch, 1990). However, it has been conjectured (Moffatt, 1985) that nonlinear interactions may be self–defeating, in the sense that they themselves produce a flow in which they become negligible : Beltrami flows in which the kinetic helicity $H^V = \int \mathbf{u} \cdot \omega \; d^3\mathbf{x}$ (with $\omega = \nabla \times \mathbf{u}$ the vorticity) is maximal. In MHD, the flow could become either force–free (vanishing Lorentz force), or fully correlated (normalized $\int \mathbf{u} \cdot \mathbf{b} \; d^3\mathbf{x}$ maximal), or both. Numerical evidence in two dimensions seems to corroborate these ideas in MHD, but the 3D problem remains open.

Finally, mention should be made of chaos and intermittency, and the ensuing spatial complexity of the flow with a possible fractal structure (see Scalo, 1990 for a review in the context of molecular clouds and also Falgarone and Phillips, 1990).

Low–dimensional dynamical systems exhibit complex behavior, with a transition that is now well mapped. However, when the number of relevant modes increases substantially, the concepts developed in the framework of chaos do not readily apply. The difficulty may lie in the way to couple many temporal scales as well as spatial scales. The wavelet technique (Combes et al., 1988) that combines local spatial information and Fourier mode analysis has been recently applied to the identification of structures in turbulent flows (Argoul et al., 1989; Farge et al., 1989; Everson et al., 1990) and in galaxy counts (Slezak et al., 1990). This technique may prove useful in the analysis of well–resolved maps of molecular clouds, the Taurus cloud (from IRAS data) being a possible candidate (Henriksen, 1990).

Chaos, however, may be relevant in fully developed turbulence as well, for example at stagnation points of the velocity and in the dynamo problem. In the latter case, it was shown (Galloway and Frisch, 1986) that the emerging magnetic structures are elongated. Numerical simulations (Meneguzzi et al., 1981) also point to an intermittency of the magnetic field, but it is not clear whether this is a dynamical effect or simply due to the proximity from the cross–over in the magnetic Reynolds number, separating the non–magnetic from the magnetic regime. The question of whether intermittency will steepen the Kolmogorov spectrum or not is yet another open problem (Kraichnan, 1990). Intermittency may have been observed in molecular clouds. Its origin may vary (fluid, MHD, gravitation, or a combination). Falgarone and Phillips (1990) have shown that there is a systematic departure from a Gaussian profile in the wings of molecular lines, that they relate to an intermittent behavior. Indeed, Anselmet et al. (1984) and Gagne and Castaing (1990) have shown that the probability distribution function of the velocity field from wind–tunnel data, as well as for the derivatives of the velocity and for a passively advected temperature, all have exponential wings with a Gaussian core. This is also the case in numerical simulations of MHD (Biskamp, 1990).

## 3. Compressible Turbulence

With increasing resolution in the observations it appears that molecular clouds are agitated by turbulent flows. Models assuming distinct clouds and quasi-equilibrium should thus be modified to account for a more dynamical vision (Scalo 1990) as suggested by observations exhibiting an enormous variety of structures with irregularities at all scales, filaments, bubbles etc...(Bally et al. 1987, Bally 1989; Falgarone 1989, Fukui, 1990). A first step in this direction can be attempted by studying compressible turbulence and the effect of the nonlinear advection term of the momentum equation on the shaping of the flow. Although interstellar cloud turbulence certainly includes magnetic fields, stellar energy sources, radiative cooling and gravitation, nonlinear advection is a major common feature to take into account.

Homogeneous compressible turbulence has not been extensively studied, partly due to the fact that the incompressible case remains unsolved. Analytical studies pertain mostly to the weakly compressible regime, either concerning the acoustic part of the flow (its generation (Lighthill, 1954), or statistical properties (Zakharov et al., 1970)) or the extension of the incompressible phenomenology for small Mach numbers (Moiseev et al., 1983). General arguments (Kraichnan, 1953) and closure schemes (Chandrasekhar 1951a, Weiss 1979, Hartke et al. 1988, Marion 1988) have been developed which are restricted to the small Mach number regime (see also Passot and Pouquet (1987) for a review).

Feireisen (1981) studied numerically the effect of a weak compressibility on the statistics of 3D turbulent shear flows. Computations on homogeneous and turbulent supersonic flows have been performed in both two dimensions and in three dimensions for decay flows (Erlebacher et al., 1990; Passot and Pouquet, 1990, and references therein) and forced flows (Kida and Orszag, 1990). Large–Eddy Simulations (Erlebacher et al., 1987; Porter et al., 1990a) have also been implemented.

In the supersonic regime, a dominant feature is the presence of shocks which are the major cause of dissipation. In the case of decaying turbulence, the flow remains globally supersonic for short times (a few turnover times of the large–scale vortices) whatever the initial value of the Mach number. However, the trace of an initially supersonic flow is still visible at late times on hot spots of temperature (assuming no radiative leaks) and entropy production. Dissipation is found to be similar for the two and three dimensional cases during this first period of supersonic evolution, but different in the subsequent part of the evolution. Whereas in 2D the Mach number almost stabilizes at *r.m.s.* values of about .6, possibly due to a remnant of the incompressible property of global squared vorticity conservation (Kraichnan and Montgomery, 1979), in the 3D case it keeps decreasing due to the usual nonlinear transfer of energy towards small scales.

When a strong pressure imbalance is present in the initial conditions it is found that the compressive component of the kinetic energy and the internal energy both oscillate periodically (in opposite phase) even for long times, indicating that large scale, large amplitude sound waves are still free to propagate into the system, being minutely influenced by the vortices interacting mostly with themselves. For *rms* Mach numbers smaller than .3, the flow can be considered incompressible, the interaction between rotational and compressive modes being weak, mostly consisting of sound production by large–scale vortices. For larger Mach numbers the compressive modes are fed more efficiently and contribute in a dominant way to the small–scale kinetic energy because of the presence of strong shocks, the large scales being mostly solenoidal. The ratio $\chi$ of the compressive over total kinetic energy is typically .2. The same transitional Mach number has been found in MHD.

The opposite interaction consisting of the production of rotational modes by the compressive ones has been observed in 2D during collisions of shocks or behind curved shocks where entropy gradients are not colinear to temperature gradients (Passot and Pouquet, 1987). It is intermittent and concentrated near the small dissipative scales. This interaction takes the form of small vortices created just behind the shocks or during the process of a Kelvin-Helmotz instability developing in contact discontinuities. It has not yet been observed in 3D.

The interaction between rotational and compressive modes is thus mostly concentrated in the large and small scales, the latter being only efficient for large Mach numbers. Consequently the flow presents a dual nature, consisting of a weakly compressible turbulence, retaining most of its characteristics of the incompressible case, on which is superimposed sound and shock waves. This weak interaction can also be observed when measuring spectra in 2D. The velocity correlation spectrum for the rotational modes still presents an inertial range whose slope is close to –3, a value observed in the incompressible case. The compressive modes, being dominated by shock waves, present a $k^{-2}$ spectrum (Passot et al. 1988). Inertial ranges cannot be observed in 3D due to a lack of resolution, although time averaging will help. When visualizing the density field it appears that there are large patches in which fluctuations are mild. The local *rms* Mach number is small but both mean velocity and density may vary greatly between patches. The existence of such patches has also been observed in the 3D case (Porter et al., 1990b) at late times.

A supersonic turbulent flow also presents striking filamentary structures both in 2D and in 3D, but for possibly different reasons. In 2D the filaments observed on the density field are strongly correlated to entropy fluctuations, created by heating due to dissipation in shocks or vortex sheets. Being passively advected by the flow (Bayly et al 1990) these fluctuations naturally form ribbons, and accumulate as time evolves. These filaments pierce from one patch to the other, revealing once more the dual nature of the flow (Passot et al., 1988). In 3D the filaments which are observed at earlier times (Porter et al. 1990b) are more likely to be associated to shock collisions and intersections, as well as over–compressions in shock bendings or vortex tubes (Vincent and Meneguzzi, 1990). These filaments are striking in the compression field, vorticity and density, and may be a locus of star formation.

The most striking difference between a 2D and a 3D compressible flow is in the density contrast defined as $\Delta\rho = \rho_{max}/\rho_{min}$. Whereas in 2D $\Delta\rho \sim 4$, at similar Reynolds and Mach numbers in 3D $\Delta\rho \sim 100$ (Passot and Pouquet, 1990).

When recasting all these results in the framework of molecular clouds dynamics, several problems emerge. The most important is linked to the rate of dissipation of supersonic turbulence, too high in comparison with estimated energy injection rates, although the gravitational potential well is omnipresent. This problem may possibly be alleviated by the presence of a magnetic field as discussed in the next Section. The structures that obtain in such neutral flows are however not in contradiction with observations and it will be interesting to see how they will be modified by gravitation and magnetic fields.

## 4. Supersonic MHD Turbulence

This Section briefly reports on some recent calculations of supersonic magnetohydrodynamic flows. Homogeneous compressible MHD flows have attracted little attention until very recently, previous works being mostly devoted to the study of reconnection processes (see e.g. Ugai, 1988 and Sonnerup 1988). The growth of correlations $\int \mathbf{v}\cdot\mathbf{b}\, d^n\mathbf{x}$ between the velocity field $\mathbf{v}$ and the magnetic field $\mathbf{b}$ which occurs in the incompressible case (see *e.g.* Pouquet, 1990) has also been shown to occur in compressible flows by Dahlburg and Picone (1988), and a study of turbulent relaxation (dynamic alignment versus selective decay) has been undertaken by Ghosh et al. (1988). Shebalin and Montgomery (1988) studied the pseudo–sound generation in an isentropic flow and more recently, Dahlburg and Picone (1989) described the influence of compressibility on the evolution of the Orszag–Tang (1979) vortex. These works are dealing with 2D subsonic flows (thus with a high value of the plasma $\beta$ value, ratio of kinetic to magnetic pressure).

Even when studying the simple case of a conducting perfect gas in a two-dimensional periodic box, we are faced with a large free–parameter space. It has been chosen here to concentrate on a comparative study of the overall aspects of a supersonic flow when permeated with a random magnetic field at the same scale as that of the velocity field but with differing magnitudes.

When shocks are present, kinetic energy dissipates into heat within a few non-linear times. One of the goals of this work is to identify the regimes where strong shocks are inhibited but where supersonic linewidths could nevertheless be observed. Does this regime exist and does it correspond necessarily to a state of Alfvénic wave turbulence as suggested by Falgarone and Puget (1986) and Lizano and Shu (1987) ? Does this require a smooth large–scale magnetic field ?

New characteristic times appear in the evolution of an MHD flow. Certainly one of the most important is the Alfvén time $T_a = L/V_a$ where $L$ is a characteristic scale and $V_a$ is the Alfvén speed $B_0/\sqrt{4\pi\rho}$ which governs the propagation of transverse waves in the direction of the magnetic field $B_0$ in an average density field $\rho$. There are also characteristic times based on the speed of the compressive waves which propagate into the system (i.e. the slow and fast magneto-acoustic waves). For reasons of simplicity we will classify the flows with respect to their character (sub or super–sonic and sub or super–Alfvénic), comparing the *rms* velocity with the sound speed and the Alfvén speed respectively. This defines in turn the Mach number $M_a$ and the Alfvénic Mach number $M_{alf}$. The Alfvén speed is based on the *rms* magnetic field since no mean field is considered here.

Before describing the new numerical simulations on supersonic MHD flows, it will be useful for the discussion to recall some properties of discontinuities in a compressible magnetic fluid. Beside transverse waves which are non–compressive and remain smooth, there are also magneto-acoustic waves which can steepen into shocks. Transverse shocks and contact layers (examples of which are current sheets) are not formed by steepening but have to result from topology, breaking of equilibrium, *etc*.... Current sheets can be present regardless of the properties of the velocity field. They do not propagate and in contrast to pure hydrodynamical contact discontinuities, there cannot be a non–zero component of the magnetic field perpendicular to the plane of the discontinuity, in which case only jumps in density and entropy are allowed. Thus in 2D the center of a current sheet is a neutral line. These discontinuities are known to dissipate through various mechanisms, in particular because they are subject to internal instabilities, such as the tearing mode.

In contrast, the dissipation in a shock is only due to molecular transport coefficients and depends on its strength as measured by the entropy or pressure jump. Weak shocks necessarily propagate at the velocity of the corresponding linear wave. Stationary shocks in a medium of zero mean velocity are then moderately strong and we want to discuss their existence in some particular cases. In absence of magnetic fields, their existence requires a supersonic flow. Numerical simulations in two dimensions (Passot and Pouquet, 1987) indicate that a supersonic velocity fluctuation produces strong shocks that dissipate rapidly until the *rms* Mach number attains values close to .6. It thus seems that within a factor of two the above mentioned criterion gives good approximations for the upper bound Mach number compatible with small dissipation. We refer here only to dissipation due to shocks, leaving aside the dissipation due to the cascade of eddies in the three-dimensional case or due to current sheets. In three dimensions, indeed, the Mach number may drop to substantially lower values (Porter et al., 1990b) due to the usual turbulent eddy viscosity. In the presence of a magnetic field, shocks propagate anisotropically. It is particularly true for slow shocks which propagate efficiently only in a cone centered on the local direction of the magnetic field. It will be useful to consider two limit cases of shocks. In the first one corresponding to the propagation perpendicular to a constant magnetic field, slow shocks degenerate into contact discontinuities and the existence of fast shocks requires the velocity of the fluid $V$ to be greater than $\sqrt{B_0^2/(4\pi\rho)+c^2}$. In the case of the propagation parallel to the magnetic field, the pure gas limit is recovered but the evolutionary conditions are different since in presence of small perturbations transverse waves can be induced by the longitudinal magnetic field. In particular for strong shocks, the flow has to be super–Alfvénic behind the shock. What is important to point out is the general trend that in the presence of a magnetic field the strength of the shocks is reduced. A shock propagating in a medium permeated with a magnetic field parallel to its plane can eventually degenerate into a sound wave if the strength of the magnetic field increases beyond a critical value (Ferraro and Plumpton, 1966, p. 105). From the preceding discussion, it seems also that a sub–Alfvénic (but possibly supersonic) flow will develop less strong shocks and it is also generally admitted that turbulence is inhibited by magnetic fields.

The following description of some of our numerical simulations reveals indeed this tendency, but points out to some other interesting features of a magnetic turbulent flow. The three runs presented now are for decaying two–dimensional flows, starting with a *rms* Mach number of unity, with excursions up to 2. The velocity field has most of its energy at a length scale $L = \pi$ corresponding to one–half the computational box. Both the usual and magnetic Prandtl numbers are unity, $\gamma = 5/3$ and the Reynolds number is 150. All primitive variables are initialized randomly. We define $\chi = E^c/E^v$, with $E^c + E^s = E^v$ the kinetic energy decomposed into its solenoidal $E^s$ and compressive $E^c$ components. Initially, $\chi = 15\%$, a deliberate choice since we want to start our computation in a fully compressible regime.

The case without magnetic field is analogous to the one described in the previous Section, developing numerous elongated shocks. When taking initially a random magnetic field at the same scale and such that the ratio $r_m = E^m/E^v$ of the magnetic over kinetic energy is equal to .9, the shocks are already completely absent although the flow is not yet sub–Alfvénic. It has to be mentioned however that the compressibility of the flow measured by the ratio $\chi$ is enhanced, attaining values of 30% or 1.5 times the peak value of the neutral case. Density fluctuations are also higher. This is due to the increase in the total pressure gradients induced by the presence of the magnetic field. We see at this point that the effect of the magnetic field on the velocity will depend on its scale and not only on its magnitude, the smoothing mechanism being valid only if the field is at a scale larger than or comparable to that of the velocity.

Increasing the magnitude of the ratio $r_m$ to 3.45 and leaving all other parameters constant, the aspect of the flow changes drastically. The smoothing mechanism does not persist when increasing the magnetic field strength, leaving its scale constant; indeed the initial desequilibrium in pressure is large enough to produce shocks

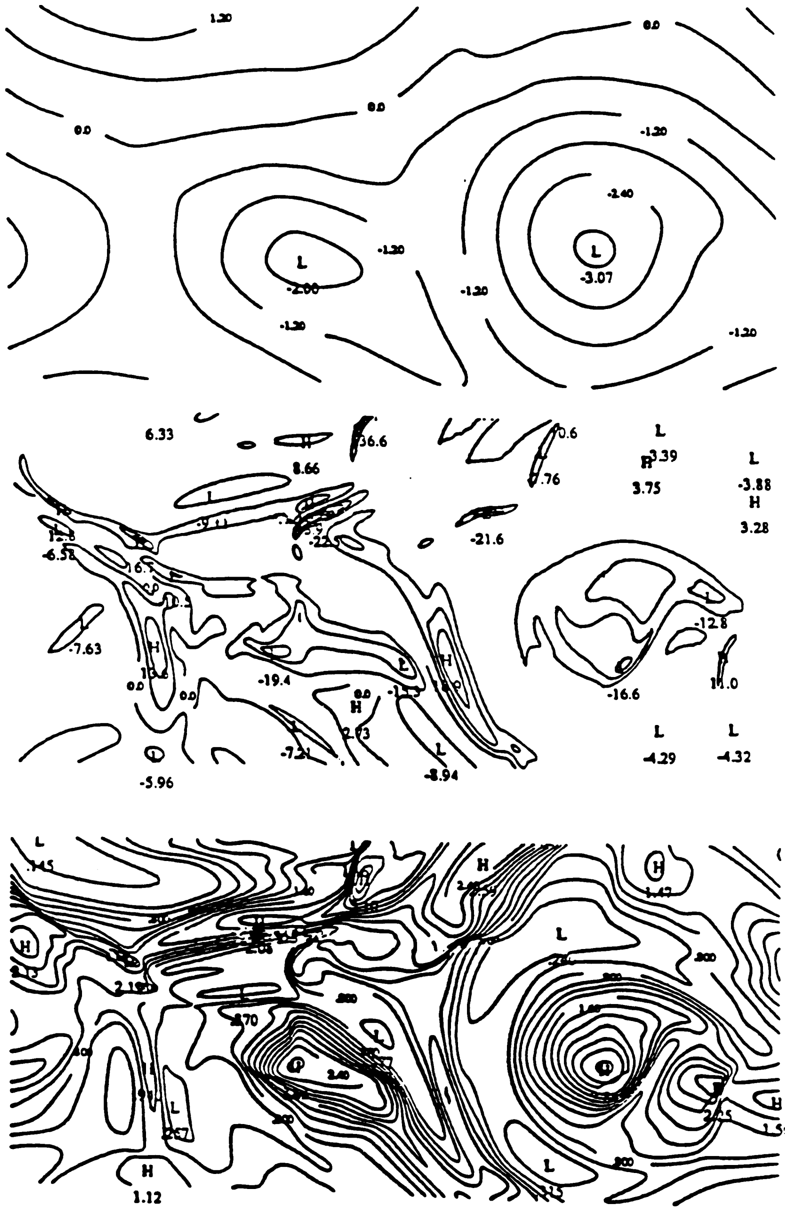

**Figure 1:** (top) magnetic potential, (middle) magnetic current, (bottom) density, at $t = 1.5$ for a flow with an initial Mach number of 1 and $r_m = 3.45$.

https://doi.org/10.1017/S0074180900198833 Published online by Cambridge University Press

**Figure 2:** (top) density, (middle) temperature, (bottom) vorticity, at $t = 2.5$ for a flow with an initial Mach number of 2, and $r_m = 1.7$.

https://doi.org/10.1017/S0074180900198833 Published online by Cambridge University Press

although the flow is now sub–Alfvénic. In a short time, violent processes occur whose dynamics is mostly governed by the magnetic field. There is initially a large transfer of energy from magnetic to kinetic. Shocks form and current sheets develop near X-type neutral points, releasing a large quantity of heat. This heat causes the fluid to expand, producing bubbles. When growing, these bubbles form circular shocks which smooth out rapidly while expanding. The discontinuities present in this flow have therefore a shape very different from the neutral case. Shocks are thin but much shorter, their life time being also much smaller as we could predict. Current sheets are more long-lived and thicker than shocks since they arise from a topological and not a dynamical constraint; they have a certain amount of magnetic flux to dissipate which can be quite large. We show in Figure 1 the vertical component of the magnetic potential (top), the current density (middle) and the density (bottom) for the lower half of the flow at $t = 1.5$. The current sheet, corresponding to the magnetic X–point (hyperbolic neutral point), is thicker than the filamentary structures observed both in the current and the density, associated with angular points of the potential. The bubbles in density correspond to previous current structures, and the thinner compressions to the magnetic shocks. The Joule heating mechanism, more important in MHD shocks than in current sheets, is now able to decorrelate efficiently the temperature field from the density thus increasing the strength of the baroclinic term $\frac{(\nabla p \times \nabla \rho)}{\rho^2}$ responsible for vorticity production; vorticity is also created by the purely magnetic term leading for example to quadrupolar structures centered on the current sheets, as in the incompressible case. Thus, a polytropic approximation becomes questionable when the Alfvén time is shorter than the characteristic radiative cooling time. Ambipolar diffusion will also provide an efficient dissipation mechanism.

Reconnection is seen to be more efficient than in the incompressible case. Current sheets are broken and dissipated rapidly by *e.g.* tearing modes, when high–speed flows converge onto it (fast reconnection). This is clearly observed in a simulation of the Orszag–Tang vortex performed at Mach one, with constant initial pressure. In this case, the lateral (as opposed to central) current sheets are the strongest. This is due to the combined action of the centrifugal force (not compensated by the initial pressure) and the periodic boundary conditions which direct a flow onto the sheet and enforce reconnection in an intrinsic dynamical way.

Similar results obtain when other runs with initially smaller scales and/or higher Mach numbers are performed. In a run with an initial Mach number $M = 1.8$ and with $r_m = 1.7$, the Mach number is equal to .46 after ten turn–over times, as opposed to .4 when starting with $M = 1$. The ratio $\chi = 48\%$ is much larger in the high Mach number case, and yet the flow appears smooth at that Reynolds number (computation on a $256^2$ grid). In Figure 2, we show the density (top), temperature (middle) and vorticity (bottom) for the higher half of the flow at $t = 2.5$ for the Mach 2 run. We see both the bubbles linked to current and thin structures in the density, we see the decorrelation between density and temperature, and we see the breaking of the vorticity in small often roundish quadrupoles.

Common to all these runs is the fact that the ratio $r_m$ increases with time after an initial decaying transient and that the scale of the magnetic field seems to diminish. Here it is important to note that the initial conditions of these runs are violent. For later times, the flow is gentler and discontinuities almost inexistent. The dissipation is then much smaller. This is consistent with the observations on the evolution of the strength and scale of the magnetic field. Finally, we note that, in agreement with the results of Dahlburg and Picone (1988), the alignment factor $\frac{<\mathbf{V}.\mathbf{B}>}{<\mathbf{V}^2>^{1/2}<\mathbf{B}^2>^{1/2}}$ increases, starting from .33 and rising to .6.

The properties of the spectra are also noteworthy. There is quasi–equipartition between the solenoidal component of the kinetic energy and the magnetic energy in the small scales, with a slight excess for the latter as for incompressible flows. On the other hand, the correlation spectrum of the compressible part of the velocity itself dominates in the small scales both the $E^s$ and $E^m$ modes for sufficiently high Mach numbers.

Summarizing these results, we can say tentatively that in sub–Alfvénic but still supersonic flows, shocks are hindered. When the magnetic field is tangled, reconnection takes place, but this process dissipates less energy than would do shocks. If the flow is perpendicular to the magnetic field (possibly due to gravitation), we could imagine the flow to be less turbulent. Also, the formation of bubbles of density is striking.

It is planned to study these dissipation mechanisms with different topologies of the magnetic field and different initial conditions, trying to test the stability of regimes consisting of nonlinear Alfvén waves.

Magnetic fields may play an essential role in the collapse of molecular clouds (Mouschovias, 1987; Lada and Shu, 1990). The numerical treatment of a turbulent MHD compressible flow undergoing gravitational collapse, is scanty (Dorfi, 1982; Pouquet et al., 1990). In the next part we shall discuss the simpler case (although still not resolved) of the interplay of gravitation with a compressible turbulence in a barotropic flow. Works performed directly in the astrophysical context are reviewed for example in Scalo (1988).

## 5. Turbulence and Gravitation

In molecular clouds, large scales are found to be relatively stable over times long compared to the free-fall time of the cloud, whereas small scales are clumpy. The generalization of the Jeans' stability analysis to account for turbulent kinetic energy has been considered by Chandrasekhar (1951b), Sasao (1973), and by Bonazzola et al. (1987); magnetic fields (Lizano and Shu 1987; Pudritz, 1990) have also been taken into account. A phenomenological argument (Léorat et al. 1989,1990) which encompasses the interaction between turbulent eddies, the phase coherence of shocks, waves and gravitational collapse is briefly exposed below. It is confirmed by numerical simulations assuming an isothermal flow and bidimensionality.

Gravitation acts directly on the compressive modes and competes with pressure and turbulent dissipation. At a given scale, the comparative strength of these processes can be evaluated by estimating their characteristic times. For turbulence, the ratio $r_g = \tau_{tr}/\tau_{ff}$ of the transfer time of the compressible kinetic energy towards small scales $\tau_{tr}$ to the free-fall time $\tau_{ff}$ is the relevant parameter to consider. In the Jeans' case where only pressure is considered, the relevant parameter is obtained by replacing $\tau_{tr}$ by the acoustic time $\tau_{ac} = \ell/c_s$ where $c_s$ is the sound speed. At small Mach number, the transfer time is obtained by considering sound wave coupling as in Zakharov et al. (1970). The parameter $r_g$ increases with scale and eventually collapse will take place for a critical length, turbulence resulting in a shift of the Jeans' length towards larger scales (see also Chandrasekhar, 1951b).

However, $r_g$ is found to be very sensitive to the value of the Mach number $M$. With increasing $M$, dissipation tends to occur mainly in shocks, with an equal strength for all scales. The concept of inertial range and energy cascading becomes meaningless and $\tau_{tr}$ must be modified accordingly. Taking the Burgers' equation limit, $\tau_{tr}$ is found independent of scale. Assuming a constant local mean density, the parameter $r_g$ will thus also be scale–independent and reads in the simplest case :

$$r_g = \tau_{tr}/\tau_{ff} = (L/L_J)c_s/u_c$$

where $L_J$ is the Jeans' length and $u_c$ the compressive component of the velocity at scale $L$. There is a critical value $\tilde{r}_g$ of order unity below which the flow becomes stable. This leads plausibly to a global marginal equilibrium between turbulence and collapse at all scales. This equilibrium can be broken at scales smaller than the Jeans' length, when local peaks in density are formed by turbulence, *e.g.* in the vicinity of shocks. Large density enhancements can be expected in shocks when radiative transfer is included (Zeldovich and Raizer, 1966), which will favor this kind of local mechanism for collapse. Observations do support clumpiness of the medium on a wide range of scales. An unmagnetized medium will have planar shocks, and clumps might occur at their intersections, on filaments. Furthermore, in the MHD case, bubbles form in the density leading to arched–like shocks. Also, shocks may be unstable through a hernia–type effect.

Numerical simulations support the predictions based on the preceding phenomenology, with $\tilde{r}_g \sim 0.3$ in two dimensions. Although the Reynolds numbers of the computed flows are not large, and thus no complete inertial range is exhibited, it has nevertheless been observed that the large scales of the flow tend to tear the collapsing clumps into pieces and fragment them into stable entities. When the value of $r_g$ is small enough, the gravitational collapse is stopped for at least ten free-fall times. For larger values of $r_g$, collapse occurs but strongly structured by turbulence. Clumps tend to form on filaments of denser matter (Passot, 1987). Their scale decreases with the Mach number and hierarchical structures are also found when several Jeans' masses are present in the cloud (Léorat et al., 1990). However, no successive fragmentation is observed, possibly due to a lack of numerical resolution. The hierarchical structures observed are formed one at a time through the influence of the turbulent velocity field; it happens also that small structures are formed first before collapsing together. In three dimensions, the situation is not clear. Chantry et al. (1990) find that density clumping tends to form initially at hyperbolic points of the velocity, where the vorticity is weak, through the centrifugal force. This effect is amplified when two vortices meet. However, as both the Reynolds number and the Mach number are increased and vortex interactions become embedded in a turbulent flow, the preceding coherent mechanism may be swamped by shock formation and ensuing strong density contrasts in their vicinity.

Finally, when coupling gravity and MHD, one may conjecture that pressure is modified by both the turbulent pressure, and the magnetic one (see also Pudritz, 1990). A somewhat *ad hoc* modification of the phenomenological argument presented above is given in Pouquet et al. (1990).

## 6. Conclusion

It has been mentioned in Section II, that in a supersonic flow, there are regions where density fluctuations as well as the local *rms* Mach number are small. It is thus of interest to consider the limit of quasi-incompressible flows. This has been studied rigorously for isentropic fluids (Klainerman and Majda 1981, 1982). Entropy fluctuations should nevertheless be considered as soon as forcing or dissipative processes are no longer negligible. We do not review this rather large subject but rather discuss the observations of Armstrong et al. (1981). They have measured the density fluctuations in the large scale interstellar medium, where the flow can be considered quasi-incompressible and found that they exhibit a power law close to $k^{-5/3}$. Assuming a barotropic fluid, density fluctuations are correlated with pressure fluctuations and thus should obey a $k^{-7/3}$ law if a Kolmogorov-like cascade is assumed. Recently Montgomery et al. (1987) and Matthaeus and Brown (1988) suggested that the presence of a magnetic field in equipartition with the velocity field could possibly explain a $k^{-5/3}$ law. By systematically deriving the incompressible limit for a gas whose equation of state depends on two thermodynamic variables, Bayly et al. (1990) showed both analytically and numerically that a $-5/3$ law can also be expected as soon as entropy fluctuations are non-negligible. The basis of the argument is that when taking the incompressible limit two parameters have to be taken to zero, namely the Mach number and the size of temperature fluctuations. Depending on the relative size of these two parameters, different physical limits can be identified. When the latter is larger than $M^2$, the density and temperature both obey at first order a passive scalar advection equation. This in turn can also explain the filamentary structures as discussed in Section II at long times, with relatively small density fluctuations within them.

Very few high resolution numerical simulations of compressible turbulence exist. Although turbulence *per se* is certainly still far from being solved, the results described here may lead to some insight in the dynamics of the interstellar medium. The overall aspect of the flow, when compared to observations, indicate that nonlinearities are certainly at play in the interstellar medium. Filaments are observed both in molecular clouds and numerical simulations, with density clumps along them. In the latter we saw three different occurrences for such structures, showing that they are general patterns produced by nonlinear advection terms. Sheets also occur in three dimensions, and bubbles obtain when strong magnetic fields reconnect. Finally, hierarchical structures appear with gravitational collapse. Virialisation of the medium may end-up being a consequence of its dynamical evolution when few density gradients occur and there is an equilibrium, albeit marginal, between turbulence and collapse. Furthermore, nonlinear build-up of intermittent structures and fractal behavior have been shown to take place in molecular clouds.

## ACKNOWLEDGEMENTS

The three-dimensional computations with the PPM code reported here were done in collaboration with D. Porter and P. Woodward on the Minnesota Super-Computer

Institute (MSI) computers. The other numerical simulations were performed at the Centre de Calcul Vectoriel pour la Recherche (Palaiseau, France) at NCAR and at the Pittsburgh Supercomputing Center under N.S.F. grant D.M.S. 8703397. We received partial support from the DRET grant 89–236.

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