C

A few more details

C.1 Nobel Prizes

Success in science is, strictly speaking, measured only in ells of time: Democritus' and Leucippus' idea of elementary particles, even after two and a half millennia, serves successfully as a guiding thought and Leitmotif, and Newton's and Leibniz's calculus still forms the basis of the mathematical formulation of the laws of Nature. The fact that more than a third of twentieth century Nobel Prizes were awarded to discoveries relating to the physics of elementary particles and fundamental physics is probably foreordained by the selection effect: in a field where one knows less, the probability of discovering something fundamentally new is higher. Nevertheless, I hope that this, perhaps even pompous, review of major successes in the past century will serve as a convenient reminder.

Table C.1 Nobel Prizes awarded for discoveries and contributions in fundamental physics

Year	Awardee	Award for [*paraphrase*; T.H.]
1901	Wilhelm C. Röntgen	discovery of the remarkable rays subsequently named after him, also known as X-rays
1903	A. Henri Becquerel $\left(\frac{1}{2}\right)$	discovery of spontaneous radioactivity
	Pierre Curie, Marie Curie, née Sklodowska	their joint researches on radiation phenomena
1906	Joseph J. Thomson	investigations on the conduction of electricity by gases [*i.e., discovery of the electron*; T.H.]
1918	Max K. E. L. Planck	advancement of physics by his discovery of energy quanta [*quantization of electromagnetic radiation* ***emission***; T.H.]
1921	Albert Einstein	discovery of the law of the photoelectric effect [*not the discovery that electromagnetic radiation* ***exists*** *in quanta – photons*; T.H.]
1922	Niels H. D. Bohr	investigation of the structure of atoms and of the radiation emanating from them

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Year	**Awardee**	**Award for** [*paraphrase*; T.H.]
1923	Robert A. Millikan	work on the elementary charge of electricity and on the photoelectric effect
1925	James Franck, Gustav L. Hertz	discovery of the laws governing the impact of an electron upon an atom [*confirming the quantization of atomic states*; T.H.]
1927	Arthur H. Compton	discovery of the effect named after him
	Charles T. R. Wilson	method of making the paths of electrically charged particles visible by condensation of vapor [*invention of the cloud chamber*; T.H.]
1929	Prince Louis-Victor P. R. de Broglie	discovery of the wave nature of electrons [*and not the universal wave–particle duality*; T.H.]
1932	Werner K. Heisenberg	creation of quantum mechanics
1933	Erwin Schrödinger, Paul A. M. Dirac	discovery of new productive forms of atomic theory
1935	James Chadwick	discovery of the neutron
1936	Victor F. Hess	discovery of cosmic radiation
	Carl D. Anderson	discovery of the positron
1939	Ernest O. Lawrence	invention and development of the cyclotron
1945	Wolfgang Pauli	discovery of the exclusion principle
1949	Hideki Yukawa	prediction of the existence of mesons
1950	Cecil F. Powell	development of the photographic method of studying nuclear processes and his discoveries regarding mesons made with this method
1954	Max Born	statistical interpretation of the wave-function
	Walther W. G Bothe	the coincidence method
1955	Willis E. Lamb	discoveries concerning the fine structure of the hydrogen spectrum
	Polykarp Kusch	precision determination of the magnetic moment of the electron
1957	Chen-Ning Yang, Tsung-Dao Lee	penetrating investigation of the so-called parity laws [*i.e., of C-, P- and CP-violation*; T.H.]
1958	Pavel A. Cherenkov, Il'ja M. Frank, Igor Ye. Tamm	discovery and the interpretation of the Cherenkov effect
1959	Emilio G. Segrè, Owen Chamberlain	discovery of the antiproton
1960	Donald A. Glaser	invention of the bubble chamber
1963	Eugene P. Wigner $(\frac{1}{2})$	discovery and application of fundamental symmetry principles [$\frac{1}{2}$: *Maria Goeppert-Meyer and J. Hans D. Jensen, nuclear shell structure*; T.H.]
1965	Shin-Ichiro Tomonaga, Julian Schwinger, Richard P. Feynman	fundamental work in quantum electrodynamics, with deep-ploughing consequences for the physics of elementary particles [*renormalization in QED; Freeman Dyson showed the equivalence of the methods of Tomonaga, Schwinger and Feynman*; T.H.]
1968	Luis W. Alvarez	discovery of a large number of resonance states (hadrons)

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Year	**Awardee**	**Award for** [*paraphrase*; T.H.]
1969	Murray Gell-Mann	classification of elementary particles and their interactions
1976	Burton Richter, Samuel Chao-Chung Ting	discovery of a heavy elementary particle of a new kind
1979	Sheldon L. Glashow, Abdus Salam, Steven Weinberg	theory of the unified weak and electromagnetic interaction between elementary particles, including, inter alia, the prediction of the weak neutral current
1980	James W. Cronin, Val L. Fitch	discovery of violations of fundamental symmetry principles in the decay of neutral K-mesons [*CP-violation*; T.H.]
1982	Kenneth Wilson	theory for critical phenomena in connection with phase transitions [*this theory contains the approach to renormalization that is built into the foundations of contemporary field theory*; T.H.]
1984	Carlo Rubia, Simon van der Meer	decisive contributions to the large project that led to the discovery of the field particles W and Z, communicators of the weak interaction
1988	Leon M. Lederman, Melvin Schwartz, Jack Steinberger	neutrino beam method and the demonstration of $\nu_e \neq \nu_\mu$
1990	Jerome I. Friedman, Henry W. Kendall, Richard E. Taylor	pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, of essential importance for the development of the quark model
1992	Georges Charpak	invention and development of particle detectors, in particular the multiwire proportional chamber
1995	Martin L. Perl	discovery of the tau lepton
	Frederick Reines	detection of the neutrino [*already in 1956 – 39 years earlier! C. Cowan died in 1974, and was not awarded*; T.H.]
1999	Gerardus 't Hooft, Martinus Veltman	elucidating the quantum structure of electroweak interactions in physics [*renormalization in models with Higgs fields*; T.H.]
2002	Raymond Davis Jr., Masatoshi Koshiba;	pioneering contributions in astrophysics: detection of cosmic neutrinos and the solar neutrino problem (the Homestake Experiment)
	Riccardo Giacconi	pioneering contributions in astrophysics: cosmic X-rays
2004	David J. Gross, H. David Politzer, Frank Wilczek	discovery of asymptotic freedom in the theory of the strong interaction
2006	John C. Mather, George D. Smoot	discovery of the blackbody form and anisotropy of the cosmic microwave background radiation
2008	Yoichiro Nambu $(\frac{1}{2})$	discovery of the mechanism of spontaneous broken symmetry in subatomic physics
	Makoto Kobayashi, Toshihide Maskawa	discovery of the origin of the broken symmetry that predicts the existence of at least three families of quarks in Nature
2011	Saul Perlmutter $(\frac{1}{2})$, Brian P. Schmidt, Adam G. Riess	discovery of the accelerating expansion of the universe through observations of distant supernovae

It is worth noting that several physicists with very important contributions to fundamental physics were awarded for their contributions in other areas, instead of their main discoveries: For example, Ernest Rutherford was awarded the 1908 prize in chemistry, while his work on classifying radioactivity, identifying α-particles as helium ions, establishment of the exponential decay law and its use as a clock, and – most importantly – the discovery of the atomic nuclei were not so awarded. Similarly, Enrico Fermi was awarded in 1938 for "demonstrations of the existence of new radioactive elements produced by neutron irradiation, and for his related discovery of nuclear reactions brought about by slow neutrons," while his theoretical model of β-decay and his other contributions to fundamental physics remained not so awarded; Vitaly L. Ginzburg was awarded in 2003, together with Alexei A. Abrikosov and Anthony J. Legett, "for pioneering contributions to the theory of superconductors and superfluids," but not for the groundbreaking work with Lev Landau on spontaneous magnetization, which eventually led to the general idea of spontaneous symmetry breaking and the so-called Higgs mechanism [☞ Section 7.1]. Bohr's principle of complementarity, Pauli's prediction of the neutrino, and even Einstein's theory of relativity, among others, remained similarly un-awarded by the Nobel committee. After all, Nobel Prizes are also a testament to the socio-political milieu. Finally, it is important to keep in mind the defined limitations: "In no case may a [Nobel] prize amount be divided between more than three persons." Also, "a [Nobel] Prize cannot be awarded posthumously, unless death has occurred after the announcement of the Nobel Prize" [517].

C.2 Some numerical values and useful formulae

While following the narrative in this book, numerical values of various constants are mostly unnecessary, but it is useful to have an idea about the relative numerical values of the various results, so that the Reader is expected to work through the derivations and complete the skipped steps, as well as to complete the exercises. Tables C.2, C.3 and C.4 should help in this endeavor.

When including electromagnetic phenomena in a study, note that the electric charge (divided by the natural constant $\sqrt{4\pi\epsilon_0}$) may be measured in purely "mechanical" units, as shown in equations (1.12). However, it is frequently useful to extend the unit system based on the measurement of the physical quantities of mass, length and time (M, L, T) by adding, minimally, the measurement of electric charge, C, and then consistently retaining all factors of $\sqrt{4\pi\epsilon_0}$. Owing to the identity $c^2 = 1/\epsilon_0\mu_0$, the constant μ_0 may always be expressed as $\mu_0 = 1/\epsilon_0 c^2$. However, in order to emphasize the electro-magnetic duality, Table C.4 on p. 527 retains both ϵ_0 and $\mu_0 = 1/\epsilon_0 c^2 = 4\pi \times 10^{-7}\,\text{kg m/C}^2$.

Table C.2 Natural constants and some useful characteristic values

$\hbar$	$1.054\,572 \times 10^{-34}$ J s	$6.582\,119 \times 10^{-16}$ eV s	M_P	$2.176\,45 \times 10^{-8}$ kg	$1.220\,90 \times 10^{19}$ GeV$/c^2$
c	$299,792,458$ m/s		m_e	$9.109\,382 \times 10^{-31}$ kg	$0.510\,999$ MeV$/c^2$
ϵ_0	$8.854\,187\,817 \times 10^{-12}\,\frac{\text{C}^2\,\text{s}^2}{\text{kg}\,\text{m}^2}$		m_μ	$1.883\,531 \times 10^{-28}$ kg	105.658 MeV$/c^2$
e	$1.602\,176 \times 10^{-19}$ C		m_τ	$3.167\,772 \times 10^{-27}$ kg	$1.776\,99$ GeV$/c^2$
G_N	$6.674\,2 \times 10^{-11}\,\frac{\text{m}^3}{\text{kg}\,\text{s}^2}$	$6.708\,7 \times 10^{-39}\,\frac{\hbar\, c^5}{\text{GeV}^2}$	m_p	$1.672\,621 \times 10^{-27}$ kg	938.272 MeV$/c^2$
N_A	$6.022\,141\,5 \times 10^{23}$/mol		m_n	$1.674\,927 \times 10^{-27}$ kg	939.566 MeV$/c^2$
k_B	$1.380\,650\,5 \times 10^{-23}$ J/K	$8.617\,343 \times 10^{-5}$ eV/K	m_W	$1.433\,3 \times 10^{-25}$ kg	80.403 GeV$/c^2$
θ_w	$(28.74 \pm 0.01)^\circ$	("weak" mixing angle, θ_w)	m_Z	$1.625\,57 \times 10^{-25}$ kg	$91.187\,6$ GeV$/c^2$
δ_{13}	$(1.20 \pm 0.08)^\circ$	(the CKM matrix phase, δ_{13})	m_H	2.244×10^{-25} kg	125.9 GeV$/c^2$

Table C.3 Some useful abbreviations and numerical values

α_e	$\frac{e^2}{4\pi\epsilon_0\,\hbar c} = \frac{g_e^2}{4\pi}$	$\frac{1}{137.035\,999}$	fine structure constant
r_e	$\frac{e^2}{4\pi\epsilon_0 m_e\, c^2}$	$2.817\,940\,325 \times 10^{-15}$ m	classical electron radius
Ry	$\frac{m_e\, e^4}{2(4\pi\epsilon_0)^2\hbar^2} = \frac{\alpha_e}{2} m_e c^2$	$13.605\,692\,2$ eV	Rydberg, H-atom ion. energy
$\bar{\lambda}_e$	$\frac{\hbar}{m_e c} = \frac{r_e}{\alpha_e}$	$3.861\,592\,678 \times 10^{-13}$ m	Compton electron wavelength
μ_B	$\frac{e\hbar}{2m_e}$	$5.788\,381\,804 \times 10^{-11}$ MeV/T	Bohr magneton
a_0	$\frac{4\pi\epsilon_0\hbar^2}{m_e e^2} = \frac{\hbar}{\alpha_e m_e c} = \frac{r_e}{\alpha_e^2}$	$5.291\,772\,108 \times 10^{-11}$ m	Bohr radius

Other electromagnetic units (farad, tesla, volt, ampere, etc.) are expressed in terms of N, m, s, C. The unit C and the constants ϵ_0 and μ_0 may be eliminated by using the relation $c = 1/\sqrt{\epsilon_0\mu_0}$, and by redefining the electric charge $q \to q/\sqrt{4\pi\epsilon_0}$, which then is expressed in purely "mechanical" units. In general, note that precisely three base units are required in any system of units, and it is merely a tradition to choose units of mass, length and time.

Alternatively, as practiced in fundamental physics, one chooses a unit of speed (c), a unit of the Hamilton action or angular momentum ($\hbar$) and a unit of the gravitational force per product of the gravitating masses times the square of the distance between them (G_N). In addition to adopting this choice, the first two of these units are not even written in high energy particle physics practice, which is often phrased by stating (somewhat confusingly) that "$\hbar = 1 = c$." Every physical quantity is now expressible in terms (and units) of, say, energy – which is convenient in particle physics, since energy is in most cases the measured and controlled quantity [☞ Table 1.2 on p. 25]; Table C.5 could be helpful in this.

This practice is in fact no different than if one chose to adhere to a limited version of the SI system of units where (**1**) all distances are expressed in meters and all masses in kilograms, (**2**) no derivative units are ever used, and (**3**) one agrees to not even write the powers of 'm' and 'kg.' Every physical quantity would then be expressed in terms of time, and measured in units of suitable powers of seconds. In this system, length, mass and volume-specific mass (density) would have no written dimensions, speed and linear momentum would be measured in s^{-1} alike, while s^{-2} would be the appropriate (written) unit for acceleration, force and energy.

The ultimately natural (and parsimonious) unit system is then the one attributed to Planck, in which the natural constants c, $\hbar$ and G_N are implied but never written. This results, for example,

Table C.4 Comparative listing of primary (mechanical) SI units, minimally extended by the unit of electric charge, coulomb (C), and the dimensions of some oft-used electromagnetic quantities

	ϵ_0	$\vec{E}, F_{\mu\nu}$	Φ, A_μ	ρ_e	$\vec{\jmath}_e$	μ_0	$\vec{B}$	$\vec{A}$	ρ_m	$\vec{\jmath}_m$
Primary SI units	$\frac{s^2\,C^2}{kg\,m^3}$	$\frac{kg\,m}{s^2\,C}$	$\frac{kg\,m^2}{s^2\,C}$	$\frac{C}{m^3}$	$\frac{C}{s\,m^2}$	$\frac{kg\,m}{C^2}$	$\frac{kg}{s\,C}$	$\frac{kg\,m}{s\,C}$	$\frac{C}{s^2\,m}$	$\frac{C}{s^3}$
SI units (kg→N s^2/m)	$\frac{C^2}{N\,m^2}$	$\frac{N}{C}$	$\frac{N\,m}{C}$	$\frac{C}{m^3}$	$\frac{C}{s\,m^2}$	$\frac{N\,s^2}{C^2}$	$\frac{N\,s}{m\,C}$	$\frac{N\,s}{C}$	$\frac{C}{s^2\,m}$	$\frac{C}{s^3}$
Dimensions	$\frac{T^2\,C^2}{M\,L^3}$	$\frac{M\,L}{T^2\,C}$	$\frac{M\,L^2}{T^2\,C}$	$\frac{C}{L^3}$	$\frac{C}{T\,L^2}$	$\frac{M\,L}{C^2}$	$\frac{M}{T\,C}$	$\frac{M\,L}{T\,C}$	$\frac{C}{T^2\,L}$	$\frac{C}{T^3}$

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Table C.5 Dimensions of some oft-used physical quantities, in the general $M^x L^y T^z$ format (first row), and the power-of-energy (particle physics) convention where $\hbar$ and c are implied and unwritten units (second row); e.g., $[\mathscr{L}] = 4$ means $[\mathscr{L}] = \text{MeV}^4$ up to powers of $\hbar$ and c

Basic units			In Lagrangian densities						Feynman calculus			
c	$\hbar$	G_N	$\mathscr{L}^a$	ϕ	$\mathbb{A}_\mu$	$\mathbb{F}_{\mu\nu}$	$\mathbb{J}_\mu$	$(\overline{\Psi}\,\Psi)$	$(\overline{u}\,u)$	$\mathfrak{M}$	Γ	σ
$\frac{L}{T}$	$\frac{ML^2}{T}$	$\frac{L^3}{MT^2}$	$\frac{M}{L^2T}$	$\frac{M^{1/2}}{T^{1/2}}$	$\frac{ML^2}{T^2}$	$\frac{ML}{T^2}$	$\frac{M}{T^2}$	$\frac{T}{L^4}$	$\frac{ML}{T}$	—	$\frac{1}{T}$	L^2
0	0	2	4	1	1	2	3	3	1	0	1	−2

[a] Relativistic Lagrangian densities $\mathscr{L}$ are normalized so that $[\int \mathrm{d}^4\mathrm{x}\,\mathscr{L}] = [\hbar]$, with $x^0 = ct$ and $[\mathrm{d}^4\mathrm{x}] = [L^4]$. Similarly, $[\int \mathrm{d}^4\mathrm{x}\,\overline{\Psi} m c^2 \Psi] = [\hbar]$, and Feynman calculus uses $u \propto \sqrt{\hbar c^3}\int \mathrm{d}t\, e^{-i\omega t}\Psi(\mathrm{x})$; see also equation (5.53).

Table C.6 Natural (Planck) units and their SI equivalent value

Name	Expression	SI equivalent	Practical equivalent
Length	$\ell_P = \sqrt{\frac{\hbar G_N}{c^3}}$	$1.616\,25 \times 10^{-35}$ m	
Mass	$M_P = \sqrt{\frac{\hbar c}{G_N}}$	$2.176\,44 \times 10^{-8}$ kg	$1.220\,86 \times 10^{19}$ GeV/c^2
Time	$t_P = \sqrt{\frac{\hbar G_N}{c^5}}$	$5.391\,24 \times 10^{-44}$ s	
Charge[a]	$q_P = \sqrt{4\pi\epsilon_0\,\hbar c}$	$1.875\,55 \times 10^{-18}$ C	$e\sqrt{\alpha_e} \approx 11.706\,2\,e$
Temperature	$T_P = \frac{1}{k_B} M_P c^2$	$1.416\,79 \times 10^{32}$ K	

[a] $\alpha_e \approx 1/137.035\,999\,679$ in low-energy scattering experiments, but grows to about 1/127 near ∼200 GeV energies [☞ Section 5.3.3].

in the units for physical quantities that are listed in Table C.6 on p. 528, and the Reader is invited to compute many more along the lines of the computations practiced in Section 1.2. Notice, however, that once all physical quantities are expressed in units of $\hbar, c, G_N$ – which are not written explicitly – all physical quantities appear to have no (written) dimensions/units! Note that the Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/K is clearly simply a unit conversion factor, from temperature to energy, and need be written only if one wishes to emphasize the statistical nature of a certain quoted energy (temperature).

Table C.7 lists a few symbols used in this book, many of which are fairly standard in formal logic and set theory, but are not as frequently used in the physics literature. The symbols: $\propto$ ("proportional"), $\cong$ ("isomorphic"), $\simeq$ ("equivalent"), $\approx$ ("approximate," but "homomorphic" for groups and algebras), $\sim$ ("asymptotic" for functions, but "of the order of" for numbers), $\times$ (Cartesian or direct product, but "vector product" for 3-vectors and the usual product of a decimal number and a power of ten), $\otimes$ (Kronecker, i.e., tensor product), $\ltimes$ (semidirect product), $\hookrightarrow$ (injection), $\twoheadrightarrow$ (surjection) and $\mapsto$ ("maps/assigns to") are probably more familiar, but are listed here for completeness; see also the lexicon of jargon in Section B.1.

Finally, Table C.8 lists symbols that have been constructed for their specific indicated purpose in this book, and which to the best of my knowledge do not appear elsewhere in the literature.

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Table C.7 Symbols borrowed from formal logic and set theory

Symbol	**Meaning of the symbol as used in this book**
$\subset$	"subset"; e.g., "$A \subset B$" means "A is a subset[a] of B"
$\subsetneq$	"proper subset"; e.g., "$A \subsetneq B$" means "A is a subset[a] of B and $A \neq B$"
$\cup$	"union"; an element belongs to $A \cup B$ if it belongs to A or B (inclusively)
$\cap$	"intersection"; an element belongs to $A \cap B$ if it belongs to *both* A and B
$\smallsetminus$	"minus"; an element belongs to $A \smallsetminus B$ if it belongs to A but not to B
$\in$	"in" or "is an element of"; e.g., "$x \in X$" means "x is an element of X"
$\varnothing$	"empty set", i.e., the formal set that has no element at all
$\forall$	"for all"; e.g., "$\forall x$" means "for every x"
$\exists$	"exists"; e.g., "$\exists x$" means "there exists an x"
$\Rightarrow$	"implies"; e.g., "$x \Rightarrow y$" means "x implies y" (said of claims x, y)
$\Leftrightarrow$	"is equivalent"; e.g., "$x \Leftrightarrow y$" means "x is equivalent to y" (said of claims x, y)

[a] If B has a structure (of an algebra, a group, ...), A inherits this structure from B – unless noted otherwise.

Table C.8 The definition of some less frequently used or here constructed mathematical symbols

Symbol	**Meaning of the symbol as used in this book**
$:=$	the left-hand symbol is defined to equal the right-hand expression
$=:$	the previously undefined right-hand symbol is defined so as to make the equality hold for all values of the remaining symbols
$:\simeq$	the left-hand symbol is defined to be equivalent (by an implicit equivalence, such as integration by parts) to the right-hand expression
$\mathrel{+\!\!\!\!=}$	need not be equal – in distinction to the "(certainly) not equal" symbol, $\neq$
$\overset{!}{=}$	required to be equal
$\overset{\text{“...”}}{=}$	equals, owing to (by use of) the relation/property "$\cdots$"
$\niplus$	semidirect sum of two algebras $\mathfrak{a} \niplus \mathfrak{b}$, the first summand maps $\mathfrak{a} : \mathfrak{b} \to \mathfrak{b}$; e.g., for Lie algebras, $[a, b] \in \mathfrak{b}$, for $a \in \mathfrak{a}$ and $b \in \mathfrak{b}$.
$\wedge$	antisymmetric product of two forms [☞ Digression 5.8 on p. 184]

C.3 Answers to some exercises

A successful solving of the end-of-section exercises should confirm the understanding of the material of that section. For assistance and orientation, some partial and final results to these exercises are listed here.

Ex. 1.2.1 and 1.2.3 Admittedly, these are trick exercises. Let a standing person's *horizontal* linear dimensions be scaled down by a factor of λ_h while the vertical measurements scale by λ_v, and let W denote the person's weight, A the cross-section area of the bones in the legs (femur, tibia, fibula, etc.) and $P = \frac{W}{A}$ the pressure of the person's own weight on these bones. Then,

$$W \propto \lambda_v \cdot \lambda_h^2, \quad A \propto \lambda_h^2, \quad P \propto \lambda_v, \tag{C.1}$$

so that the vertical pressure in the bones is, in this rough estimate, independent of the horizontal scaling factor and only depends on the vertical scaling factor. Therefore, in part 1 of this exercise, for this pressure to be about the same as in ordinary humans, $\lambda_v \sim 1$ and not $\lambda_v = 40$ as stated.

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This then implies that, in Lilliputians and small animals, the structure and even chemical composition of bones may be proportionally weaker than in ordinary humans. In turn, in animals larger than humans, bones must support greater pressures than in ordinary humans. Since the structure and chemical composition of bones cannot vary too much, this provides a strong limitation on the height of land-dwelling animals. Sorry: there can exist no 25-foot, 20-ton gorillas.

Ex. 1.2.6 The principal quantum number n becomes continuous.

Ex. 2.4.1 $\triangle y(\ell) = \frac{1}{2}\frac{q}{m}\,\ell^2\,\frac{B_0{}^2}{E_0}$. $\triangle z = 0$.

Ex. 3.2.4 $T_2 - T_1 = (m_1 - m_2)(1 - \frac{m_1+m_2}{M})c^2$, so that $T_2 - T_1 = \frac{m_1}{M}(M - m_1)c^2$ when $m_2 = 0$.

Ex. 4.2.1 With only the orthonormal states $|a\rangle$ and $|b\rangle$ given, eigenstates must be of the form $\alpha|a\rangle + \beta|b\rangle$. Then $P\big[\alpha|a\rangle + \beta|b\rangle\big] = \pi_P\big[\alpha|a\rangle + \beta|b\rangle\big]$, where π_P is the eigenvalue, so that

$$\pi_P\big[\alpha|a\rangle + \beta|b\rangle\big] = P\big[\alpha|a\rangle + \beta|b\rangle\big] = \big[\alpha|b\rangle + \beta|a\rangle\big]. \tag{C.2}$$

Projecting with $\langle a|$ and $\langle b|$ yields

$$\pi_P\,\alpha = \beta, \quad \pi_P\,\beta = \alpha, \qquad \Rightarrow \quad \pi_P{}^2 = 1, \;\; \pi_P = \pm 1. \tag{C.3}$$

From that,

$$\pi_P = +1, \qquad |+\rangle := \tfrac{1}{\sqrt{2}}\big(|a\rangle + |b\rangle\big), \qquad P|+\rangle = (+1)|+\rangle; \tag{C.4}$$

$$\pi_P = -1, \qquad |-\rangle := \tfrac{1}{\sqrt{2}}\big(|a\rangle - |b\rangle\big), \qquad P|-\rangle = (-1)|-\rangle. \tag{C.5}$$

Ex. 5.3.2 Using the relations from Digression 5.9 on p. 191, we have

$$\begin{aligned}
\partial_\alpha \frac{\partial \mathscr{L}_{QED}}{\partial(\partial_\alpha A_\beta)} &= \partial_\alpha \frac{\partial}{\partial(\partial_\alpha A_\beta)}\Big[-\tfrac{4\pi\epsilon_0}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)\eta^{\mu\rho}\eta^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)\Big]\\
&= -\tfrac{4\pi\epsilon_0}{4}\partial_\alpha\Big[(\delta^{\alpha\beta}_{\mu\nu} - \delta^{\alpha\beta}_{\nu\mu})\eta^{\mu\rho}\eta^{\nu\sigma}(\partial_\rho A_\sigma - \partial_\sigma A_\rho)\\
&\qquad\qquad + (\partial_\mu A_\nu - \partial_\nu A_\mu)\eta^{\mu\rho}\eta^{\nu\sigma}(\delta^{\alpha\beta}_{\rho\sigma} - \delta^{\alpha\beta}_{\sigma\rho})\Big]\\
&= -\tfrac{4\pi\epsilon_0}{4}\partial_\alpha\Big[(\delta^{\alpha\beta}_{\mu\nu} - \delta^{\alpha\beta}_{\nu\mu})(\partial^\mu A^\nu - \partial^\nu A^\mu) + (\partial^\rho A^\sigma - \partial^\sigma A^\rho)(\delta^{\alpha\beta}_{\rho\sigma} - \delta^{\alpha\beta}_{\sigma\rho})\Big]\\
&= -\tfrac{4\pi\epsilon_0}{4}\partial_\alpha\Big[(\delta^{\alpha\beta}_{\mu\nu} - \delta^{\alpha\beta}_{\nu\mu})F^{\mu\nu} + F^{\rho\sigma}(\delta^{\alpha\beta}_{\rho\sigma} - \delta^{\alpha\beta}_{\sigma\rho})\Big]\\
&= -\tfrac{4\pi\epsilon_0}{4}\partial_\alpha\Big[F^{\alpha\beta} - F^{\beta\alpha} + F^{\alpha\beta} - F^{\beta\alpha}\Big] = -4\pi\epsilon_0\partial_\alpha F^{\alpha\beta}.
\end{aligned} \tag{C.6}$$

Similarly,

$$\begin{aligned}
\frac{\partial \mathscr{L}_{QED}}{\partial A_\beta} &= \frac{\partial}{\partial A_\beta}\Big[-\overline{\Psi}(\mathrm{x})\,\Big[i\boldsymbol{\gamma}^\mu\big(\hbar c\,\partial_\mu - iq_\Psi A_\mu\big) - mc^2\Big]\,\Psi(\mathrm{x})\Big]\\
&= -\overline{\Psi}(\mathrm{x})\,\Big[i\boldsymbol{\gamma}^\mu\big(-iq_\Psi\delta^\beta_\mu\big)\Big]\,\Psi(\mathrm{x}) = -q_\Psi\overline{\Psi}(\mathrm{x})\boldsymbol{\gamma}^\beta\Psi(\mathrm{x}).
\end{aligned} \tag{C.7}$$

The relation (5.120f) follows upon equating these two results.

Ex. 5.4.4 Using definition $m_i := z_i M$, the property $\delta(ax) = \delta(x)/a$ and that $x_i = x/z_i$ yields

$$W_1^i = \frac{Q_i^2}{2(Mz_i)}\,\delta\Big(\frac{x}{z_i} - 1\Big) = \frac{Q_i^2}{2M}\,\delta\Big(z_i\frac{x}{z_i} - z_i\Big) = \frac{Q_i^2}{2M}\,\delta(x - z_i). \tag{C.8}$$

Also, using that $\delta(x-1) = x^2\delta(x-1)$ yields

$$\begin{aligned}
W_2^i &= -\frac{2m_i c^2 Q_i^2}{\mathrm{q}^2}\,x_i{}^2\delta\Big(\frac{x}{z_i} - 1\Big) = -\frac{2(Mz_i)c^2 Q_i^2}{\mathrm{q}^2}\,x_i{}^2 z_i\,\delta(x - z_i)\\
&= -\frac{2Mc^2 Q_i^2}{\mathrm{q}^2}\,x^2\delta(x - z_i).
\end{aligned} \tag{C.9}$$

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Ex. 6.1.3 Write the equation $\partial_\mu F^{a\,\mu\nu} = J^{a\,\nu}_{(c)}$ in matrix notation, $\partial_\mu \mathbb{F}^{\mu\nu} = \mathbb{J}^{\nu}_{(c)}$, where we also have equation (6.16), $\mathbb{F}'_{\mu\nu} = U_{\boldsymbol{\varphi}}\,\mathbb{F}_{\mu\nu}\,U_{\boldsymbol{\varphi}}^{-1}$. It then follows that

$$\partial_\mu \mathbb{F}'^{\mu\nu} = \partial_m \big(U_{\boldsymbol{\varphi}}\,\mathbb{F}^{\mu\nu}\,U_{\boldsymbol{\varphi}}^{-1}\big) \tag{C.10}$$

$$= (\partial_\mu U_{\boldsymbol{\varphi}})\mathbb{F}^{\mu\nu}\,U_{\boldsymbol{\varphi}}^{-1} + U_{\boldsymbol{\varphi}}(\partial_\mu \mathbb{F}^{\mu\nu})U_{\boldsymbol{\varphi}}^{-1} + U_{\boldsymbol{\varphi}}\,\mathbb{F}^{\mu\nu}(\partial_\mu U_{\boldsymbol{\varphi}}^{-1}). \tag{C.11}$$

To simplify this result, use that $\mathbb{1} = U_{\boldsymbol{\varphi}} U_{\boldsymbol{\varphi}}^{-1}$, the derivative of which gives

$$0 = (\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1} + U_{\boldsymbol{\varphi}}(\partial_\mu U_{\boldsymbol{\varphi}}^{-1}) \qquad \Rightarrow \qquad (\partial_\mu U_{\boldsymbol{\varphi}}^{-1}) = -U_{\boldsymbol{\varphi}}^{-1}(\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}. \tag{C.12}$$

Combining, we have

$$\begin{aligned}
\partial_\mu \mathbb{F}'^{\mu\nu} &= (\partial_\mu U_{\boldsymbol{\varphi}})\mathbb{F}^{\mu\nu}\,U_{\boldsymbol{\varphi}}^{-1} + U_{\boldsymbol{\varphi}}(\partial_\mu \mathbb{F}^{\mu\nu})U_{\boldsymbol{\varphi}}^{-1} - U_{\boldsymbol{\varphi}}\,\mathbb{F}^{\mu\nu}U_{\boldsymbol{\varphi}}^{-1}(\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}\\
&= (\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}(U\,\mathbb{F}^{\mu\nu}\,U_{\boldsymbol{\varphi}}^{-1}) + (U_{\boldsymbol{\varphi}}\,\mathbb{J}^{\nu}_{(c)}\,U_{\boldsymbol{\varphi}}^{-1}) - (U_{\boldsymbol{\varphi}}\,\mathbb{F}^{\mu\nu}U_{\boldsymbol{\varphi}}^{-1})(\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}\\
&= \mathbb{J}'^{\,\nu}_{(c)} + (\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}\,\mathbb{F}'^{\,\mu\nu} - \mathbb{F}'^{\,\mu\nu}(\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}\\
&= \mathbb{J}'^{\,\nu}_{(c)} + \big[\,(\partial_\mu U_{\boldsymbol{\varphi}})U_{\boldsymbol{\varphi}}^{-1}\,,\,\mathbb{F}'^{\,\mu\nu}\,\big],
\end{aligned} \tag{C.13}$$

the form of which could have been guessed from relations (6.39) and (6.6c).

Ex. 7.1.2 Motivated by the form of the result to be proven, use the polar coordinates $\phi_1 = \varrho\cos\theta$, $\phi_2 = \varrho\sin\theta$, where the potential density in the Lagrangian density (7.21) becomes

$$\mathscr{V} = -\tfrac{1}{2}\big(\tfrac{mc}{\hbar}\big)^2\varrho^2 + \tfrac{1}{4}\lambda\varrho^4, \tag{C.14}$$

so that the stationary values of the variable ϱ are given by

$$-\big(\tfrac{mc}{\hbar}\big)^2\varrho + \lambda\varrho^3 = 0 \qquad \Rightarrow \qquad \partial_0 = 0,\ \ \varrho_\pm = \pm\tfrac{mc}{\hbar\sqrt{\lambda}}. \tag{C.15}$$

It is not hard to prove that $\varrho_0 = 0$ is a maximum, and $\varrho_+ = \frac{mc}{\hbar\sqrt{\lambda}}$ a minimum; the third solution, $\varrho_- = -\frac{mc}{\hbar\sqrt{\lambda}}$, is unreasonable as a value for the radial polar coordinate. The desired result follows by transforming back into Cartesian parametrization, (ϕ_1, ϕ_2).

Ex. 9.1.4 In the extended equality (9.14) only the last one is not evident, and follows from the fact that

$$g_{\mu\nu}\,g^{\mu\nu} = 4 \quad \overset{\delta}{\Longrightarrow} \quad \delta(g_{\mu\nu}\,g^{\mu\nu}) = 0 \quad \Rightarrow \quad (\delta g_{\mu\nu})g^{\mu\nu} = -g_{\mu\nu}(\delta g^{\mu\nu}). \tag{C.16}$$

With no extra effort, we also have the general result:

$$g_{\mu\nu}\,g^{\mu\sigma} = \delta^\sigma_\nu \quad \overset{\delta}{\Longrightarrow} \quad \delta(g_{\mu\nu}\,g^{\mu\sigma}) = 0 \quad \Rightarrow \quad (\delta g_{\mu\nu})g^{\mu\sigma} = -g_{\mu\nu}(\delta g^{\mu\sigma}). \tag{C.17}$$

Contracting this last equality with $g^{\rho\nu}$ yields [☞ also Digression 9.3 on p. 329]

$$\delta g^{\rho\sigma} = -g^{\rho\nu}(\delta g_{\mu\nu})g^{\mu\sigma}. \tag{C.18}$$

Ex. 10.3.1 Direct computation yields

$$\begin{aligned}
\mathrm{Tr}\,\big[\{Q_i, Q^{\dagger j}\}\big] &= \tfrac{1}{2}\sum_i \{Q_i, Q^{\dagger i}\} + \tfrac{1}{2}\sum_i \{Q^{\dagger i}, Q_i\}\\
&= \tfrac{1}{2}\sum_i \underbrace{\{Q_i, Q_i\}}_{\equiv 0} + \tfrac{1}{2}\sum_i \{Q_i, Q^{\dagger i}\} + \tfrac{1}{2}\sum_i \{Q^{\dagger i}, Q_i\} + \tfrac{1}{2}\sum_i \underbrace{\{Q^{\dagger i}, Q^{\dagger i}\}}_{\equiv 0}
\end{aligned}$$

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$$
\begin{aligned}
&= \tfrac{1}{2}\sum_i \{\mathsf{Q}_i + \mathsf{Q}^{\dagger\, i}, \mathsf{Q}_i + \mathsf{Q}^{\dagger\, i}\}] \overset{(10.32a)}{=} \tfrac{1}{2}\sum_i \{\mathcal{Q}_i, \mathcal{Q}_i\} = \sum_i \mathcal{Q}_i \mathcal{Q}_i \\
&= \sum_i |\mathcal{Q}_i|^2 \geqslant 0,
\end{aligned} \tag{C.19}
$$

where $\mathcal{Q}_i \mathcal{Q}_i = |\mathcal{Q}_i|^2$ as the operators $\mathcal{Q}_i$ are Hermitian.

Ex. 11.3.1 The Ricci tensor is

$$
[R_{mn}] = \begin{bmatrix} -2e^{-2k|y|}[k\ \mathrm{sig}^2(y) - \delta(y)] & 0 & 0 \\ 0 & 2e^{-2k|y|}[k\ \mathrm{sig}^2(y) - \delta(y)] & 0 \\ 0 & & 2k[\delta(y) - k\ \mathrm{sig}^2(y)] \end{bmatrix}, \tag{C.20}
$$

and the scalar curvature is $R = 2\,k[4\,\delta(y) - 3\,k\ \mathrm{sig}^2(y)]$.

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