existence results and show no interest in methods of calculating the functions whose existence is asserted. They do achieve thereby some very short and slick proofs, but I question whether this is a worthwhile gain. Despite this, I unhesitatingly recommend this work of great scholarship in a fundamental and still active branch of analysis.

N. J. YOUNG

WEEKS, J. R., *The shape of space: how to visualize surfaces and three-dimensional manifolds* (Pure and Applied Mathematics Vol. 96, Marcel Dekker, New York, 1985), x + 324 pp., $59.50.

In this fascinating book the author's aim is to give the reader an intuitive understanding of three-dimensional manifolds with particular reference to the geometry and shape of the universe. It is also his hope that the book will be accessible to the "interested non-mathematician" though, recognising the dearth of nontechnical accounts of these topics, he suggests that mathematicians may also find it a useful general introduction. How accessible the book will be to the wider audience is perhaps a matter of definition but certainly all mathematicians will get from it a clear, nontechnical and intuitive exposition of current developments in the topology and geometry of 3-manifolds.

The author's starting point is E. A. Abbott's classic *Flatland: A Romance of Many Dimensions* first published in 1884. From there he proceeds to describe the construction of surfaces and 3-manifolds by glueing and takes care to distinguish between their topology and geometry. This occupies the first two-thirds of the book and this part ends with an account of the Gauss–Bonnet formula for surfaces with constant curvature. The final part starts by producing examples of 3-manifolds that admit elliptic, Euclidean or hyperbolic geometry and then contains a description of the eight homogeneous geometries that can arise on closed 3-manifolds. In the chapter the author explores possible geometries and global topologies for the universe.

The book is well supplied with clearly drawn diagrams and contains an instructive collection of exercises. At the end there is a useful set of solutions to the exercises as well as a bibliography. The latter refers the reader to a number of the standard texts for which the author has written an admirable complement.

R. M. F. MOSS

GROSSWALD, E., *Representations of integers as sums of squares* (Springer-Verlag, 1985) 251 pp., DM 148.

The study of the representations of a number as a sum of squares has a long history. Diophantus concerned himself with several problems of this type sixteen hundred years ago, but it was only towards the end of the seventeenth century that notable advances were made and valid proofs published. For a positive integer k denote by $r_k(n)$ the number of representations of the non-negative integer n as a sum of k squares of integers; thus $r_k(n)$ is the number of solutions of the Diophantine equation

$$x_1^2+x_2^2+\cdots+x_k^2=n \quad (x_i\in\mathbb{Z},\ 1\leq i\leq k). \tag{1}$$

The order of the x_i is taken into account so that, for example, $r_2(1)=4$, the solutions being $(x_1,x_2)=(\pm 1,0)$ and $(0,\pm 1)$. In his treatment the author follows the historical development of the subject, beginning with the case $k=2$.

Euler proved that, for $k=2$, (1) is soluble if and only if each prime divisor p of n, for which $p\equiv 3\,(\mathrm{mod}\,4)$, occurs in n to an even power. Later, the formula

$$r_2(n)=4\left\{\sum_{\substack{d\mid n\\ d\equiv 1\,(\mathrm{mod}\,4)}}1-\sum_{\substack{d\mid n\\ d\equiv 3\,(\mathrm{mod}\,4)}}1\right\}$$