EFFECT OF MASS GAIN ON STELLAR EVOLUTION

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ABSTRACT

In this paper we present a fully hydrodynamical treatment of the stationary isothermal accretion problem onto a moving gravitating point mass. The derivation is purely analytical. We find that the accretion rate is more than a factor of 50 higher than the accretion rate derived from the partially non-hydrodynamical treatment by Hoyle and Lyttleton (1939) or Bondi and Hoyle (1944). This result may have some bearing on the evolutionary tracks of young pre-Main Sequence stars still embedded in their parent protocluster cloud. We discuss the work by Federova (1979) who investigated the pre-Main Sequence evolution of degenerate low mass 'stars' with strong accretion of protocluster cloud material. We suggest that the stars which lie below the Main Sequence in young clusters could strongly accrete matter at the pre-Main Sequence stage.

Also we suggest that the observed lack of low mass stars in open galactic clusters (van den Bergh 1961) compared to the field may be due to the accretion of residual gas preferentially by low mass stars.

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I. INTRODUCTION

The following contribution is concerned with something that seems to be the contrary of the topic of this conference: instead of the effect of mass loss on stellar evolution, we shall be dealing with the effect of mass gain on stellar evolution.

To begin with, we give the classical accretion rate onto a moving stellar object which was first derived by Hoyle and Lyttleton (1939) and by Bondi and Hoyle (1944):

$$\dot{M} \approx 4\pi(GM)^2 \rho_\infty / v_\infty^3 \approx 10^{-11} (M/M_\odot)^2 \; n_\infty / v_\infty^3 (\mathrm{km/s}) \qquad \left[M_\odot/\mathrm{yr}\right] \qquad (1)$$

Here M is the mass of the stellar object, and G denotes the gravitational constant; v_∞ is the relative velocity of the body with respect to the gas at infinity; ρ_∞ is the mass density of the gas at infinity, while n_∞ is the number density of hydrogen atoms plus hydrogen molecules at infinity. For a given set of parameters (M, v_∞, n_∞) the accretion rate - expressed in units of solar masses per year - can be read off from the above formula.

Note that if the stellar object were at rest with respect to its surrounding gas, we would have to replace the quantity v_∞ by the isothermal sound speed c_s (Bondi 1952, Ebert 1952), which is ~0.2km/s for a molecular hydrogen cloud at temperature T = 10 K. McCrea (1953) has used formula (1) with v_∞ replaced by c_s to predict that massive O-stars may form by accretion from solar mass stars penetrating into a dense interstellar cloud after being braked by accretional drag. Now, accretion onto stars is commonly thought to be unimportant (e.g. Spitzer 1978). The object is usually surrounded by a low density gas ($n_\infty \sim 1\mathrm{cm}^{-3}$) and/or is usually moving with a rather high velocity relative to the gas ($v_\infty \sim$ 10km/s); hence the accretion rate is usually insignificant.

However, there is one exceptional situation and that is young low mass stars embedded in their dense parent protocluster cloud (as an example we refer to the Taurus/Auriga dark cloud complex). This situation persists for a long time ($\sim 10^7$ yrs, Bash et al. 1977), and the relative velocities are small in this case (~1-2 km/s, Herbig 1977)! Thus an accretion rate of 10^{-7} $M_\odot$/yr for solar mass objects in a cloud with a hydrogen number density $\sim 10^4 \mathrm{cm}^{-3}$ may result. This means that a young solar mass object could double its mass in 10^7 years, if no other effects like a stellar wind prevents it from accreting from its dense gaseous environment. (Fahr 1980 has recently given a review on some aspects of the interaction between stars and their dense gaseous environment, excluding the present one.) Therefore, we might expect significant changes in the evolutionary tracks of young low mass stars on their way to the Main Sequence (in comparison with 'isolated evolution').

In a qualitative discussion the importance of mass gain from the 'mother cloud' on the early evolution of low mass stars has been recognized already by Castellani and Panagia (1972), who called the process 'self-accretion'.

II. RECENT RUSSIAN WORK ON PRE-MS-EVOLUTION WITH ACCRETION

In a recent Russian publication A.V. Federova (1979) investigates quantitatively the pre-Main Sequence evolution of degenerate low mass stars (cf. Kumar 1963) with strong accretion of protocluster cloud material and suggested that stars which lie below the Main Sequence in young clusters could strongly accrete matter at the pre-Main Sequence stage. She assumes $\dot{M} = 10^{-7}$ $M_{\odot}$/yr for an object with an initial mass $M = 0.05$ $M_{\odot}$. The final mass of the object is taken to be 0.5 $M_{\odot}$ (Case A) or 1.0 $M_{\odot}$ (Case B). In view of the result on the accretion rate that we shall derive in the next section we think that such extreme cases may well be possible.

The most interesting outcome of these Russian evolutionary calculations is the following: If the final mass is 0.5 $M_{\odot}$ (Case A), the star after the end of accretion places 1-2 magnitudes below the Zero Age Main Sequence and evolves up to the Main Sequence almost vertically; if the final mass is 1.0 $M_{\odot}$ (Case B) the star after the end of accretion places 0.25 magnitudes below the Zero Age Main Sequence and evolves up almost along it (Fig.1). In both cases, the assumption is made that accretion begins at a relatively late stage of the Kelvin-Helmholtz gravitational contraction.

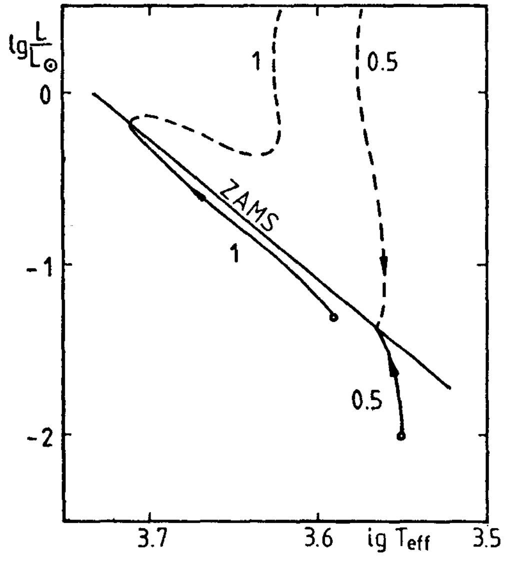

Fig. 1
Hertzsprung-Russell Diagram showing the evolutionary tracks (solid lines) of stars with mass $M = 0.5$ $M_{\odot}$ and $M = 1.0$ $M_{\odot}$ towards the ZAMS after they have stopped accreting dense protocluster cloud gas. In both cases, accretion started onto a point-like degenerate object with mass $M = 0.05$ $M_{\odot}$. Dashed lines indicate the well-known tracks of quasi static pre-MS-evolution for stars with mass $M = 0.5$ $M_{\odot}$ and $M = 1.0$ $M_{\odot}$ without accretion at the pre-MS-stage (from Federova 1979).

III. OUR WORK ON STATIONARY ACCRETION ONTO A MOVING POINT MASS

1. General Remarks And Assumptions

In order to do detailed calculations of the pre-Main Sequence evolution of low mass stars including accretion (such as the ones described in the preceding section), it is necessary to be sure about the correct order of magnitude of the accretion rate.

Use of the classical accretion rate given in equ. (1) must be considered doubtful, since it is based on the approximation of a single gas particle trajectory (as long as the gas particle is far from the downstream accretion axis), i.e. essentially on a non-hydrodynamical treatment (cf. Spiegel 1970) while it is clear that at protocluster cloud densities ($n_\infty \sim 10^4$ cm^{-3}) a hydrodynamical treatment would be appropriate. At these densities the mean free path L_{mfp} for collisions between hydrogen molecules is much smaller than the accretion radius R_{acc}:

$$L_{mfp} \ll R_{acc}, \text{ i.e. } (n_\infty \sigma_{H_2-H_2})^{-1} \ll 2GM/v_\infty^2 \qquad (2)$$

$\sigma_{H_2-H_2}$ is the collisional cross-section between hydrogen molecules which is a few times 10^{-16} cm^2. Thus, the problem is to derive the accretion rate on a fully hydrodynamical basis.

Because of the presence of interstellar dust as an effective cooling agent for the gas which is compressed and heated during the accretion process, an isothermal equation of state has to be adopted. The dust grains are thermally coupled to the gas and convert the compressional heat into infrared radiation for which the gas is optically thin. For a pressure-density relation $p \sim \rho^{5/3}$ and $p \sim \rho^{4/3}$ (adiabatic equation of state and equation of state intermediate between adiabatic and isothermal, respectively) the stationary accretion flow problem onto a gravitating point-like body moving at a constant velocity relative to a gaseous background medium which is uniform at infinity has been solved numerically by Hunt (1971) and by Hunt (1979), respectively. Dodd (1953) has tried a numerical treatment of the stationary isothermal case ($p \sim \rho$). His result for the accretion rate for a finite temperature was in agreement with the result of Hoyle and Lyttleton (1939) and Bondi and Hoyle (1944) for zero temperature. His numerical method, however, suffers from difficulties near the upstream accretion axis.

2. Derivation Of The Accretion Rate For The Isothermal Case

In what follows, we present our purely analytical treatment of the stationary isothermal case which yields an accretion rate much higher than the classical one given in equ. (1). The accretion flow can be assumed to be free of vorticity, hence we put

$$\vec{v} = \text{grad } \Phi \qquad (3)$$

Φ is the scalar velocity potential, $\vec{v}$ is the velocity vector of the flow which possesses rotational symmetry with respect to the direction of the velocity at infinity. The continuity equation reads

$$\mathrm{div}\ (\rho\,\vec{v}) = \rho\ \mathrm{div}\ \vec{v} + \vec{v}\cdot\mathrm{grad}\ \rho \tag{4}$$

which, after dividing by ρ and using equ.(3), can be written as

$$\Delta\,\Phi + (\mathrm{grad}\ \Phi)\cdot(\mathrm{grad}\ \Psi) = 0 \tag{5}$$

where we have introduced

$$\Psi := \ln\ (\rho/\rho_\infty) \tag{6}$$

Δ denotes the Laplacian. In addition to equ.(5) we have

$$(\mathrm{grad}\ \Phi)^2 + 2\ c_s^2\ \Psi = v_\infty^2 + 2\ GM/r \tag{7}$$

which is the Bernoulli equation (integrated Euler equation).
In equ. (7) c_s is the isothermal sound speed, and v_∞ is the relative velocity between the gravitating point mass and the gas at infinity; r denotes the radial distance to the point mass. In order to eliminate the scalar function Ψ from equ.(5), we have to take the gradient of the whole equ.(7) and to insert grad Ψ in equ.(5). In this manner we obtain

$$\Delta\,\Phi - \frac{1}{2c_s^2}\left[(\mathrm{grad}\Phi)\cdot(\mathrm{grad}\ \ (\mathrm{grad}\Phi)^2\ )\right] + \frac{GM}{c_s^2}\left[(\mathrm{grad}\Phi)\cdot(\mathrm{grad}\ \frac{1}{r})\right] = 0 \tag{8}.$$

An approximate solution to this differential equation for Φ in the region defined by $r \ll GM/v_\infty^2$ (near zone) and under the condition $v_\infty^2 > c_s^2$ is given by

$$v(r) = \mathrm{grad}\Phi = -\ (2\ GM/r)^{1/2} \tag{9}.$$

This is easily checked by inserting equ.(9) into equ.(8). The solution for v(r) corresponds to the free-fall solution. From equ.(9) it follows for near zone that

$$\Phi \approx \Phi_1 = -\ (8\ GMr)^{1/2} \tag{10}.$$

On the other hand, for very large distances (far zone)

$$\Phi \approx \Phi_2 = -\ v_\infty\, r\cos\vartheta \tag{11}$$

where ϑ is the inclination of the radius vector to the symmetry axis. Equ.(11) says that the flow is parallel at infinity.

We now assume that in a first approximation the velocity potential may be expressed everywhere by the sum of Φ_1 plus Φ_2, i.e.

$$\Phi = \Phi_1 + \Phi_2 = \{\ (8GMr)^{1/2} + v_\infty\, r\cos\vartheta\ \} \tag{12}.$$

For small r we get $\Phi \to \Phi_1$, for large r we get $\Phi \to \Phi_2$.

Since there is rotational symmetry, it is sufficient to consider the flow in a plane containing the symmetry axis. Each surface Φ = const crosses this plane in a line which is characterized by the equation

$$2\ (2GM)^{1/2}\ r^{1/2} + v_{\infty} r \cos\vartheta = \text{const} \tag{13 a}$$

In rectangular coordinates x, y (symmetry axis : y) equ.(13) reads

$$2\ (x^2+y^2)^{1/4} + \lambda\ y = \text{const} \tag{13 b}$$

where

$$\lambda^2 := \frac{v_{\infty}^2}{2GM} \tag{14}.$$

Now, we have to establish the differential equation for the stream lines in the x - y plane. To this purpose we first take the derivative of equ.(14) with respect to x; we find

$$\frac{dy}{dx} = - \frac{x}{\lambda(x^2+y^2)^{3/4} + y} \tag{15}.$$

Equ.(15) represents the differential equation of the lines of constant velocity potential. The streamlines then follow from the transition to the orthogonal trajectories. This requires replacing dy/dx in equ.(15) by - 1/(dy/dx); thus, the stream lines obey

$$\frac{dy}{dx} = \frac{\lambda(x^2+y^2)^{3/4} + y}{x} \tag{16}$$

Substituting y by the new variable

$$z(x) = y(x)/x \tag{17}$$

equ.(16) transforms into

$$\frac{dz}{dx} = \lambda\ x^{-1/2}\ (1 + z^2)^{3/4} \tag{18}$$

which allows for separation of the variables. Integration yields

$$\int_c^{\infty} \frac{dz}{(1+z^2)^{3/4}} = \lambda \int_0^{x_c} x^{-1/2}\ dx = 2\ \lambda\ x_c^{1/2} \tag{19}.$$

Here x_c is the impact parameter of the stream line which merges into the straight line y = cx close to the origin (point mass); arccot(c) is the angle ϑ which the tangent of the stream line approaches close to the origin (Fig. 2).

For each c, the left-hand side of equ.(19) has a certain value. Thus, for brevity we define

$$\int_c^{\infty} \frac{dz}{(1+z^2)^{3/4}} =: P(c) \tag{20}.$$

Then the critical impact parameter is given by

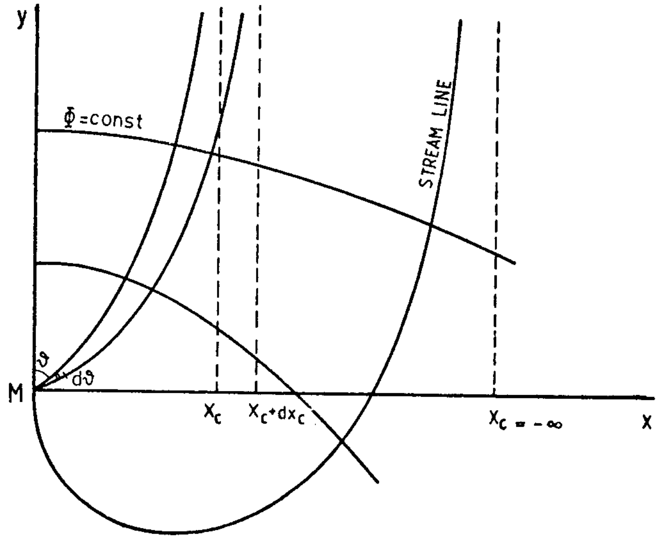

Fig. 2 Lines of constant velocity potential (Φ = const) and stream lines (orthogonal trajectories) in a plane containing the symmetry axis y. $x_{c=-\infty}$ is the critical impact parameter; the corresponding stream line hits the origin M from behind ($\vartheta = 180^o$).

$$x_{c=-\infty} = \frac{P^2(-\infty)}{4\ \lambda^2} \tag{21}$$

Given $x_{c=-\infty}$, the accretion rate is

$$M = \pi\ x^2_{c=-\infty}\ \rho_\infty v_\infty \tag{22}.$$

In order to determine $x_{c=-\infty}$, we must evaluate the elliptical integral $P(-\infty)$. From equ. (20) we immediately prove that

$$P(-\infty) = 2\ P(o) \tag{23}.$$

Introducing $(1 + z^2)^{-1/4} = \cos\alpha$ (24)
we find that

$$P(o) = \sqrt{2}\ F\ (\frac{1}{\sqrt{2}}\ ;\ \frac{\pi}{2}) = \sqrt{2}\ \cdot\ 1.845 \tag{25}$$

where $F(q;\ \beta)$ is defined by

$$F(q;\ \beta) = \int_o^\beta (1 - q^2\sin^2\alpha)^{-1/2} d\alpha \tag{26}$$

(Handbook of Mathematical Functions, page 615, Dover Publ., eds:

Abramowitz & Stegun). According to equ.(22) together with equs. (21), (23), (25), and (14) the final result is

$$\dot{M} \approx 225\ \pi\ (GM)^2\ \rho_\infty / v_\infty^3 \qquad (27).$$

This result applies for high Mach numbers ($v_\infty \gg c_s$) in the isothermal hydrodynamical regime. For low Mach numbers ($v_\infty \ll c_s$) in the same regime we know that

$$\dot{M} \approx 4\ \pi\ (GM)^2\ \rho_\infty / c_s^3 \qquad (28)$$

(Bondi 1952, Ebert 1952), as already stated in the introduction. Thus we may combine the high Mach number (velocity-limited) case [equ.(27)] and the low Mach number (temperature-limited) case [equ.(28)] to give

$$\dot{M} \approx 4\ \pi\ (GM)^2 \rho_\infty \,/\, [c_s^2 + (4/225)^{2/3}\ v_\infty^2\ ]^{3/2} \qquad (29)$$

which reduce correctly to both limits. We stress that our interpolation for the intermediate Mach number case is different from the interpolation conjectured by Bondi (1952), viz.

$$\dot{M} \approx 4\ \pi\ (GM)^2 \rho_\infty \,/ (c_s^2 + v_\infty^2\ )^{3/2} \qquad (30)$$

who combined equ.(28) which is fully based on hydrodynamics with equ.(1) which is only partially based on hydrodynamics.

3. Discussion of the Result

Our expression (27) for the accretion rate is formally the same as the one by Hoyle and Lyttleton (1939) and by Bondi and Hoyle (1944), except that the absolute value is higher by a factor 225/4: see equ.(1). Note that a higher accretion rate implies a higher accretional drag (the drag force is $F_{drag} \sim \dot{M}\ v_\infty$ approximately: cf. Dodd and McCrea 1952). One expects that a fully hydronamical treatment leads to a higher accretion rate than the largely non-hydrodynamical classical treatment. In the latter treatment the gas particles can lose their angular momentum only in a narrow cone behind the point-like object, while in the former treatment loss of angular momentum can take place everywhere through the existence and the action of pressure. The enhancement factor of 225/4 could be even larger, if we take into account that there is probably a downstream shock-front (tail shock) which would slow down the accretion flow and would lead to the capture of still more material. On the other hand, because of the crude ansatz for the total velocity potential in equ.(12), we must not overestimate our result for the accretion rate.

Furthermore, the question arises whether the accretion flow really is a stationary flow, if the gravitating mass grows markedly during accretion. Under this condition, since the mass of the object becomes time-dependent, we expect the flow to be quasi-stationary at most. Nevertheless, this should not decrease the accretion rate; on the contrary, it should make accretion a runaway process until some limiting process like a stellar wind interferes with it (Zinnecker 1980). The runaway behaviour of the accretion process would be even more pronounced, if we had taken into account the effect of the self-gravity of the gas within the accretion radius of the object. This effect is no longer negligible at the high protocluster cloud densities (Chia 1978, 1979).

If numerical calculations of high accuracy will confirm our analytical calculation, the implications would be far reaching - not only for our present problem but also in the context of other astrophysical problems of great interest (e.g. a galaxy moving through the intracluster gas in a cluster of galaxies).

IV CONCLUSIONS

The main conclusion of this paper is as follows: A young pre-Main Sequence stellar object ($M \sim 0.1 - 1.0\ M_\odot$) can increase its mass appreciably by accretion from the gas of its parent protocluster cloud. This is due to the high density of the gas in the cloud ($n_{gas} \sim 10^4\ cm^{-3}$) and the small velocity of the object relative to the gas of the cloud ($v_{rel} \sim 1$ km/s), in which case the accretion rate is very high, viz. $\dot{M} \approx (225/4) \cdot 10^{-7}\ (M/M_\odot)^2$ $[M_\odot/yr]$.

Such a high accretion rate has a tremendous influence on the pre-Main Sequence evolution of the object. Theory predicts that the object does not approach the Zero Age Main Sequence from above but from below (see Fig. 1). Observations seem to confirm this prediction. Penston et al. (1976) did photographic photometry of many stars in the Orion nebula cluster in wavebands which are free from the major nebular emission lines and compiled a (V, V-I) colour-magnitude diagram which reveals the presence of stars with V ~ 13-15 lying below the Zero Age Main Sequence. Penston et al. (1976), therefore, suggested that something is wrong with current models of low mass contracting stars. We conclude that current pre-Main Sequence models are to some extent wrong because they neglect accretion at the pre-Main Sequence stage.

In addition, in view of the high accretion rate that we found from our fully hydrodynamical treatment, we tentatively conclude that the lack of low mass stars in open galactic clusters compared with the luminosity function of the field stars (van den Bergh 1961, Scalo 1978) is due to accretion of residual gas in the cluster onto its low mass stars at the pre-Main Sequence stage. In such a cluster, since it is gravitationally bound, dense residual gas is kept for a long time ($\sim 10^7$ yrs) while this is not true for expanding OB-associations where most of the field stars are assumed to be born (Ebert et al. 1960; Miller and Scalo 1978).

In the end, we attempt to answer the question about the end of accretion onto a pre-Main Sequence object. When does accretion stop? There are two possibilities: either after the dispersal of the cloud or after the onset of a stellar wind. The first possibility is trivial; the second possibility leads into a serious dynamical problem which is unsolved. We speculate (cf. Norman and Silk 1980) that the onset of a stellar wind may be connected with the onset of nuclear burning which provides a new energy source for the star which is available to drive the wind.

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DISCUSSION

CARRASCO: A comment: I strongly disagree with your comment that the effect of premain sequence stellar winds may be neglected, given the fact that in the case of bipolar nebulae, the premain sequence stellar winds will not only prevent background accretion, but will actually blow the surroundings away.

ZINNECKER: I did not say that the effects of the winds may be neglected. I said that a pre main sequence object as long as it is embedded in a dense gaseous environment perhaps does not start a wind (cf. Ripken and Fahr in Mitt. AG 48).

KWOK: It is widely believed that once a star is born in a molecular cloud, radiation pressure on dust will drive all neighboring gas away and create a bubble. Such bubbles are widely observed in the radio as compact HII regions. High velocity gas has also been observed in such regions.

ZINNECKER: This is true for very massive stars, i.e. stars more massive than about 10 $M_{\odot}$; you must not forget, however, that I was talking about very low mass objects.

BASU: Have you checked the effect of viscosity of gas on the accretion rate? Although the temperature is not high, high density close to the star may have significant influence on the accretion rate.

ZINNECKER: In the review article by Fahr (1980) that I have cited the effect of viscosity is discussed.He concludes that the effect is very important (e.g. the mass dependence of the accretion law $\dot{M} \sim M^2$ is altered), but I am not sure about his result.

FRIEDJUNG: You do not need spherical asymmetry, one can have accretion and a wind at the same time. In a study of a very peculiar star of the Large Magellanic Cloud posted here, which certainly has a wind, we found indication of an accretion disk.

ZINNECKER: It is entirely possible that accretion and a wind may coexist. Fully three-dimensional hydrodynamical numerical calculation will be required to treat such a complex situation. Our analytical treatment of the accretion process cannot incorporate the wind.