Multiple Periodic Variable Stars
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RED VARIABLES

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1. Introduction

This paper will be concerned with those variables lying on the cool side of the cepheid strip in the region of the HR diagram occupied by the red giant and supergiant stars. The blue edge of the red variable region for old disk population stars has been found by Eggen(1973b) to be at $(R-I)_K \sim 0.9$ ($\log T_{eff} \sim 3.53$). According to Eggen(1973a,1973b), all giant stars cooler than this vary, although the amplitude is very small ($\Delta V \sim 0.2$ mag) near the blue edge. In the halo population the blue edge is considerably hotter at $(R-I)_K \sim 0.5$ ($\log T_{eff} \sim 3.62$). On the other hand, some small amplitude red variables (eg. R Dor, AK Hya, IQ Her) have been found at the low temperatures ($R_K-I_K > 1.6$) where large amplitude red (Mira) variables exist. From these results, it appears that the position of the blue edge depends on some unknown stellar parameter(s). Auman(1971) has suggested that the helium abundance may be involved.

The RV Tauri variables, which are heavily reddened by circumstellar dust shells (du Puy1973, Gehrz 1972) and possibly lie in the cepheid instability strip, are not included in the survey.

Before looking at the observational data on red variables, we will look at some theoretical results in order to try to understand the basic pulsation mechanism and to see if any secondary periodicities might be expected.

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2. Theory

A distinctive property of highly centrally condensed stars, such as red giants, is the large ratio of the fundamental period P_o to the first overtone period P_1. For a typical red giant star, $P_o/P_1 \approx 3$ to 5, $P_1/P_2 \approx 1.2$ to 1.5 and $P_2/P_3 < 1.2$; these values reoresent the oeriod ratios that may be exoected if two modes are oresent in a single star.

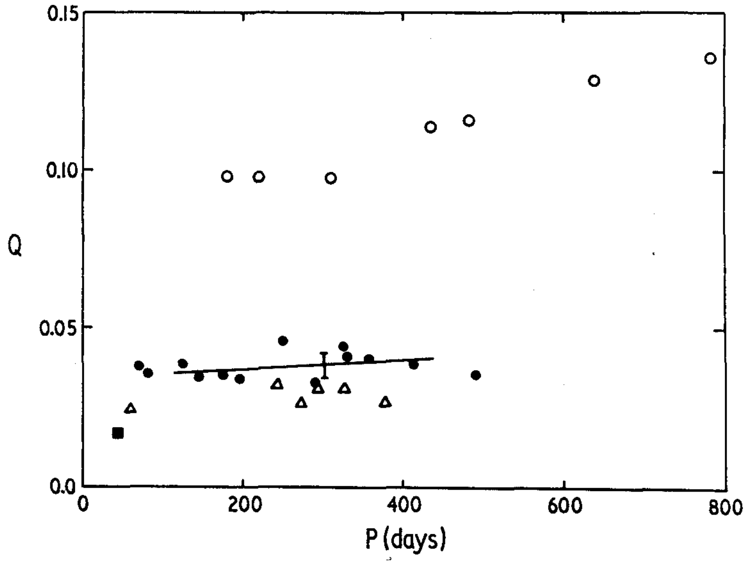

Fig. 1. The pulsation constant Q plotted against period for the fundamental (open circles), first overtone (solid circles) and second overtone (open triangles) modes from theoretical models. The continuous line is the observational result for large amplitude red variables, the error bar indicating a temperature uncertainty of $\pm$ 100^{o}K. The solid square is an average value from observations of small amplitude red variables.

The pulsation constant $Q = P\sqrt{(\frac{M}{M_\odot})/(\frac{R}{R_\odot})^3}$ plotted against period for the first three modes of pulsation from theoretical models by Keeley(1970), Langer(1971) and Wood(1974,unpublished). For such a group of red giants with different central concentrations, the fundamental mode is still well separated from the

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overtone modes. However, the overtone modes tend to be smeared together.

Full amplitude non-linear models of pulsating red variables have been published by Keeley(1970) and Wood(1974). A general result of these calculations is that as a star evolves up the giant branch and becomes more luminous, it tends to pulsate in a lower order mode. Keeley(1970) found that stable pulsation in both first overtone and fundamental modes was possible. However, in the calculations of Wood(1974,unpublished) the fundamental mode is always found to be very unstable and to have envelope relaxation oscillations superimposed upon it.[1] Model 3 of Wood(1974) is a good example of this type of behaviour. Such pulsation cannot be associated with Mira variables but could possibly be the cause of planetary nebula ejection and the behaviour of some symbiotic stars (Wood 1974).

At lower luminosities on the giant branch, the overtone modes rather than the fundamental mode dominate. Figs. 2 and 3 show light and radial velocity curves for a first overtone pulsator with luminosity $L=4000L_\odot$ and a second overtone pulsator with $L=2500L_\odot$. Both models have mass $M=1.2M_\odot$, composition $(X,Z)=(0.68,0.02)$ and core masses of asymptotic-giant-branch stars. The second overtone pulsator has a smaller light and velocity amplitude than the first overtone pulsator and it also shows a significant cycle-to-cycle variation in the shape of the light curve.

Theoretical calculations show that the luminosity at which the transition between two particular modes occurs in a star increases with the mass of the star. For stars with composition $(X,Z)=(0.68,0.02)$, asymptotic-giant-branch core masses and a mixing-length of one pressure scale height, the transition between first overtone and fundamental pulsation has been found to occur at the following luminosities (Wood, unpublished): $\log\frac{L}{L_\odot} = 3.4$ to 3.6 for $M=0.9M_\odot$, $\log\frac{L}{L_\odot} = 3.6$ to

[1]The difference in the behaviour of the models of Keeley and Wood is probably due to the different treatment of convection and possibly the larger number of mesh points used by Wood.

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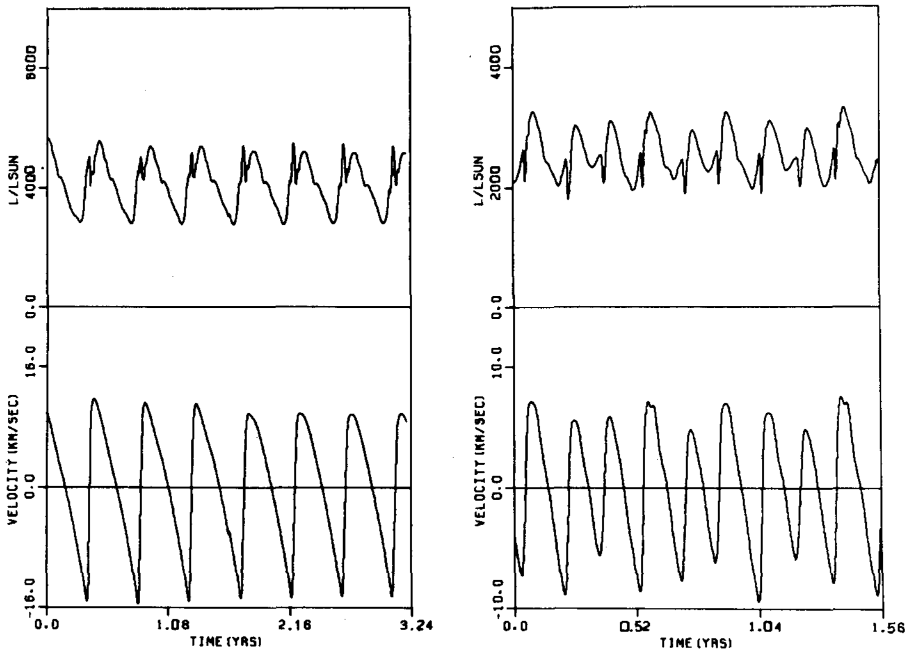

Fig. 2. (left) Light and velocity curves of a theoretical first overtone pulsator.
Fig. 3. (right) Light and velocity curves of a theoretical second overtone pulsator.

3.8 for $M=1.2M_{\odot}$ and $\log\frac{L}{L_{\odot}} > 3.8$ for $M=1.5M_{\odot}$. Although the general trend should remain unaltered, the exact value of the luminosity at which the transition between two given modes occurs will depend on the effective temperature of the model (via mixing-length, composition,etc), as shown by Keeley(1970). These results indicate that, for a given composition, increasing the mass of a star allows it to reach higher luminosities before it switches to fundamental mode pulsation with consequent planetary nebula ejection.

Strong mixtures of modes in theoretical non-linear models of pulsating red giant stars were found by Keeley(1970) in one model. In this model, there was a quasi-periodic switching between fundamental and first overtone pulsation, with a cycle length of $\sim$60 overtone periods. Some of the first overtone pulsators I have studied have a small fundamental component of pulsation present, as seen in Fig. 8 of Wood (1974). The fundamental component causes a small modulation of the first overtone period length and light curve. It is also possible that the apparently irregular component of pulsation noted in the second overtone pulsator shown in Fig. 3 is due to the presence of other modes.

One further type of modulation of the basic pulsation expected in a small number of asymptotic-giant-branch stars is a secular change in period produced by luminosity variations resulting from a shell flash of the helium burning shell. During the flash phase the stars do not deviate from the giant branch except in the last flash cycle when the envelope mass is small (Gingold 1974).

A completely different mechanism for producing irregular and semi-regular variability in red giants has been suggested by Schwarzschild(1975). In Schwarzschild's model, the luminosity variations are caused by surface temperature fluctuations in giant supergranules produced by the extensive convection zones in red giant stars.

In summary, theoretical models predict that the following multiperiodic phenomena may be observed in red variable stars: (a) gradual switching from mode to mode over many pulsation cycles, (b) in first overtone pulsators, the light curve and period length may be modulated by the fundamental mode which has a period 3 to 5 times longer, (c) the period may change continuously due to a change in luminosity resulting from a shell flash in the helium burning shell, and (d) a combination of pulsation and temperature fluctuations in supergranules could lead to luminosity variations with two different timescales.

3. Observations: The basic pulsation mode.

Before looking for multiperiodicities in red variables, we will first try to establish which mode(s) is the source of the primary pulsation. Eggen(1975) has derived the period-luminosity relation $< M_{bol} > = 0.5\text{mag} - 2.25 \log P$

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for large amplitude red variables of the Hyades and old disk groups, the giant branches of which lie along the line $< M_{bol} > = -0.65\text{mag} - 2.5 < R_K - I_K >$ in the HR diagram. Using these results, Wood(1975) has shown that the large amplitude red variables satisfy the (Q,P) relation given by the continuous line in Fig. 1. The error bar indicates a change of $\pm$ 100°K in the temperature derived from the (R-I, T_{eff}) relation of Johnson(1966). The position of the line in the (Q,P) diagram indicates that the Mira variables are first overtone pulsators. It would be difficult to shift the observed relation to agree with the Q values for fundamental pulsators, but the possibility that second overtone pulsation is dominant cannot be excluded. A further piece of evidence for first overtone pulsation is provided by the halo Mira variable V3 in 47 Tuc for which a value of Q=0.05 is derived from the observations of Eggen(1972,1975), assuming a mass of 0.85 $M_{\odot}$. The apparent absence of fundamental pulsators among the large amplitude red variables agrees with the theoretical results of Wood(1974), which showed that fundamental pulsators have no stable light curve and are probably associated with planetary nebula ejection and some symbiotic stars. However, the luminosities derived from Eggen's (M_{bol},P) relation ($\log\frac{L}{L_{\odot}}$ = 3.92 for a typical Mira with P = 300 days) indicate that the luminosity at which the transition from first overtone to fundamental pulsation occurs in the theoretical models given earlier is too low.

Some features which distinguish first overtone from second overtone theoretical pulsators are the smaller light and velocity amplitudes and the greater apparent irregularity of the second overtone pulsators. The small and intermediate amplitude variables, which lie below the large amplitude variables on the giant branch (eg. Eggen 1971), are much less regular than the large amplitude variables and are probably pulsating in higher overtone modes. An average value of Q (0.017) derived for the small amplitude variables given in Table 1 of Eggen(1973b)is shown by the solid square in Fig. 1 and indicates high overtone pulsation.

Another possible method of distinguishing between first and second overtone pulsation is provided by the position of the humps on the rising branch of the light curve. As shown in Figs. 2 and 3, the humps occur at $<\phi> \approx 0.8$ on the bolometric light curve for first overtone pulsators and at $<\phi> \gtrsim 0.7$ for second overtone pulsators. Observations of the light curves of Mira variables by Lockwood and Wing (1971) at the continuum point at 1.04μ show that humps occur in the light curves of most Mira variables in at least some cycles. The humps generally occur in the expected region but the existing observations are too sparse to allow accurate determinations of the phase of the hump relative to the phase of maximum, both at 1.04μ.

Studies of the kinematics of Mira variables by Feast (1963) showed that the variables with $P<145$ days had systematic motions similar to those with $P \gtrsim 300$ days. On this basis, Feast suggested that the shorter period stars were similar to the most common Miras with $P \sim 300$ days, but pulsating in a higher overtone. A period ratio of 2.4 is predicted in this way, which is incompatible with the two modes being successive overtones but, within the limits of error, could possibly be consistent with the 300 day Miras being fundamental pulsators while the Miras with $P<145$ days are first overtone pulsators. An interpretation of Feast's result which does not require the existence of fundamental pulsation is that the two intermediate period groups $149d < P < 200d$ and $200d < P < 250d$ contain all the halo variables and consequently have higher systematic motions. This is consistent with the observation that the Mira variables in globular clusters generally have periods $P \approx 200$ days (Feast 1972), although two Mira variables of longer period have been identified with the metal rich globular clusters NGC 5927 and NGC 6553 by Andrews et al. (1974). The semi-regular variables in the solar neighbourhood have similar (old disk) systematic motions regardless of period (Feast et al. 1972).

In summary, the observations indicate that the basic pulsation mode of large amplitude red variables is the first overtone. The smaller amplitude semi-regular variables, which lie below the Mira variables on the giant branch, appear to be second or higher overtone pulsators.

4. Observed secondary periodicities.

It has long been recognized that in red variable stars the period length and magnitudes at maximum and minimum vary from cycle to cycle, possibly indicating the presence of secondary periodicities. An analysis of the period length variation in a group of long period variables by Eddington and Plakidis(1929) and Plakidis(1932) led them to conclude that in over 75% of the stars studied, the variations could be explained in terms of (a) errors in the determination of the date of maximum and (b) random and independent deviations of the period length about a mean value. Plakidis(1932) found that of the remaining stars, R Hya and R Aql had continuously decreasing periods while R UMa and R Aur showed a decrease in period in sudden jumps. Sterne and Campbell(1937) looked for period changes in 377 well observed long period variables and found decreases in period of R Hya and R Aql and possibly sinusoidal changes in R Cnc, U Boo and S Ser.

Some examples of O-C diagrams of Mira variables are shown in Fig. 4; the data is taken from the tables of Campbell (1926,1955), together with more recent data kindly supplied by J. A. Mattei of the AAVSO. As well as a random scatter about the calculated date, there appear to be sudden changes in the period, which remains relatively constant before and after the change. Student's statistic

$$t = (\bar{P}_1 - \bar{P}_2)/\sqrt{\left(\frac{1}{n_1}+\frac{1}{n_2}\right)\frac{(n_1-1)\sigma_1^{\,2} + (n_2-1)\sigma_2^{\,2}}{(n_1+n_2-1)}}\quad,\text{ where}$$

(P_1, n_1, σ_1) and (P_2, n_2, σ_2) are the (mean period, number of cycles, standard deviation of the period) before and after a preselected date, was used to test the significance of apparent period changes in a group of 45 well-observed red variables using data from the above sources. It was found that 14 stars

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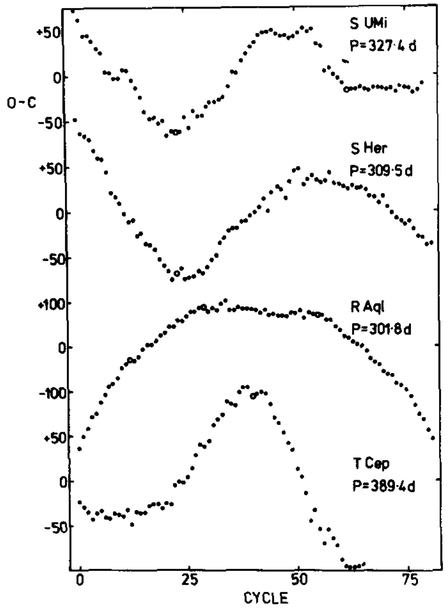

Fig. 4. O-C diagrams for the variables S UMi, S Her, R Aql and T Cep. Open circles indicate period changes at > 99.5% significance as described in the text.

showed changes at the 99.5% significance level with a further 6 having changes at greater than 99% significance. These results indicate that period changes occur in approximately half the red variable stars investigated.[2] Although 14 stars show two or more period changes at greater than 95% significance, the duration of observation is not long enough to show whether these changes represent some quasi-periodic phenomenon or whether the changes occur at random times.

The period changes found at >99.5% significance amount to 2-5% of the period of the star (R Aql excepted), which is too small to be attributed to mode changes. A likely cause of these small period changes is an alteration in the envelope structure near radius $r \simeq 0.8R$, which is the region having most influence on the length of the first overtone

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[2] Preselection of the date at which the test for period change is made is the reason for the large number of significant period changes found here compared with the number found by Sterne and Campbell (1937).

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period in red giant stars (Epstein 1950). As the shortest thermal decay timescales for typical red giant envelopes are ≈ 3 years (three or four pulsation cycles), the changes in envelope structure appear to be maintainable over ~ 5 thermal timescales.

An example of a red variable which has probably changed its mode of pulsation is the SRd variable Z Aur, whose period has changed twice from 110.5d to 113.5d and then to 134.8d (Lacy 1973). The latter change, with a period ratio of 1.19, is typical of the period ratio P_2/P_3. This result is consistent with the evidence in the last section which suggested that the small amplitude red variables are pulsating in the second or higher overtones.

The two Mira variables R Hya and R Aql, both of which show a continuous decrease in period, are possible examples of stars undergoing secular luminosity changes resulting from a shell flash in the helium burning shell. Since asymptotic-giant-branch stars do not deviate from the giant branch during a shell flash, the relation between M_{bol} and period (Eggen 1975) given above can be used to calculate the luminosity change in these two stars. Using the periods for R Hya given in the GCVS (Kukarkin et al. 1969) and the periods derived from Fig. 4 for R Aql, the luminosities given in Table 1 result. In R Hya the luminosity is declining on a

Table 1

Star	Date	Period	$\log\frac{L}{L_\odot}$	Timescale
R Hya	1700	500d	4.117	1200 yrs
	1975	388d	4.018	
R Aql	1915	320d	3.943	550 yrs
	1975	284d	3.896	

timescale $L/\frac{dL}{dt}$ of ~ 1200 years while in R Aql the timescale is ~ 550 years. It is difficult to compare these timescales with theoretical ones because we are not sure of the exact phase of the flash cycle which each of the two stars is in. However, if it is assumed that the stars lie in the phase of luminosity decline immediately following the surface luminosity peak of a flash cycle, then the data of Schwarzschild and Härm(1967) and Sweigert(1971) give timescales $\sim$ 2000 years at $\log\frac{L}{L_\odot} \approx 3.3$ and $\sim$ 50 years at $\log\frac{L}{L_\odot} \approx 4.3$. These theoretical timescales probably represent lower limits as the luminosity decline is relatively rapid in the phase selected. The observed timescales and luminosities are consistent with those derived from the theoretical models. As a further test, the observed fraction of red variables undergoing shell flashes ($\sim$2/400) can be compared with the theoretically expected fraction. A minimum for the expected fraction f is very roughly the time between the main and secondary peaks of a flash cycle divided by the cycle length. In the models of Schwarzschild and Härm with $\log\frac{L}{L_\odot} < 3.5$, $f \approx \frac{1}{10}$ to $\frac{1}{300}$ while for Sweigert's models with $\log\frac{L}{L_\odot} \approx 4.3$, $f \approx \frac{1}{1000}$. These theoretical values bracket the observed value. It thus appears that R Hya and R Aql are in the active phase of a helium flash cycle; they should be checked for past and present abundance changes such as those found in FG Sge by Langer et al.(1974).

Many semi-regular variables are known to vary with a secondary quasi-period $\sim$10 times longer than the primary characteristic period. Some examples of light curves of this type are given for TX Dra and Z Eri by Sacharow(1953), while Payne-Gaposhkin(1954) lists a group of these variables. The N-type carbon star V Hya, for which a long light curve is given by Mayall(1965), may be an extreme example of this phenomenon. The longer of the two periods (typically 500-1500 days) is not inconsistent with this period being the fundamental period of pulsation, which would require the shorter period to be one of the higher overtones ($\sim$5th). Other possible explanations of the longer periodicity are (a) mode switching between overtones in a manner similar to that found by Keeley(1970), (b) some kind of oscillation of the envelope on a thermal timescale which is typically 1000 days, (c) a combination of pulsation and temperature fluctuations in supergranules as suggested by Schwarzschild(1975), and (d) in the carbon stars, periodic grain formation in a circumstellar envelope in a manner similar to that suspected in the R Coronae Borealis stars (Feast and Glass 1973).

Secondary periodicities have been reported in a number of Mira and well-observed SR variables. Van der Bilt (1934) has reported a secondary period of 930 days in the Mira variable SV And, whose primary period (hereafter called P) is 316 days. Fritzová et al.(1954) looked for secondary periodicities in the primary period length and magnitudes at maximum and minimum brightness of a group of 45 long period variables, and found (a) a secondary period ~9 P in V Boo and S Boo (b) a secondary period ~2.7 P in T UMa, and (c) long period (20-50 years) irregular changes in amplitude in R Tri, R Aur, R Cam, T Cas, U Per, S UMa and R Vir. Fischer(1969) has analysed changes in period length (defined in three different ways) in Mira Ceti over 69 cycles using power spectra. Although he found peaks at 2 P and ~10 P in some of his power spectra, his results for different definitions of the period length conflict.

Preliminary results from some calculations of power spectra of the magnitudes at maximum, and of the dates of maximum, are now given. The data used are that described previously. Power spectra (three point running means of the raw power spectra) for three variables in which secondary periodicities have been reported in the literature quoted above are given in Fig. 5. The error bars indicate 99% significance limits for a power spectrum of a white noise process, using a χ^2 test with 6 degrees of freedom. The first ten frequency points in the power spectra of the dates of maximum are joined by dotted lines and should be regarded with caution as the large amount of power generally found in this region is due to the sudden period changes discussed earlier.

In the power spectrum of SV And, there is little evidence for a secondary period at 3 P as suggested by van der Bilt(1934). The light curve of T UMa shows a general excess of power in the region 2-4 times P with possible peaks at 2.2 P and 2.4 P. No peak at ~2.7 P is evident, in disagreement with the

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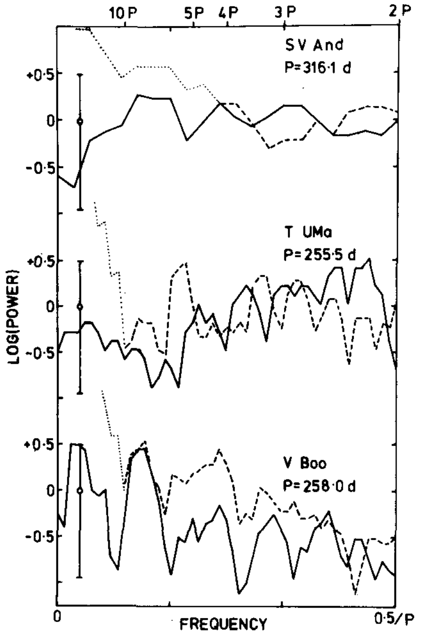

Fig. 5. Power spectra of the magnitudes at maximum (continuous lines) and dates of maximum (dashed lines and dotted lines). Error bars indicate 99% confidence limits.

result of Fritzová et al.(1954). There appears to be a peak in the power spectrum of the dates of maximum of T UMa near 5.5 P. Of the 2 by 46 power spectra calculated, that of V Boo shows the most significant peaks. The two power spectra of V Boo appear well correlated for periods > 3 P and both show a large peak at 8-9 P, in agreement with the secondary period found by Fritzová et al.(1954). No satisfactory explanation of this secondary period is known to me. If it results from beating between two modes then a period ratio of ~1.14 is required, which would require overtones around the fourth to be involved. This appears too high a mode for a star of the amplitude of V Boo.

Some general features which have been noted to occur in the power spectra are (a) strong peaks at 2 P indicating alternating high and low maxima (eg. R And, R Gem), (b) an excess of power in the region 2 to ~3 P (eg. T UMa, χ Cyg, R Cyg), (c) an excess of power at low frequencies in the power spectra of the magnitudes at maximum, indicating long term (20+ years) changes in luminosity (eg. X Cas, T Cas, T Cam), and (d) discrete periods (eg. V Boo). The periods do not appear to occur in both power spectra except in V Boo and probably represent statistical fluctuations. There is no evidence for a predominance of secondary periods in the region 3-5 P which might be expected if the primary period was producing a small modulation of the first overtone. Full results will be published elsewhere when the data has been full analysed.

In summary, the following secondary periodicities have been found in red variables (a) sudden changes in period which may occur on some quasi-periodic timescale in Mira variables, (b) a change in period on a secular timescale in R Hya and R Aql, probably due to a shell flash in the helium burning shell of each of these stars, (c) a quasi-periodic variation in the magnitude of some of the small amplitude red variables on a timescale $\sim$10 P, and (d) a well defined secondary period of 8-9 P in V Boo. There is no general evidence for secondary periods which would occur at 3-5 P if a small amplitude fundamental component of pulsation was superimposed upon the basic first overtone pulsation, as suggested by some theoretical models of Wood(1974,unpublished).

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Discussion to the paper of WOOD

COX: How do you explain the predicted fundamental mode instability which is not ever observed? How are the observed modes ordered with luminosity, and are they predicted theoretically?

WOOD: If the fundamental mode pulsation leads to planetary nebula ejection, then we don't see the stars as red giants any more since they have become planetary nebula nuclei and nebula shells. If fundamental mode pulsation is associated with symbiotic star behavior, then we do observe fundamental pulsators. At earlier times, when the luminosity was smaller, only overtone pulsation would have occurred.

BATESON: Has any investigation been made of stars at the short end of Mira periods, e.g. T Cen which has large changes in amplitude over many years?

WOOD: No, so far as I know there has been no investigation.

BATESON: In discussing period changes, has attention been paid to the lack of homogeneity in investigations of periods, as investigators use different observational material?

WOOD: Apart from using Campbell's published data, no check has been made on the homogeneity of material.

DZIEMBOWSKI: Do you have any interpretation for those stars that have the period ratios around 9, e.g. V Boo?

WOOD: No.

DZIEMBOWSKI: What is the maximum order of the overtone that may be excited in the red giant region, according to theory?

WOOD: I have calculated only up to the second overtone.

GEYER: In the case of V Hya with a long period variation of 18 years, one should reconsider the old idea of star spot activity. This is also the case where the power spectrum analysis gives a period ratio of 2. Here the stars spots are more concentrated on a part of the stellar hemisphere, and the star rotates with a period of about twice the fundamental pulsation period.

STOBIE: In your models, a large fraction of the luminous flux will be carried by convection. How sensitive do you think your results will be to the theory of the interaction of convection and pulsation which you have used?

WOOD: I think the results are probably qualitatively correct - in particular, the finding that lower order mode pulsation occurs as luminosity increases or temperature decreases. Amplitudes seem to agree with those observed. However, the critical luminosities and temperatures at which the transitions between different modes occur are not reliable.