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DISTURBING FUNCTION

Meffroy (**1**) computed the periodic part of the disturbing function to find new secular terms of the third order in the expansion for the semi-major axis, in connection with Poisson's theorem on the invariability of the major axes of the planetary orbits.

Heinrich (**2**) writes that he has succeeded in eliminating the reciprocal distance of planets from the right-hand sides of the equations of motion by the operation, which he has previously discovered. The operation leads to an integral equation for a simple linear coupling of the major axes, which can be solved without the intervention of a small divisor. He has in mind to apply the theory to the motion of the Moon, the Trojans, Gauss's elliptic ring, and some of the characteristic asteroids.

Izsak (**3**) has published tables for the Laplace coefficients and their Newcomb derivatives. Izsak, Barnett, Efimba and Gerard (**4**) worked on the construction of Newcomb operators on a digital computer.

Mulholland (**5**) invented a method of computing the Laplace coefficients on electronic computers. He transformed the infinite series for the Laplace coefficients into forms better suited for computation by means of Gamma functions.

Kaula (**6**) and Musen, Bailie, Upton (**7**) published the analytical expansion of the lunar and solar disturbing functions for use in the theory of a close Earth satellite along the line of Kozai (**8**), Groves (**9**) and Kaula (**10**). The expansion will be naturally of use for the motion of a natural satellite with arbitrary inclination and eccentricity, when the ratio of the mean distances of the disturbed and the disturbing bodies is small enough. Kaula's (**6**) development is advantageous when it is desired to conserve computer storage space or to include the luni-solar perturbations in the same computation with perturbations due to tesseral harmonic terms of the Earth's potential. A quasi-potential for the solar radiation pressure effect for use in the equations of motion is also written in terms of the Keplerian elements.

Elenevskaya (**11**) has obtained a development of the disturbing function for an eccentricity approaching unity, by expanding in powers of $(1 - e)$.

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PLANETARY THEORY

For facilitating the ephemerides computation by means of an electronic computer the method of Strömgren has been improved by using a set of elements other than the Keplerian, since

https://doi.org/10.1017/S0251107X00014322 Published online by Cambridge University Press

indeterminacy comes in for an orbit with vanishingly small eccentricity or inclination. Garafalo (1), Pines (2), Cohen and Hubbard (3), and Herget (4) discussed this point. A new method published by Herget (4) is greatly enlightening the works of the Central Bureau of the International Co-operation at Cincinnati in the ephemerides computation of asteroids. Instead of the usual concept of the elliptic elements M_0, n, a, e, ω, Ω and I the following are dealt with: $\mathbf{c} = p^{\frac{1}{2}}\mathbf{R}$, $X = e \cos \omega$, $Y = e \sin \omega$ and U_0, where U_0 is the sum $M + \omega$ for $t = 0$ and the vector $\mathbf{R}$ is directed to the normal of the orbital plane. Strömgren, Kaula (5), Musen (6) and Popovič (7) went farther in referring to matrices and dyadics.

Danby (8) used a kind of matrix notation for computing special perturbations and the matrizant of Keplerian motion in matrix method in the calculation and analysis of orbits, especially for Hill-Brouwer's method of computing the perturbation in rectangular co-ordinates. The solution is obtained directly in the form of the complementary function and the particular integral.

Musen and Carpenter (9) developed a new theory of general planetary perturbations in rectangular co-ordinates. Their theory has certain characteristics common with Hill's theory. They decomposed the perturbations along $\mathbf{r}$, $\mathbf{v}$ and $\mathbf{R}$, in contrast to Hill's decomposition along $\mathbf{r}$, $\mathbf{R} \times \mathbf{r}$ and $\mathbf{R}$. This decomposition leads to a direct method of integration and the final formulae are in a convenient form for programming for electronic computers. Further the six integration constants are determined in a direct manner, in contrast to the seven constants appearing in Hill's double and triple integrals. The final expressions are obtained in the form of trigonometric series with the number of arguments equal to three times the number of planets in the problem. The potential is expanded in terms of multipoles, which facilitates the computation of the perturbations of any order. For the computation of the perturbations the components of the disturbing force are expanded into trigonometric series by means of numerical double harmonic analysis.

Musen (10) modified Strömgren's method for including the effects of higher orders. Although Strömgren's method has a mathematical elegance by using the vectorial elements, it has a disadvantage of taking into account only the first order perturbations, because Strömgren obtained the Gibbs rotation vector indicating the integrated value of the angular velocity of rotation of the osculating ellipse only in the first approximation. Musen obtained an accurate form for the rotation matrix in terms of the Gibbs vector and gave the differential equation for the perturbation of this vector.

Tables for the method of the perturbation of elements have been published by Merton (11). The tables contain the quantities required to facilitate the calculation of special perturbations in the orbits of comets by the rigorous method proposed by Merton (11) himself, in which the mean anomaly is taken as the independent variable.

P. Stumpff, Schubart and others are working, according to a letter from Schubart, on a program for obtaining the motion of up to ten bodies in a planetary system with high precision to be provided for the computer at Heidelberg. Simultaneous integration of the equations of motion with a constant step-length is used, although the step-length can be changed without stopping the computer (compare Commission 20).

Sehnal (13) applied Gauss's method for computing the secular variation to the computation of the secular perturbation of the quadrantid meteoric swarm, and then modified it (14) by expanding the disturbing function around its value for two circular orbits.

The long-range effects caused by the Moon and the Sun are of primary importance for proving the stability of highly eccentric orbits of an Earth satellite and for obtaining its lifetime (15). Musen (16) applied Halphen's method of computing the secular variation based on Gauss's idea. The method is also used by Musen (17) in computing the long-range or the secular effects in the motion of asteroids when there is no sharp commensurability between the mean motions of the asteroid and Jupiter. The disturbing function is averaged over the short-periods by Gauss's concept. Musen describes the method by referring to vectors, matrices and dyadics. The properties of the elliptic functions of Weierstrass and of the hypergeometric functions of Gauss are fully utilized. The formulae are more suitable for electronic computers than the usual analytical expressions for computing the long-period perturbations.

Clemence (**18**) applied Hansen's theory modified by Hill with further improvements and corrections for facilitating the use of a computing machine with very high accuracy to the motion of Mars. The long-period inequalities appear with arguments, among others, l (Venus) $- 8\ l$ (Mars), $15\ l$ (Mars) $- 8\ l$ (Earth), l (Mars) $- 6\ l$ (Jupiter).

Clemence (**19**) remedied the discrepancy between the theory and observations in the longitudes by giving 1/3499·7 to the mass of Saturn.

The comparison of Clemence's new theory of the motion of Mars with observations extending from 1750 to 1960 to determine the definitive values of the constants has been continued by Duncombe. A preliminary solution of the equations of condition indicates a provisional value for the reciprocal of the mass of Venus of 408 945 ± 470.

Rectangular co-ordinates of Mars referred to the mean equinox and equator of 1950·0 with an interval of 4 days for the period 1800 – 1950 have been completed on the new theory. They are based on provisional elements and are consistent with the co-ordinates from 1950 to 2000, published in U.S. Naval Observatory Circular no. 90, 1960.

Clemence has completed the first-order portion of a new general theory of the heliocentric motion of the Earth and is working on higher order contributions.

Morando is studying a general theory on the motion of Vesta without secular or mixed secular terms.

Roemer (**20**) and Brandt (**21**) discussed the residuals of the acceleration in the motion of periodic comets in connection with the interaction of comet tails with the interplanetary medium.

In order to draw geophysical and geodetic conclusions from the motion of artificial satellites we need an accurate theory which permits easy inclusion of any gravitational term and which is suitable for machine computation.

Musen (**22**) has given Hansen's lunar theory in a form which permits a purely numerical treatment of the perturbations on artificial satellites. However, Musen for the purpose of numerical computation refers rather to Hansen's planetary theory and uses the fictitious mean anomaly instead of Hansen's W function, and sets up the process of iteration in a convenient form. After the process of iteration is completed, the function is formed and the perturbations in the mean anomaly and in the radius vector are determined. The orbit of the disturbing body is supposed to be a moving ellipse. The orbital plane of the disturbing body is taken as the basic reference plane according to Hansen. However, Musen (**23**) uses instead of the latitude four parameters in order to make all basic arguments linear in time from the outset, for applying to satellite orbits with high inclination. Bailie and Bryan (**24**) computed the osculating elements from this modified Hansen's theory, based on matrix transformation, for the motion of an artificial satellite.

The tendency of celestial mechanics of including the eccentricity in the disturbing function without expanding it in powers of the eccentricity as in the classical theory is one of the recent progresses never thought of in the past years due to the small eccentricities of the natural heavenly bodies. The discussion of the motion of asteroids with high eccentricity and inclination by Kozai (**25**) is one of the outcomes of this nature. Secular perturbations of an asteroid with high values of inclination and eccentricity have been studied by Kozai by referring to von Zeipel's theory. Since the conventional technique for developing the disturbing function cannot be adopted, Kozai expanded it in powers of the ratio of the semi-major axes of the asteroid and Jupiter. Short-period terms depending on the two mean anomalies are eliminated from the disturbing function by von Zeipel's transformation.

Hori is now engaged in the computation of the secular variations of periodic comets by avoiding the usual expansion in powers of eccentricities, inclinations and the ratio of the major-axes.

It should be mentioned that Jeffreys (**26**) pointed out the similarity of von Zeipel's theory and Brown's theory for eliminating all short-period terms simultaneously from the disturbing function.

Miachin (**27**) has presented a general technique for estimating an error in numerical methods of integrating the differential equations of celestial mechanics, with specific reference to the methods of Cowell and of Runge. Kulikov (**28**) has worked out a procedure of integrating the equations of motion in celestial mechanics by using electronic computers and Cowell's quadrature method with automatic pitch selection.

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CHARACTERISTIC ASTEROIDS

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