This department welcomes short notes and problems believed to be new. Contributors should include solutions where known, or background material in case the problem is unsolved. Send all communications concerning this department to I. G. Connell, Department of Mathematics, McGill University, Montreal, P. Q.

ON THE EXACTNESS OF THE HOMOTOPY SEQUENCE

P. Scherk

This note is based on Hu's Homotopy Theory (New York, 1959), pp. 115 f.

The theorem that the homotopy sequence is exact splits into six statements. The fifth in Hu's exposition can be stated as follows: Let $f : I^n, I^{n-1}, J^{n-1} \to X, A, x_o$,

(1) $$f \mid I^{n-1} \sim 0 : I^{n-1}, \partial I^{n-1} \to A, x_o .$$

Then there exists a map

(2) $$g : I^n, \partial I^n \to X, x_o$$

such that $f \sim j\, g : I^n, I^{n-1}, J^{n-1} \to X, A, x_o$.

Hu - like Hilton in his Cambridge Tract - uses homotopy extension. It may be useful to have a direct alternative available. The following proof can readily be shortened to a couple of lines.

By (1) there exists a map $F : I^{n-1} \times I, J^{n-1} \times I \to A, x_o$ such that

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(3) $$F(t_1,\ldots,t_{n-1};0) = f(t_1,\ldots,t_{n-1},0)$$

and

$$F(t_1,\ldots,t_{n-1};1) = x_o .$$

We need a map

$$H : I^n \times I,\ I^{n-1} \times I,\ J^{n-1} \times I \to X,\ A,\ x_o$$

such that

$$H(t_1,\ldots,t_n;0) = f(t_1,\ldots,t_n)$$

and that the map

$$g(t_1,\ldots,t_n) = H(t_1,\ldots,t_n;1)$$

satisfies (2).

Define

$$H(t_1,\ldots,t_{n-1},t_n;t) = \begin{cases} f(t_1,\ldots,t_{n-1},-\varphi) & \text{if } \varphi \leq 0 \\ F(t_1,\ldots,t_{n-1};\varphi) & \text{if } \varphi \geq 0 . \end{cases}$$

Here $\varphi = t - t_n - t_n t$; thus $-1 \leq \varphi = t(1-t_n) - t_n \leq 1$.
On account of (3), the map $H : I^n \times I \to X$ is uniquely defined.

It is now an easy matter to verify that all the required conditions are satisfied.

University of Toronto

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