Photoelectric Astrometry

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I. *INTRODUCTION*

Improvement of astrometric accuracy is a major need of astrophysics today. A significant increase in the amount of information about the masses of stars, parallaxes, and proper motions is to be gained. Some improvements have been accomplished in the last few years such as the astrometric reflector like the 61-inch at Flagstaff, (K. Strand, 1967), and the McCormick 40-inch (under construction). Another direction of improvement has been automation of measuring devices such as the Strand Measuring Engine and the Grant Measuring Engine; but one of the large improvements of the future ought to be observations outside the Earth's atmosphere to the level of diffraction limited images instead of seeing limited images. The concentration of light in a smaller image will then mean both a fainter magnitude reached and a high astrometric accuracy.

Space astrometry cannot be realized by using the present techniques because the photographic plate is used as an intermediary to carry the positional information between the focal plane of the telescope and the measuring engine. The use of photographic plates in space will be prohibitive. They are bulky, must be protected, and require revisitation of the space craft with material return. Furthermore, photographic plates are undesirable even in ground based telescopes. They have a limited dynamic range, there is the possibility of emulsion deformation which eventually will limit the astrometric accuracy, and they have low quantum efficiencies. It is therefore necessary for space astrometry that we replace the photographic plate by some other kind of device to carry the positional information.

The angular discrimination required is of the order of or better than 10^{-7}, or about 1 micron, for an effective focal length of 10 m. The use of television systems, as available today, is out of the question because of the loss of signal in the point-by-point scanning, and in the analog-digital-analog conversion which adds further to the loss of information. What we require is a two-dimensional area scanner able to resolve a limited number of quasi point sources.

A technique that can be used to carry positional information is to geometrically modulate the light of the star field with a carrier function uniquely dependent on its position in the star field. After modulation, the light is detected in a photomultiplier; the voltage output will then be the superposition of a number of carrier functions, one for each of the stars in the field. After the necessary recording or integration and telemetry, the signal can be processed to extract the positional information contained in the output.

The general technique which we have pictured above is what can be called "photoelectric astrometry" and some successful steps in the field have already been taken. The strip integrated one-dimensional scanner of K. D. Rakos (1965) and O. G. Franz (1966) allows scanning in one direction while it integrates all the light passing through the slit oriented in the perpendicular direction. In this device profiles of double stars are obtained and very promising astrometric information derived. Meridian transit observations of a group of stars behind systems of wires or through properly placed slits have been essayed by V. E. Brandt (1968), E. Høg (1970), and others.

Measurement of stellar radii by the occultation method is another example of application of this technique. Here, the light of the star is modulated into a diffraction pattern by the limb of the Moon. The recorded light intensity fluctuations are then analysed through the carrier function which in this case is the diffraction pattern. The power of this technique is shown by D. S. Evans (1951, 1955, 1957), P. A. G. Scheuer (1962), R. Berg (1970), and others. The method for extracting the positional information from the output waveform depends on the particular carrier function used. In the one-dimensional

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scanner of Rakos and Franz, and the slit meridian circle transit, position and time are linearly related and the extraction of the information is immediate. In the lunar occultation method the carrier function is more complex and convolution techniques are used to extract the information. In the two-dimensional case complex carrier functions are to be expected.

II. *THE DEVICE*

Imagine (Fig. 1) a Ronchi grating in the focal plane of the astrometric telescope rotating around an axis parallel to the optical axis at a few times ten revolutions per sec and such that the center of rotation is outside the star field. (A Ronchi grating consists of a series of parallel equally spaced transparent and opaque slits). The rotation of the grating must be highly controlled in speed through a reliable frequency standard. A triggering device is to be provided to produce a position angle reference for the rotation of the Ronchi grating. Behind the focal plane there is a large optical transfer system to bring the light of the star field onto the photocathode for detection and further amplification. The output of the photomultiplier is amplified, integrated, digitized and stored at a data sample rate of the order of thousands of points for 2π radians; this is equivalent to a few hundred microns per channel for a distance to the center of rotation of 50 mm; the imperfections of the optics behind the Ronchi grating do not impair the astrometric accuracy since it does not change the geometrical information contained in the modulated light.

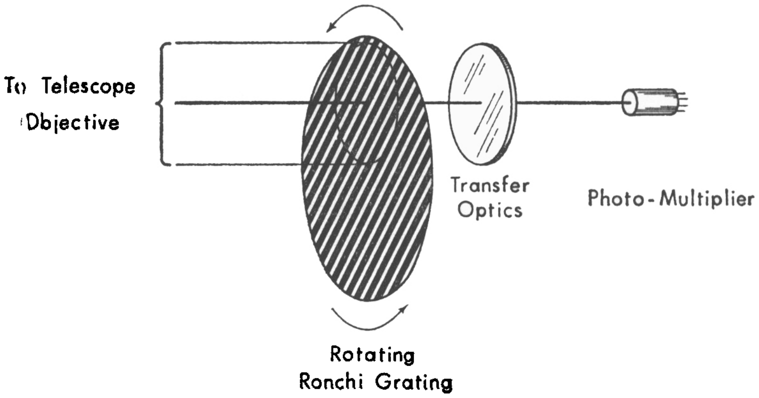

Fig. 1 Schematic Concept of Device

The instrument to be assembled onto the telescope would be light in weight and smaller than a conventional photometer.

Figure 2 shows the output wave form for a single star produced numerically by using a Ronchi grating of 0.150 mm width for a star situated at a radial distance of 15 mm from the rotation center, and 0° phase angle. The distance to the center is very small in this case, which permits one to show the general form of the modulated wave output.

This and all the figures of wave forms have been built in our numerical experiment by assuming a parabolic profile of the stars and an equal image diameter of 0.100 mm; the peak of the profile could be changed. Every point in the wave-generating procedure was obtained by two-dimensionally integrating the "volume" of the paraboloid which represents the image; we believe these parabolic profiles are good approximations of the actual profiles of star images for the purpose of predicting the general characteristics of this instrumentation. The star image, we are assuming, is the result of integrating over a long period of observation, with a ground based telescope, and it is the convolution of the Airy profile with the seeing motions. In space, atmospheric effects will be eliminated if the pointing stability of the space telescope can be kept to the low values expected. If this is the case then a large improvement over the behaviour deduced in our numerical experiments will be attained.

The carrier function for a single star shown in Figure 2 resembles an F.M. signal. In one turn of the grating the star image crosses from a maximum number of slits per unit time to zero slits per unit time 90° later; to the same maximum 90° later, etc. The maximum number of slits crossed is proportional to the distance from the star to the center of rotation. Each star in the field is characterized by its maximum frequency which we might call its characteristic frequency (f_c). Therefore, the carrier function for a given star will contain all the frequencies from f_c to zero and will be a band limited function in the Fourier transform sense. f_c is given by

$$f_c = 2\pi r/w\,T$$

where w is the width of the grating and r the distance to the center.

Schematic Characteristic Wave Pattern for a Single Star (Full Cycle)

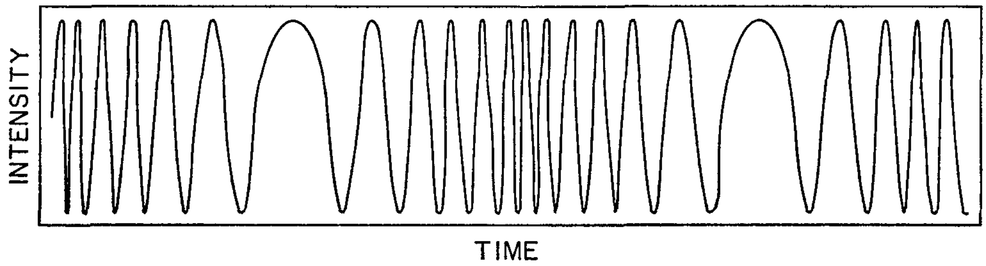

Fig. 2

We have chosen the Ronchi grating for this experiment because it can be produced easily with a high degree of accuracy but any other moiré pattern, or any other mask can be essayed for producing geometrical modulation.

III. *APPLICATION TO DOUBLE STARS*

A first application of this device is to the study of double stars, not only to provide astrometric information but, at the same time, photometric data. In this application the device can be considered a one-dimensional strip-integrating scanner, comparable to the one described by Rakos (1965), and by Franz (1966), but with two differences. Here the position angle of scanning is continuously changed between 0° and 360° making the extraction of the positional information more complete. In our case the slit width is much larger than the diameter of the star. Therefore, our raw data will be the integral of the light going through the slit from a large portion of the star image. This will make the analysis more complex. In particular, it might increase the signal to noise ratio in an individual profile.

If one pictures the double star observation as the scanning of the image profile with such an extremely large slit, one comes to the conclusion that there will be a very low resolution for double stars. However, by interpreting the data-gathering process as a digital one in which the sample data gives us the amount of light going through for a given position of the grating, then the successive differences will represent a coarse profile from which positional information is readily obtained. We intend to obtain the equivalent of five data points within one star diameter. As a function of position angle, we will get a consinusoidal variation of the relative position of the components of the double star superposed with some other variations dependent on the position angle of the pair. From it, the separation of the pair and position angle in the sky is easily deduced.

IV. *ASTROMETRY OF A STAR FIELD*

We are interested in a limited area scanning for a few quasi-point sources for which we would like to obtain positional accuracy of the order of 1 micron. With our device the light for every star in the field will be modulated with a different carrier function. The result of the modulation of the entire field will consist of the superposition of the individual carrier functions. This superposition property and the band limited spectrum of the carrier function caused us to consider the application of Fourier transform techniques. In the frequency domain every carrier function will be characterized by a maximum frequency f_c.

For continuous signals the Fourier transform relations are written in the form

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}dt$$

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{i2\pi ft}df$$

which transform the description of an event from the time domain to the frequency domain and vice versa. In our numerical experiment we have used a discrete fast Fourier transform algorithm presented by Cooley and Tukey (1965) which allows a quick and economical application of the Fourier transform techniques.

By the superposition theorem of the Fourier transform, the transform of a superposition of functions is the superposition of the transform of each function; therefore, the frequency description of a number of stars will be a combination of spectra. Unfortunately, the spectrum for a single star extends all the way from f_c down to zero frequency and therefore confusion might occur. One way out is to reduce the low frequency components of the stars so that the characteristic frequency of the other stars might be seen more clearly. This can be done physically in the instrument by the use of an opaque mask so that only the high frequency components of the grating modulate the light. On the other hand, one can apply some filtering techniques through the digital computer. We have tried to "tune" a half-gaussian window through the data in the time domain such that we can cause the high frequency component of some star to be enhanced and low frequencies from other stars to be reduced. The gaussian window used is given by the formula

$$G(t) = \exp\left(-\pi u^2\right)$$

where

$$u = 4\,\frac{t(N) - t(N/2)}{t(N) - t(1)}$$

At the same time the tuning to a high frequency component allows one to find the position angle for a given star while the characteristic frequencies provide the radial distances. This filtering process in the computer is equivalent to the use of a physical neutral density filter at the telescope with a transmission varying in a gaussian fashion as a function of time.

Figure 3 shows the instrumental output for two stars at the same radius and 10° apart in phase. Figure 4 shows the output for three stars at radius 35, 45, and 55 mm and position angles 80°, 90°, and 115°. Figure 5 shows the representation in the frequency domain for the two star case and Figure 6 for the three star case. As can be seen, the stars are easily resolved and the radial distances can be easily obtained with this procedure. Tuning of the gaussian window for different times will provide the position angle. This approach is based on the characteristic frequency for each star, therefore part of the information contained in all of the other frequencies is discarded to avoid confusion with the frequency component of some other star.

It seems appropriate then to look for some other technique for extracting the positional information making use of all the data available. It is a general problem of measurement where the type of function is known and the value of the parameters are to be determined from the signals available. The common technique of low pass filtering meets with difficulties because of the non-orthogonality of the carrier functions. Fitting of the observations to a convenient model for the device can be essayed but a high initial approximation is needed (better than half of a fringe). The Fourier transform technique should provide the required first approximation. But all those techniques are just some of the different procedures that might be used to extract the information about the geometry of the problem which is contained in the data.

The power spectra for the different stars are a function of the brightness of the stars and therefore photometry of several stars can be done with this device. It imitates the panoramic property of the photographic plate while avoiding the restricted dynamical range, non-linearities, and low quantum efficiency. The possibility of observing more than one star at one time points to its application for differential photometry, in which greater accuracy must be expected for the same amount of observing time.

One can also conceive of applying this geometric modulator to the scanning of spectra if the spectrographic plate is replaced by our device. The dispersed spectrum is oriented radially from the center of rotation of the pattern. In this case every wavelength in the spectrum is modulated with a different carrier frequency which is linearly dependent on the radial distance. In this case, the Fourier transform of the observed waveform will produce the spectral profile allowing applications in broad spectral scanning. This is just a particular case of multiplexing, which is presently widely applied through

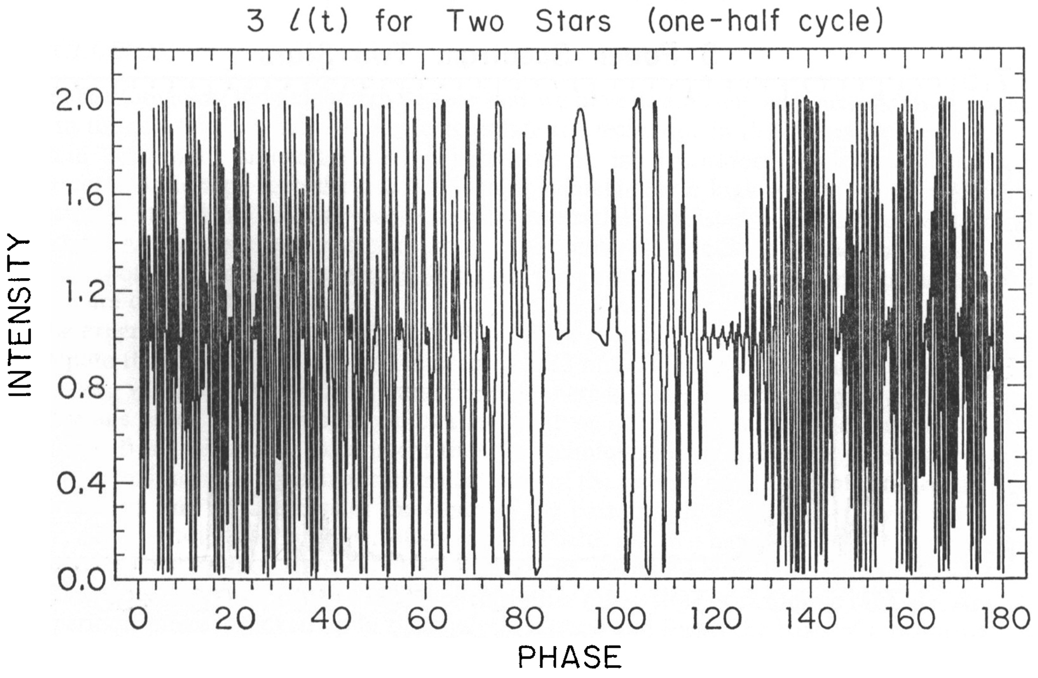
3 l(t) for Two Stars (one-half cycle)
INTENSITY
0.0
0.4
0.8
1.2
1.6
2.0
PHASE
0
20
40
60
80
100
120
140
160
180

Fig. 3

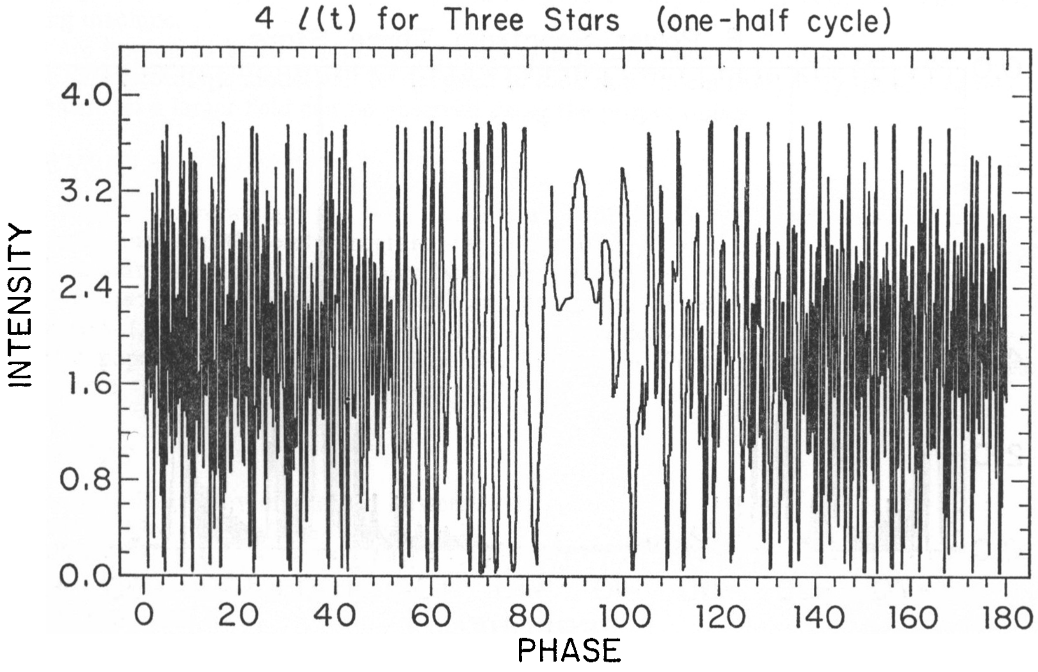
4 l(t) for Three Stars (one-half cycle)
INTENSITY
0.0
0.8
1.6
2.4
3.2
4.0
PHASE
0
20
40
60
80
100
120
140
160
180

Fig. 4

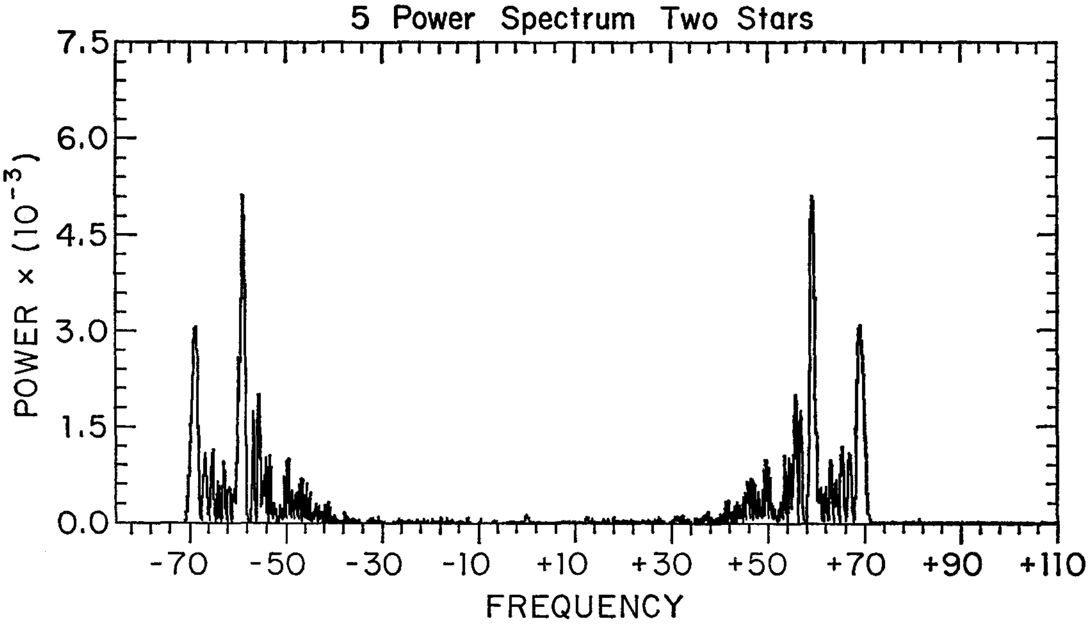
5 Power Spectrum Two Stars
POWER × (10^{-3})
7.5
6.0
4.5
3.0
1.5
0.0
-70
-50
-30
-10
+10
+30
+50
+70
+90
+110
FREQUENCY

Fig. 5

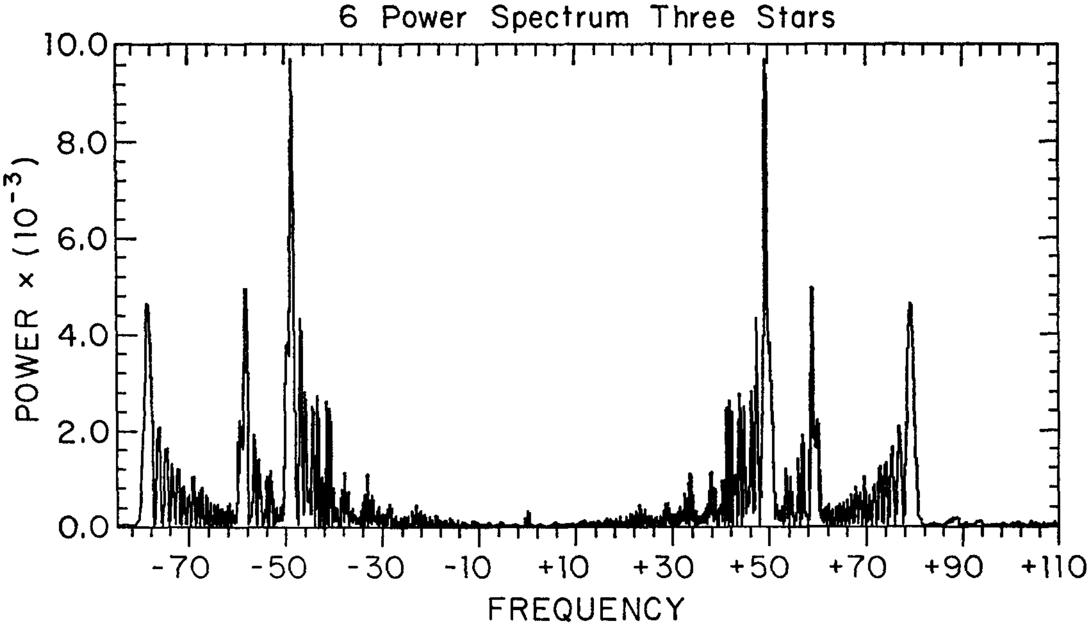
6 Power Spectrum Three Stars
POWER × (10^{-3})
10.0
8.0
6.0
4.0
2.0
0.0
-70
-50
-30
-10
+10
+30
+50
+70
+90
+110
FREQUENCY

Fig. 6

the Michelson interferometer to infrared spectroscopy. We have some reservations about the applicability of this last technique due to the low spectral resolution attainable.

V. *CONCLUSION*

From the foregoing the reader can deduce that we have completely and successfully simulated the problem in the computer. It remains to demonstrate the technique in the laboratory.

Certain "external" difficulties were encountered in the laboratory model; we do not have a computer compatible tape recorder, no funds were available for a high quality moiré pattern, photomultiplier tube, etc. Therefore, the laboratory demonstration consisted of a simulated star, a celluloid moiré pattern, an inexpensive d.c. motor, and an inexpensive photodiode whose output was amplified and fed as input to an oscilloscope that displayed the pattern by synchronizing to the motor speed. The results are displayed in Figure 7.

In the experiment a moiré pattern of 10 lines to the inch was used. The oscilloscope trace allowed us to compute that the pattern was not centred by 0.38 mm. Repeated "estimates" by eleven different students gave a value of 41.072 ± 0.008 mm. The point here is that even a coarse grid yielded an accuracy that is only one order of magnitude poorer than what we accept by standard techniques.

It is clear from this simple experiment that the technique works and holds great promise. Limitations on the technique arise from inaccurate centering of the pattern on the axis of rotation, inaccuracies in the parallelism and straightness of the edges of the pattern, the signal to noise ratio of the photomultiplier, sky brightness, the number of stars in the field, etc. We have looked at these in detail and can suggest methods around all but the parallelism, etc. of the pattern.

Consideration has been given to seeing effects, jitter and a slow or fast drive rate, etc. Any periodic or quasi periodic effect is picked up in the analysis routine and forms the basis of a restoring function. Any essentially linear drift through the field is revealed as a progressive change in the individual coordinates but not in the differences. Thus we have a way of restoring effects from all of the common problems and high frequency progressive scintillation effects as well.

The simulated models allow us to make some comparisons with present ground based efforts. A 200 line per inch fringe pattern will yield an accuracy of 0.5 microns with a 1 min integration on a 13th magnitude (visual) star in a 26-inch telescope. Photographic plates with automatic measuring yield an accuracy of 1 micron and require a 3 min exposure. They also require about 30 min time at the measuring machine.

We are hoping to build a prototype model of this instrument and make full scale tests on the telescope. The prototype model will be designed to look at a 7.5 cm field. Only the size of relay optics limit the field so a larger field can be observed using the proper optics.

VI. *ACKNOWLEDGEMENTS*

We wish to acknowledge Dr. S. J. Goldstein's contributions in discussions concerning power spectra techniques and the early programming efforts of J. K. Estes. This work has been supported in part by a grant from the United States National Aeronautics and Space Administration.

Added note: Since developing this work and presenting the results a paper by H. W. Schnopper and associates (*Ap.J.*, **161,** L161–167, Sep. 1970) has revealed several prior papers by L. Mertz (for example: a paper in Modern Optics, Brooklyn Polytechnic Press, 1967) containing the basic idea presented here.

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DISCUSSION

W. A. SHERWOOD: What is the loss of limiting magnitude, as compared with direct photography?

L. W. FREDRICK: There is an immediate loss of about 0.75 magnitude because half of the field is blanked out, but there is no real loss, because you can integrate, and in fact you can go very faint by this technique.

L. GRATTON: What is the angular resolution, compared with that of a photographic plate taken with the same telescope?

L. W. FREDRICK: We should be able to do as well as the area scanner does, and it gets down to 0.6 arcsec right now. It depends on how bright the two stars are.

R. E. NATHER: Have you evaluated the systematic errors yet? I noticed your curves were a bit asymmetric in frequency space after you'd done your transform.

L. W. FREDRICK: There are all sorts of pitfalls along the way; one glibly says that none of them are insurmountable, but they may very well prove to be insurmountable in the end. Clearly the tilt of the disk, the non-uniformity of rotation that the disk has, any wobble or axial run-out that is in the disk is going to cause trouble unless it is strictly periodic. If it's strictly periodic you can then fold it out in your analysis, for example periodic drive errors, periodic seeing errors or quasi-periodic seeing effects, and so on, can be folded out in the smoothing portion of the program. I think the asymmetry in the waveform was simply that I was only showing one-half of the waveform.

M. J. SMYTH: Spectroscopists may recognize a similarity to the "mock inteferometer" invented by Mertz; so this is really "mock astrometry", and you needn't apologize for introducing it into a colloquium on astrophysics!

L. W. FREDRICK: We have applied this to low-resolution spectra by displaying the spectrum and scanning with the disk, but the resolution is low and varies across the field. In fact, I forgot to point out that you have to measure the star field in four positions because of this, so you need four integrations to get the stated accuracy.

D. S. BROWN: What is the practical upper limit on the number of stars that can be measured in one field?

L. W. FREDRICK: The upper limit is actually in the computer, but it's clear that if you get into a very crowded field you get white noise. We have simulated up to 31 stars at 0.5 magnitude intervals between the stars, and this worked out all right. You can have three or four stars of the same magnitude as long as they're not too close together in the disk. If they get close together then you can't fold them out because they tend to widen and suppress everything. However, it seems that the Creator always puts at least one star much brighter than the others in the field, and you can get rid of it right away, by duplicating your tape and then taking out that star which obviously appears. This leaves you a new tape with that star gone, and then there's another star that sticks out, and you can take it out, and then pretty soon you sort them all out and you get down to the noise statistics.

N. N. MIHEL'SON: On a photographic plate the image is integrated in time; but what is the influence of seeing in your device?

L. W. FREDRICK: You get an integral, depending on the frequencies that are occurring in the seeing. The frequencies above your chopping frequency integrate, and so you get a star image profile that's more parabolic than gaussian, but frequencies right near your chopping frequency just tend to smear, and well below your chopping frequency they add in a periodic effect, which you take out.

W. LILLER: At what frequency is the Ronchi grating rotated?

L. W. FREDRICK: We've been chopping at 25 Hz, and if you have say a 500-line grating then you get the highest possible frequency at the edge of the disk where the maximum frequency is around 300KHz.