where A is a unit-vector (say $A = \cos\lambda + \iota\sin\lambda$) and B, B′ are conjugate vectors. Or, writing $B = b + \iota\beta$, $B' = b - \iota\beta$, the constants are λ, b, β; 3 constants as it should be."

Quaternion Synopsis of Hertz' View of the Electrodynamical Equations.

By Professor TAIT.

Note on Menelaus's Theorem.

By R. E. ALLARDICE, M.A.

§ 1. The object of this note is, in the first place, to show that Menelaus's Theorem, regarding the segments into which the sides of a triangle are divided by any transversal, is a particular form of the condition, in trilinear co-ordinates, for the collinearity of three points; and, in the second place, to point out an analogue of Menelaus's Theorem in space of three dimensions.

§ 2. In the usual system of areal co-ordinates, the x-co-ordinate of P (fig. 52) is $\Delta PBC/\Delta ABC$, that is PD/AD. Now let D, E, F, be three points in BC, CA, AB, respectively, dividing these sides in the ratios l_1/m_1, l_2/m_2, l_3/m_3; then the co-ordinates of D, E, F, are proportional to $(0, m_1, l_1)$, $(l_2, 0, m_2)$, $(m_3, l_3, 0)$. Hence the condition that D, E, F, lie on the straight line $Ax + By + Cz = 0$ is

$$\begin{vmatrix} 0 & m_1 & l_1 \\ l_2 & 0 & m_2 \\ m_3 & l_3 & 0 \end{vmatrix} = 0,$$

that is, $l_1 l_2 l_3 + m_1 m_2 m_3 = 0$, which is Menelaus's Theorem.

§ 3. In space of three dimensions we may use the corresponding system of tetrahedral co-ordinates, and obtain a theorem analogous to that of Menelaus.

Let BCD (fig. 53) be one of the faces of the tetrahedron; and put $a_2 = PB'/BB' = \Delta PCD/BCD$, $a_3 = PC'/CC' = \Delta PDB/\Delta CDB$, etc. Then the co-ordinates of P, Q, R, S, points in the four faces of the tetrahedron, may be written $(0, a_2, a_3, a_4)$, $(b_1, 0, b_3, b_4)$, etc.; and the condition that these four points be coplanar is

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$$\begin{vmatrix} 0 & a_2 & a_3 & a^4 \\ b_1 & 0 & b_3 & b_4 \\ c_1 & c_2 & 0 & c_4 \\ d_1 & d_2 & d_3 & 0 \end{vmatrix} = 0,$$

where a_2, a_3, and a_4 may be taken to be the three areas into which the point P divides the face BCD; and this condition is the analogue of Menelaus's Theorem for space of three dimensions.

Historical notes on a geometrical problem and theorem.

By J. S. Mackay. M.A., LL.D.

The problem is

Between two sides of a triangle to inflect a straight line which shall be equal to each of the segments of the sides between it and the base.

This problem was brought before the Society at the January meeting in 1884, and a solution of it by Mr James Edward will be found in our *Proceedings*, Vol. II, pp. 5–6, a second by myself in Vol. II., p. 27 (10th April 1884), a third by Mr R. J. Dallas in Vol. III., pp. 41–2 (9th January 1885). Solutions of a slightly more general problem were also given by myself in Vol. III., pp. 40–1, and reference made to the *Educational Times*, Vol. 37, p. 328 (1st October 1884), and to Vuibert's *Journal de Mathématiques Élémentaires*, 9e année, p. 45 (15th December 1884).

I have since found that the more general problem was proposed by Monsieur J. Neuberg in the *Nouvelle Correspondance Mathématique*, Vol, I., p. 110 (1874–5), and solved by him in Vol. II, p. 248 (1876); and quite recently I have discovered the first problem to go as far back as 1773–4. Here is how it occurs.

In the *Ladies' Diary* for 1773, Mr Thomas Moss proposes for solution the following: —

*The difference of the sides including a known angle of a plane triangle being given, and also the sum of one of those sides and that opposite the given angle, to construct the triangle.**

In 1774 the question is thus answered by Mr John Turner: —

"*Analysis.* Suppose the thing done, and that ABC is the

---

* Thomas Leybourn's *Mathematical Questions proposed in the Ladies' Diary*, Vol. II., p. 377 (1817).