SHORT NOTE

FINITE-ELEMENT STRESS ANALYSIS OF AVALANCHE SNOWPACKS

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ABSTRACT. The elastic stresses have been determined, in a single-layer homogeneous snowpack on a realistic avalanche slope, by a two-dimensional finite-element analysis. Calculation of the state of stress throughout the 0.96 m snow layer on a slope approximately that of the Lift Gully at Berthoud Pass, Colorado, resulted in reasonable stress values. In particular, both field experience and the calculated shear stresses predict avalanching in the lower-density snows. Also, tensile stresses were present only in the area of the observed fracture line.

RÉSUMÉ. *Analyse par éléments finis des efforts dans les manteaux neigeux à avalanches.* Les efforts élastiques ont été déterminés dans une couche unique homogène du manteau neigeux sur une pente à avalanche réelle, par une analyse bidimensionnelle aux éléments finis. Le calcul de l'état de contrainte à travers une couche de neige de 0,96 m d'épaisseur sur une pente voisine de celle du Lift Gully au Berthoud Pass, Colorado, aboutit à des valeurs raisonnables pour les efforts. En particulier, aussi bien l'expérience que les efforts de cisaillement calculés prédisent l'avalanche dans les niveaux de neige à plus faible densité. Aussi, les efforts de traction étaient présents seulement dans la zone de la ligne fracturée observée.

ZUSAMMENFASSUNG. *Spannungsanalyse in begrenzten Elementen von Lawinenschnee.* Durch eine zweidimensionale Analyse von begrenzten Elementen werden die elastischen Spannungen in einer homogenen, einfachen Schneeschicht an einem Lawinenhang bestimmt. Die Berechnung des Spannungszustandes innerhalb der gesamten 0,96 m dicken Schneeschicht auf einem Hang, der dem von Lift Gully am Berthoud Pass, Colorado, gleicht ergab vernünftige Spannungswerte. Im einzelnen ergaben sowohl die Erfahrung im Feld wie auch die berechneten Scherspannungen, dass die Lawinengefahr von den Schneeschichten mit geringerer Dichte ausgeht. Ebenso waren Zugspannungen nur in dem Gebiet der beobachteten Abrisslinie vorhanden.

INTRODUCTION

A significant goal in avalanche research is to predict the mechanical failure of a snowpack lying in an avalanche track. One requirement of an adequate model for snow failure is an accurate knowledge of the stress state throughout the snowpack. The geometric complexity of realistic snowpacks is a serious impediment to the calculation of stresses. Stress-analysis models which have been considered (Mellor, 1968) are extremely limited in their applicability because they are restricted to very simple geometries which cannot accurately approximate a realistic avalanche snowpack.

The finite-element method was used to provide the necessary flexibility to model a realistic geometry. The preliminary results presented are restricted to linear elasticity, but the analysis method is not. With little trouble, the effects of non-homogeneity and time-dependent properties may be incorporated. Also, time-dependent and non-linear constitutive laws may be adapted to the finite-element method.

FINITE-ELEMENT METHOD

The details of this technique have been fully discussed elsewhere (Zienkiewicz, 1967) so a brief description will suffice here. The two-dimensional snowpack was divided into many small elements as indicated in Figure 1. The stresses were assumed to be constant in each element, thus approximating the smoothly varying stress distribution with a series of steps. The conditions of equilibrium, compatibility and the stress–strain law are satisfied within each element and, in addition, overall equilibrium and compatibility between the elements are satisfied.

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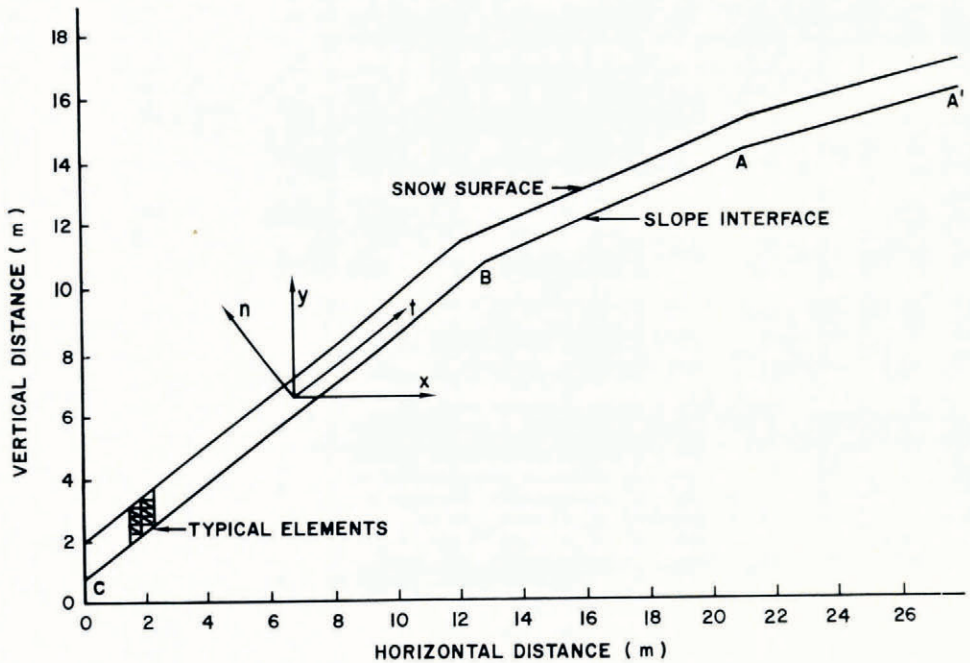
Fig. 1. Berthoud Pass element configuration.

Overall equilibrium and compatibility between elements leads to a matrix equation of the form

$$\{F\} = [K]\{u\} \tag{1}$$

which relates the forces to the displacements at all the nodes (triangle vertices) in the structure. In Equation (1), $\{F\}$ and $\{u\}$ are column matrices containing the components of the net force and the displacement, respectively, at each node. The matrix $[K]$ is called the stiffness matrix and depends only on the elastic constants and the size and shape of each element.

If the forces or displacements at the boundary node points are known, it is possible to solve Equation (1) for the unknown node-point displacements. Then the stress in each element may be computed from

$$\{\sigma\} \equiv \left\{\begin{array}{c} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{array}\right\} = [D]\{u\} \tag{2}$$

where $[D]$ depends only on the size and elastic constants of each element. A computer program presented by Wilson and Clough (unpublished) was adapted to this problem and the results computed using a CDC 6400 computer.

PROBLEM STATEMENT

The following assumptions were made to simplify the analysis:

   i. The problem is one of plane strain because the slope is reasonably flat along any contour and the stresses could only vary slowly in that direction.

  ii. The snowpack had a uniform depth of 0.96 m measured perpendicular to the slope.

 iii. The ground profile was as shown in Figure 1. This was measured in a rough survey of Lift Gully at Berthoud Pass, Colorado, with a hand level.

iv. On the upper snow surface, the normal and shear stresses were zero.
 v. At the snow–ground interface, the displacements were zero (zero glide).
 vi. The snowpack was homogeneous.
vii. The density was varied between 100 and 400 kg m^{-3} for different cases.
viii. Young's modulus varied between 0.2×10^9 and 1.2×10^9 N m^{-2}.
 ix. Poisson's ratio was varied between 0.1 and 0.3 for different cases.

RESULTS

Figure 2 presents a map of the tangential and shear stresses at representative points throughout the model. They were calculated for the following conditions: density $= 100$ kg m^{-3}; Young's modulus $= 0.2 \times 10^9$ N m^{-2}; and Poisson's ratio $= 0.3$.

The effect of varying the density and Poisson's ratio is shown in Figure 3. The stresses were found to vary linearly with density over the range 100–400 kg m^{-3}. The stresses were found to be independent of Young's modulus. These results are for an element at B in Figure 2, but the variations are similar throughout the model.
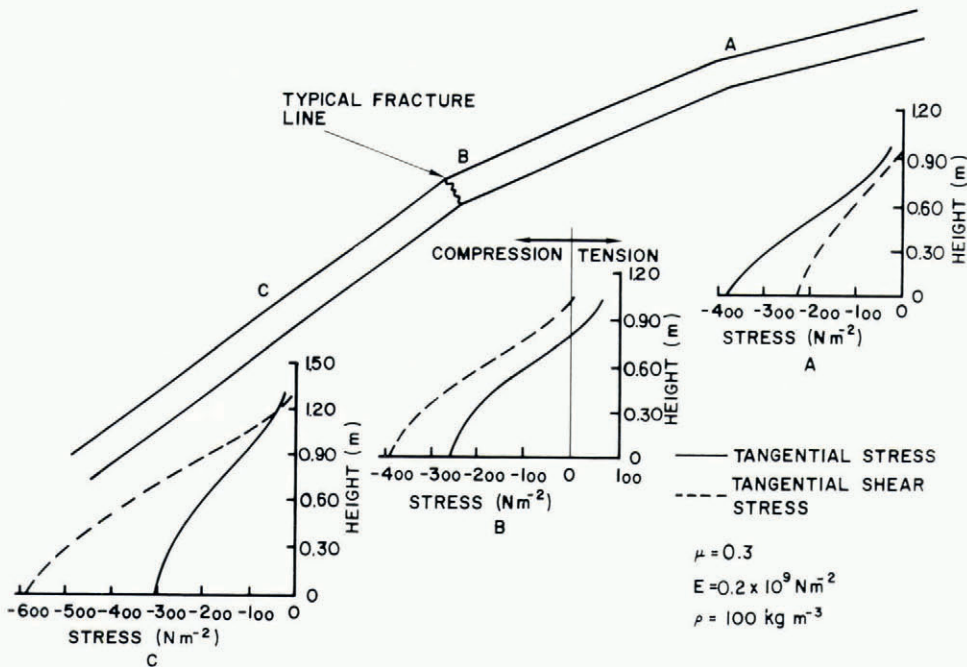

Fig. 2. Variation of stress with height at three slope locations.

Figure 4 compares the maximum shear stress found in the snowpack with a shear-strength envelope presented by Mellor (1966). The dashed lines at the lower end of the shear-strength envelope represent a slight extrapolation of Mellor's curve which did not go below shear strengths of 980.7 N m^{-2}. Figure 4 indicates two things: i. For snows with densities of 100 to 200 kg m^{-3} the maximum shear stresses are well within the failure envelope, indicating a high probability of avalanche. ii. The figure indicates that more dense snows provide a more stable situation which corresponds with field experience.

The maximum shears plotted on Figure 4 were found in the elements at the bottom of the snowpack close to B, but the shears were large and of nearly constant magnitude all along the bottom from B to a considerable distance below C. This indicates that failure might occur by shearing in this region which would precipitate cracking near B where the tensile stresses occur.
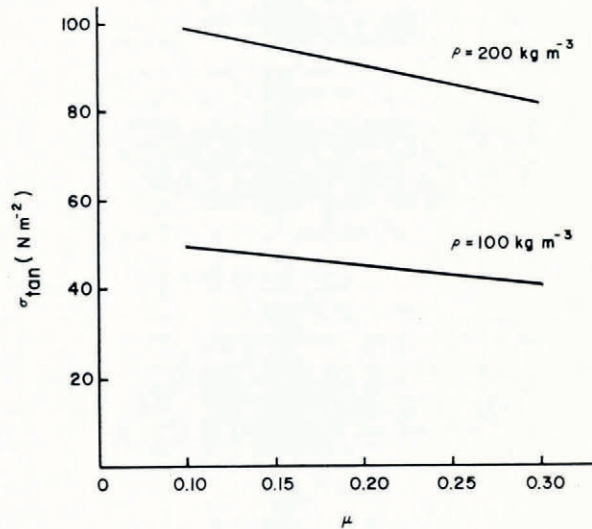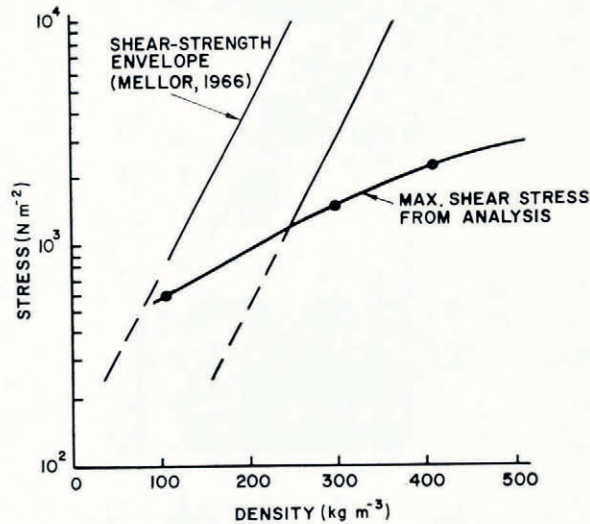
Fig. 3. *Tangential stress versus Poisson's ratio.*

Fig. 4. *Comparison of maximum shear with shear strength.*

CONCLUSION

The applicability of the present results is severely limited by the assumptions. However, even under these extreme simplifications, realistic stress values were obtained. Experience has shown that a layer of the type modeled would have avalanched from this track. The calculated stresses indicate shear-failure initiation. Furthermore, the fact that tensile stresses occur only in the region of the observed fracture line is supported by observations of avalanches in the track. These results indicate that fairly simple mechanical characterizations of snow may give results of practical accuracy by using the finite-element method.

NOMENCLATURE

$\{F\}$ Matrix of forces at vertices of all triangular finite elements (node points).
$\{u\}$ Matrix of displacements at node points.
$[F]$ Stiffness matrix.
$\{\sigma\}$ Matrix of stress components in each element.
ρ Density (kg m^{-3}).
E Young's modulus (N m^{-2}).
μ Poisson's ratio.

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