Brane Universes Tested by Supernovae

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Summary. We discuss observational constrains coming from supernovae Ia imposed on the behavior of the Randall-Sundrum models. It is interesting that brane models predict brighter galaxies for such redshifts which is in agreement with the measurement of the $z = 1.7$ supernova. We also demonstrate that the fit to supernovae data can also be obtained, if we admit the "super-negative" dark energy $p = -(4/3)\varrho$ on the brane, where the dark energy in a way mimics the influence of the cosmological constant.

1 Introduction

In recent times a lot of effort has been done on the idea that our Universe is a boundary of a higher-dimensional spacetime manifold [1]. Among superstring theories which may unify all interactions M-theory is a strong candidate for description of the real world. In this theory, gravity is a higher-dimensional theory, becoming effectively 4-dimensional at lower energies. In the standard model matter fields are confined to the 3-brane while gravity can, by its universal character, propagate in all extra dimensions. In the brane world models inspired by string/M theory [3, 4] new two parameters are introduced, namely brane tension λ and dark radiation U.

We should mention that before Randall and Sundrum (R-S) work [3, 4], where they proposed a mechanism to solve the hierarchy problem by a small extra dimension, large extra dimensions were proposed to solve this problem [1]. This gives an interesting feature because TeV gravity might be realistic and quantum gravity effects could be observed by next generation particle colliders. The Newtonian gravity potential on the brane is recovered at lowest order $V(r) = \frac{GM}{r}(1 + \frac{2l^2}{3r^2})$.

In this paper we demonstrate that if the brane world in the R-S version is realistic, we may find some evidence of higher-dimensions.

2 Brane Universes

In [7] we gave the formalism to express dynamical equations in terms of dimensionless observational density parameters Ω_i. In this notation the Friedman equation for brane universes takes the form:

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$$\frac{1}{a^2}\left(\frac{da}{dt}\right)^2 = \frac{C_\gamma}{a^{3\gamma}} + \frac{C_\lambda}{a^{6\gamma}} - \frac{k}{a^2} + \frac{\Lambda_{(4)}}{3} + \frac{C_{\mathcal{U}}}{a^4}, \tag{1}$$

where $a(t)$ is the scale factor, $k = 0, \pm 1$ the curvature index (here we use natural system of units in which $8\pi G = c = 1$), $\Lambda_{(4)}$ the 4-dimensional cosmological constant, and γ the barotropic index ($p = (\gamma - 1)\varrho$ (p = the pressure, ϱ = the energy density). The constants are $C_\lambda = 1/6\lambda \cdot a^{6\gamma}\varrho^2$ and $C_{\mathcal{U}} = 2/\lambda \cdot a^4\mathcal{U}$; C_λ comes as a contribution from the brane tension λ, and $C_{\mathcal{U}}$ as a contribution from the dark radiation.

Because the ρ^2 term and dark radiation term do not appear in the standard cosmology, such terms could provide a small window to see the extra dimensions. It is useful to rewrite Eq. 1 to the dimensionless form. The brane universe can be interpreted as the standard Universe filled with the mixture of matter with the equation of state $p_i = (\gamma_i - 1)\rho_i$. Then we obtain a basic equation in the form:

$$\frac{\dot{x}^2}{2} = \frac{1}{2}\Omega_{k,0} + \frac{1}{2}\sum_i \Omega_{i,0}x^{2-3\gamma_i} = -V(x) \tag{2}$$

$$\ddot{x} = -\frac{1}{2}\sum_i \Omega_{i,0}(2 - 3\gamma_i)x^{1-3\gamma_i} = -\frac{\partial V(x)}{\partial x} \tag{3}$$

where $i = (\gamma, \lambda, \Lambda, U)$, and $x \equiv \frac{a}{a_0}$, $T \equiv |H_0|t$, $\dot{} \equiv \frac{d}{dT}$, t is the original cosmological time, and V is the potential function. Therefore the dynamics of the considered model is equivalent to introducing fictitious fluids which mimic the ρ^2 contribution and the dark energy term.

The above relations allow one to write down an explicit redshift-magnitude relation for the brane models to study their compatibility with astronomical data which is the subject of the present paper. Obviously, the luminosity of galaxies depends on the present densities of the different components of matter content, Ω_i, and their equations of state reflected by the value of the barotropic index, γ_i.

If the apparent luminosity of the source as measured by the observer is $l = L/4\pi d_L^2$, then the luminosity distance d_L of the source is defined by the relation

$$d_L = (1+z)a_0 r_1 \equiv \frac{\mathcal{D}_L(z)}{H_0}, \tag{4}$$

and $\mathcal{D}_L$ is the dimensionless luminosity distance.

The observed and absolute luminosities are defined in terms of K-corrected apparent and absolute magnitudes m and M. When written in terms of m and M, we obtain

$$m(z) = \mathcal{M} + 5\log_{10}[\mathcal{D}_L(z)], \tag{5}$$

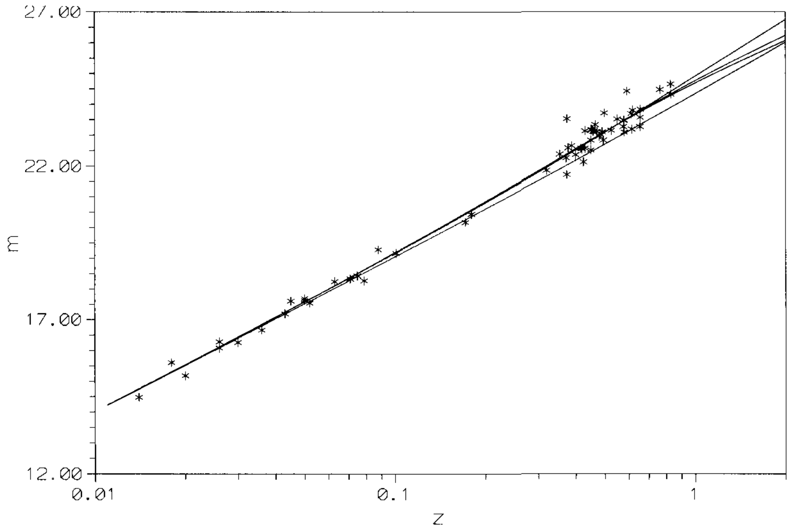

Fig. 1. The Redshift-magnitude relation for $\gamma = 1$ brane universes (dust on the brane). The top line is the best-fit Perlmutter model. The bottom line is a flat model with $\Omega_{m,0} = 1$. Between these two lines there are brane models with $\Omega_{\lambda,0} \neq 0$: lower–the best-fit non-flat model; higher–the best-fit flat model.

where $\mathcal{M} = M - 5\log_{10} H_0 + 25$. For the homogeneous and isotropic Friedman models one gets

$$\mathcal{D}_L(z) = \frac{(1+z)}{\sqrt{\mathcal{K}}} S(\chi) \tag{6}$$

where $S(\chi) = \sin\chi$ for $\mathcal{K} = -\Omega_{k,0}$; $S(\chi) = \chi$ for $\mathcal{K} = 1$; $S(\chi) = \sinh$ for $\mathcal{K} = \Omega_{k,0}$. The dimensionless parameters are $\Omega_k = -\frac{k}{H^2 a^2}$, $\Omega_\gamma = \frac{1}{3H^2}\varrho$, $\Omega_\lambda = \frac{1}{6H^2\lambda}\varrho^2$, $\Omega_{\Lambda_{(4)}} = \frac{\Lambda_{(4)}}{3H^2}$, and $\Omega_{\mathcal{U}} = \frac{2}{H^2\lambda}\mathcal{U}$.

3 Testing Brane Models

Now we test brane models using the Perlmutter sample [2]. In order to avoid any possible selection effects, we use the full sample called sample A (usually, one excludes two data points as outliers and another two points, presumably reddened, from the full sample of 60 supernovae).

First of all, we estimated the value of $\mathcal{M}$ from the sample of 18 low redshift supernovae, also testing our result by the full sample of 60 supernovae. We should note that, in fact, we have an ellipsoid of admissible models in a three-dimensional parameter space $\Omega_{m,0}$, $\Omega_{\lambda,0}$, $\Omega_{\Lambda_{(4)},0}$ at hand. Then, we have more freedom than in the Perlmutter et al. [2] analysis where they had only an ellipse in a two-dimensional parameter space $\Omega_{m,0}$, $\Omega_{\Lambda_{(4)},0}$. For a flat model

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$\Omega_{k,0} = 0$ we obtain "corridors" of possible models. Formally, the best-fit flat model is $\Omega_{m,0} = 0.01$, $\Omega_{\lambda,0} = 0.09$ $\chi^2 = 94.7$ which is again unrealistic. In the realistic case we obtain for a flat model $\Omega_{m,0} = 0.25$, $\Omega_{\lambda,0} = 0.02$, $\Omega_{\Lambda_{(4)},0} = 0.73$ with $\chi^2 = 95.6$. One should note that all realistic brane models require also the presence of the positive 4-dimensional cosmological constant ($\Omega_{\Lambda_{(4)},0} \sim 0.7$).

In Fig. 1 we present plots of the redshift-magnitude relation against the supernovae data. One can observe that in both cases (best-fit and best-fit flat models) the difference between brane models and a flat model with $\Omega_{\Lambda_{(4)},0} = 0$ is largest for $0.6 < z < 0.7$ while it significantly decreases for the higher redshifts. It gives us a possibility to discriminate between the Perlmutter model and brane models when the data from high-redshift supernovae $z > 1$ are available. It is interesting that brane models predict brighter galaxies for such redshifts which is in agreement with the measurement of the $z = 1.7$ supernova [5, 6]. In other words, if the distant ($z > 1$) supernovae were brighter, the brane universes would be favored.

For completeness, we also made our analysis using samples B and C. It emerged that it does not significantly changes our results, although it increases the quality of the fit.

Formally, the best fit for the sample B is (56 supernovae) ($\chi^2 = 57.3$): $\Omega_{k,0} = -0.1$, $\Omega_{m,0} = 0.17$, $\Omega_{\lambda,0} = 0.06$, $\Omega_{\Lambda_{(4)},0} = 0.87$. For the flat model we obtain ($\chi^2 = 57.3$): $\Omega_{m,0} = 0.12$, $\Omega_{\lambda,0} = 0.06$, $\Omega_{\Lambda_{(4)},0} = 0.82$, while for a "realistic" model we obtain ($\chi^2 = 57.6$): $\Omega_{m,0} = 0.25$, $\Omega_{\lambda,0} = 0.02$.

Formally, the best fit for the sample C (54 supernovae) ($\chi^2 = 53.5$) gives $\Omega_{k,0} = 0$ (flat) $\Omega_{m,0} = 0.21$, $\Omega_{\lambda,0} = 0.04$, $\Omega_{\Lambda_{(4)},0} = 0.75$, while for a "realistic" model ($\chi^2 = 53.6$): $\Omega_{m,0} = 0.27$, $\Omega_{\lambda,0} = 0.02$.

One should note that we have also separately estimated the value of $\mathcal{M}$ for sample B and C. We obtained $\mathcal{M} = -3.42$. However, if we use this value in our analysis it does not change significantly the results (χ^2 does not change more than 1, which is a marginal effect for χ^2 distribution for 53 or 55 degrees of freedom).

In Fig. 2 we present a redshift-magnitude relation for brane models with dark energy ($\gamma = -1/3$). Note that the theoretical curves are very close to that of Perlmutter's which means that the dark energy cancels the positive-pressure influence of the ϱ^2 term and can simulate the negative-pressure influence of the cosmological constant to cause cosmic acceleration. From the formal point of view the best fit is ($\chi^2 = 95.4$): $\Omega_{k,0} = 0.2$, $\Omega_{d,0} = 0.7$, $\Omega_{\lambda,0} = -0.1$, $\Omega_{\mathcal{U}} = 0.2$, $\Omega_{\Lambda_{(4)},0} = 0$, which means that the cosmological constant must necessarily vanish. From this result we can conclude that the dark energy $p = -(4/3)\varrho$ can mimic the contribution from the $\Lambda_{(4)}$-term in standard models. For the best-fit flat model ($\Omega_{k,0} = 0$) we have ($\chi^2 = 95.4$): $\Omega_{d,0} = 0.2$, $\Omega_{\lambda,0} = -0.1$, $\Omega_{\mathcal{U}} = 0.2$, $\Omega_{\Lambda_{(4)},0} = 0.7$.

Finally, let us study the angular diameter test for brane universes. The angular diameter of a galaxy is defined by

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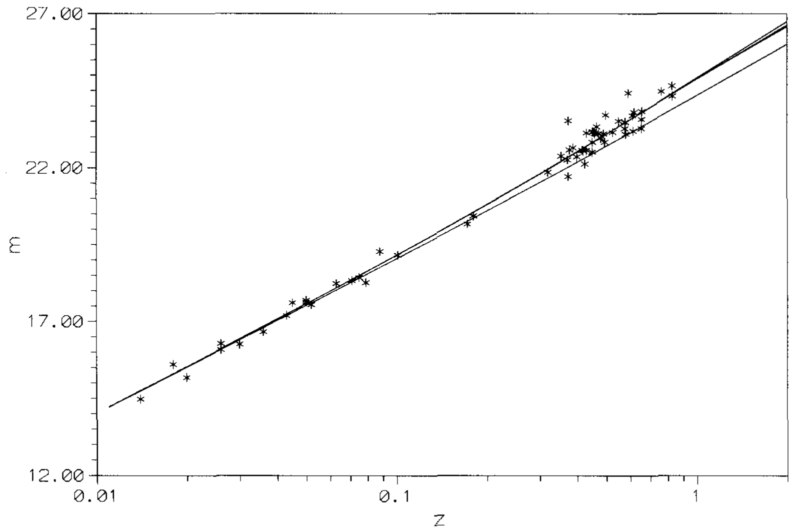

Fig. 2. The redshift-magnitude relation for $\gamma = -1/3$ brane universes (phantom on the brane). The top line is the Perlmutter model and the bottom line is the Einstein-de Sitter model. In the middle are two overlapping lines for the best-fitted and best-fitted flat brane models.

$$\theta = \frac{d(z+1)^2}{d_L}, \tag{7}$$

where d is a linear size of the galaxy. In a flat dust ($\gamma = 1$) universe θ has the minimum value $z_{min} = 5/4$. The dark radiation can enlarge the minimum value of θ while the ordinary radiation lowers this value $z_{min} = (\Omega_{\mathcal{U}} - 1 + \sqrt{3\Omega_{\mathcal{U}} + 1})/2\mathcal{U} \geq \frac{5}{4}$ for $\Omega_{\mathcal{U}} \leq 0$. This is a general influence of negative dark radiation onto the angular diameter size for brane models.

More detailed analytic and numerical studies show that the increase of z_{min} is even more sensitive to negative values of $\Omega_{\lambda,0}$ (ϱ^2 contribution). Similarly as for the dark radiation $\Omega_{\mathcal{U}}$, the minimum disappears for some large negative Ω_λ. Positive $\Omega_{\mathcal{U}}$ and Ω_λ make z_{min} decrease.

Finally, we also obtain that at high redshifts the expected luminosity of supernovae Ia should be brighter than in Perlmutter model. For the best fit value we obtain $\Omega_{\lambda,0} \simeq 0.01$ which seems to be unrealistic.

This occurs because, in the R-S model, there is a constraint on the parameter $\Omega_{\lambda,0}$ from the requirement of not violating the four-dimensional gravity on sufficiently large spatial scale. This constraint requires that the value of λ is less than about $(100\ \mathrm{GeV})^4$, which means that during the late epoch the ρ^2 term in the model should be small. Thus, the obtained value of $\Omega_{\lambda,0} \sim 0.01$ is the observational limit which is not based on theoretical model assumptions.

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