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Distribution functions for resonantly trapped orbits in our Galaxy

G. Monari[1], **B. Famaey**[2], **J.-B. Fouvry**[3] **and J. Binney**[4]

[1]The Oskar Klein Centre for Cosmoparticle Physics, Dept. of Physics, Stockholm University
AlbaNova, 10691 Stockholm, Sweden
email: giacomo.monari@fysik.su.se

[2]Université de Strasbourg, CNRS UMR 7550, Observatoire astronomique de Strasbourg
11 rue de l'Université, 67000 Strasbourg, France

[3]Institute for Advanced Study
Einstein Drive, Princeton, NJ 08540, USA

[4]Rudolf Peierls Centre for Theoretical Physics
Keble Road, Oxford OX1 3NP, UK

Abstract. We show how to capture the behaviour of the phase-space distribution function (DF) of a Galactic disc stellar population at a resonance. This is done by averaging the Hamiltonian over fast angle variables and re-expressing the DF in terms of a new set of canonical actions and angles variables valid in the resonant region. We then assign to the resonant DF the time average along the orbits of the axisymmetric DF expressed in the new set of actions and angles. This boils down to phase-mixing the DF in terms of the new angles, such that the DF for trapped orbits only depends on the new set of actions. This opens the way to quantitatively fitting the effects of the bar and spirals to Gaia data in terms of distribution functions in action space.

Keywords. Galaxy: kinematics and dynamics, Galaxy: disk, Galaxy: structure

1. Orbits near a resonance

Let us consider stars on the Galactic plane and a non-axisymmetric perturbation in the form of Fourier m-modes, e.g. a quadrupole bar like Dehnen (2000). Let us also work here within the epicyclic approximation, in which radial oscillations are harmonic with angular frequency $\kappa(R)$, so the radial action $J_R = E_R/\kappa$, where $E_R = E - E_\mathrm{c}$ is the energy of these oscillations, E_c being the energy of a circular orbit with the same angular momentum J_ϕ. We can use this approximation to rewrite the perturbing potential in the Galactic plane in terms of actions and angles.

We have three main resonances when a star's (unperturbed) orbital frequencies Ω_R and Ω_ϕ respect

$$l_R\Omega_R + m(\Omega_\phi - \Omega_\mathrm{b}) = 0, \tag{1.1}$$

where Ω_b is the pattern speed of the perturbation: $l_R = \pm 1$ for the inner and outer Lindblad resonances, $l_R = 0$ for the corotation resonance. We can study the response of the DF to the perturbation linearizing the collisionless Boltzmann equation, see e.g. Monari *et al.* (2016) and the proceeding by B. Famaey *et al.* in the same volume. However, this treatment diverges at the resonances.

To study the response of the DF at the resonances, we start from the action and angle variables $(J_R, J_\phi, \theta_R, \theta_\phi)$ [see Binney & Tremaine (2008)], and we can approximate the behaviour of orbits near the resonances with the canonical transformation $(J_R, J_\phi, \theta_R, \theta_\phi) \to (J_\mathrm{s}, J_\mathrm{f}, \theta_\mathrm{s}, \theta_\mathrm{f})$:

$$\theta_\mathrm{s} = l\theta_R + m(\theta_\phi - \Omega_\mathrm{b}t), \quad \theta_\mathrm{f} = \theta_R, \quad J_\mathrm{s} = J_\phi/m, \quad J_\mathrm{f} = J_R - lJ_\mathrm{s}. \tag{1.2}$$

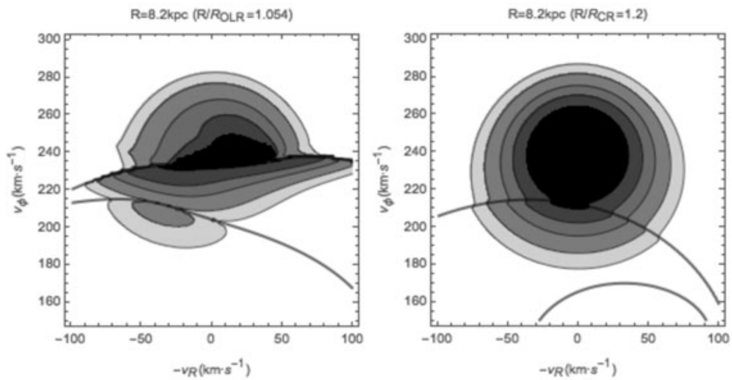

Figure 1. Velocity distribution functions for stars nearby the Sun for models with fast and slow pattern speed bar and a flat circular velocity curve. The thick lines correspond to zones of trapping ($k < 1$). Left: $\Omega_b = 1.8\Omega_0$, nearby outer Lindblad resonance. Right: $\Omega_b = 1.2\Omega_0$, nearby corotation. From Monari *et al.* (2017).

The angle θ_s is called 'slow angle' because near the resonance $\Omega_s \equiv \dot{\theta}_s \approx 0$, and quantifies the azimuth of the apocentra of the orbit in the reference frame corotating with the bar.

We can define $J_{s,res}$ as the J_s satisfying $\Omega_s(J_s, J_f) = 0$ at a certain J_f (in the axisymmetric potential). We can average the star's Hamiltonian H along θ_f (because it evolves much faster than θ_s and J_s), and expand in J_s around $J_{s,res}$ near the resonances. The action J_f is a constant of motion, and we obtain a pendulum Hamiltonian

$$H \approx \frac{1}{2}G(J_s - J_{s,res})^2 - F\cos(\theta_s + g), \tag{1.3}$$

where $G \equiv (\partial\Omega_s/\partial J_s)(J_{s,res}, J_f)$, and F depends on the amplitude of the perturbing potential. The pendulum has energy $E_p = H/G$, natural frequency $\omega_0^2 = FG$, and action/angle variables (J_p, θ_p), and $J_s(J_p, \theta_p)$. The quantity $k = [1/2(1 + E_p/\omega_0^2)]^{1/2}$ determines if the orbits are trapped or not to the resonance. For $k < 1$ an orbit is trapped, i.e. θ_s librates up to a maximum value $\theta_{s,max}$, while for $k > 1$ it is circulating, i.e. θ_s covers the whole $[0, 2\pi]$ range.

2. Distribution functions near a resonance

We assume that the distribution function for trapped and circulating orbits is:

• for $k < 1$ (trapped orbits): $f_{tr}(J_f, J_p) = \overline{f_0} \equiv \frac{1}{2\pi}\int_0^{2\pi} f_0(J_f, J_s(J_p, \theta_p))\mathrm{d}\theta_p$ [Binney (2016)], i.e. we phase-mix along the trapped orbits,

• for $k > 1$ (circulating orbits): $f_{circ}(J_f, J_p) = f_0(J_f, \overline{J_s}(J_p, \theta_p))$, i.e. the value of the correspondent unperturbed orbital torus,

where f_0 is the unperturbed DF. In Fig. 1 we show an application related to the velocity distribution of stars in the Solar neighbourhood in the case of a fast and slow rotating bar.

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