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# Nonspherical dust in exterior resonances with Neptune

**M. Kocifaj[1], J. Klačka[2] and H. Horvath[1]**

[1]Institute for Experimental Physics, University of Vienna, Boltzmangasse 5, 1090 Wien, Austria, email: kocifaj@ap.univie.ac.at

[2]Department of Astronomy, Physics of the Earth, and Meteorology, Faculty of Mathematics, Physics and Informatics, Comenius University, 842 48 Bratislava, Slovak Republic, email: klacka@fmph.uniba.sk

**Abstract.** Nonspherical dust grains orbiting the Sun are influenced by solar electromagnetic radiation. Interaction of electromagnetic radiation with nonspherical grains is complex and analogy with spherical grains may not be physically justified. As a consequence, equation of motion for nonspherical grain is more general than the equation of motion for spherical particle. Application of this more general interaction to possible trapping of the grain in resonances with planet Neptune is investigated. Approximation of the planar circular restricted three-body problem with action of solar electromagnetic radiation on nonspherical grain is used.

The orbital evolution of nonspherical dust grains of radius ≈ 2 micrometers is numerically calculated. Attention is payed to exterior resonances with the Neptune, and the numerical experiments are concentrated on their possible high capture efficiency. Physical difference between possibilities for resonant captures for spherical and nonspherical dust particles is pointed out.

**Keywords.** Dust grain, electromagnetic radiation, Celestial Mechanics

## 1. Introduction

Scientists have intensively studied the effects of resonances with solar system planets during the last two decades. The orbital evolution of dust grains near resonances with planets was also investigated. In addition to gravitational forces, the influence of solar electromagnetic radiation in the form of the Poynting-Robertson effect is usually taken into account (Robertson 1937, Klačka 2004), see also Jackson & Zook (1992), Marzari & Vanzani (1994), Šidlichovský & Nesvorný (1994), Liou & Zook (1995), Liou *et al.* (1995).

In reality Poynting-Robertson effect (P-R effect) holds only if a special condition is fulfilled (see Eq. (120) or Eq. (122) in Klačka 1992; compare Eqs. (40) and (48) in Klačka 2004). The nonspherical particles do not fulfill this condition. Thus, we have to take into account effect of solar electromagnetic radiation in an appropriate form.

## 2. Model and equation of motion

All our numerical simulations are based on equation of motion written in the form (see Klačka & Kocifaj 2001 for more details)

$$\frac{d\vec{v}}{dt} = - \frac{4\,\pi^2}{r^2}\,\vec{e}_R - 4\,\pi^2\,\frac{m_P}{M_\odot}\left\{\frac{\vec{r}-\vec{r}_P}{|\vec{r}-\vec{r}_P|^3} + \frac{\vec{r}_P}{|\vec{r}_P|^3}\right\} + \beta\,\frac{4\,\pi^2}{r^2}\sum_{j=1}^{3}\frac{Q'_{prj}}{Q'_{pr1}}\left\{\left(1 - 2\,\frac{\vec{v}\cdot\vec{n}_1}{c} + \frac{\vec{v}\cdot\vec{n}_j}{c}\right)\vec{n}_j - \frac{\vec{v}}{c}\right\},$$

$$\vec{n}_1 \equiv \vec{e}_R \equiv \vec{r}/|\vec{r}|\ ;\quad \beta = 7.6\times 10^{-4}\,Q'_{pr1}\,\frac{A'[m^2]}{m\,[kg]}, \qquad (2.1)$$

where $\vec{r}$ is the position vector of the particle and $\vec{r}_P$ is the position vector of the planet (Neptune) which moves in circular orbit ($m_P$ is mass of the planet, $M_\odot$ is mass of the Sun). As for the model of the cosmic dust particle we have used particle morphologically identical to U2015B10 (Clanton *et al.* 1984, Kocifaj *et al.* 1999). The particle shape appears to be representative of cosmic dust, since its aspect ratio (equal to 1.4) coincides with results obtained by means of the mid-infrared spectropolarimetry (Hildebrand & Dragovan 1995). The characteristics of radiation scattered by the cosmic dust particle were calculated using the discrete dipole approximation (DDA; Draine (1988)). We have made simulations for effective radius of the particle corresponding to 2 microns. Several samples of materials with different optical properties were considered: ice (mass densities 1 $g/cm^3$ and 2 $g/cm^3$), magnesium-rich silicate (mass densities 2 $g/cm^3$ and 4 $g/cm^3$), and, iron-rich silicate (mass densities 6 $g/cm^3$ and 8 $g/cm^3$). Central acceleration $- 4\pi^2(1-\beta)\ \vec{e}_R/r^2$ is used in calculations of orbital elements.

## 3. The most important results

Simulations for various optical properties were done by sampling the initial positions of nonspherical particle with respect to the Neptune in all angles. It was assumed that initial position of the grain was in the plane of the planet's orbit. In our computations we sampled the initial distances from the Sun in the range ( $\approx 1.0$, 1.5)-times the distance between the Sun and the planet. Various initial orientations of the particle's rotation axis were considered (rotation axis was fixed during the particle's motion).

Application of Eq. (2.1) for the case $Q'_{pr2} = Q'_{pr3} = 0$ yielded standard positions for resonances (see Eq. (4.1) below: the fraction of orbital periods $T/T_P$ defines type of the resonance). However, real particles exhibit nonzero values for $Q'_{pr2}$ and $Q'_{pr3}$ and their values may be much more important than the ratio $v/c$ present in the P-R effect (Klačka & Kocifaj 2001). Our simulations for orbital evolution of micron-sized dust particles show several temporary captures of grains in exterior resonances with the planet Neptune (capture with time larger than 4 $\times 10^4$ years was not found for ice particles). Results for 2 micron sized dust grains are presented in Table 1. One has to take into account that particle's orientation (with respect to the incident radiation) change during its motion and the presented values of $\beta$ may change within a few percent.

**Table 1.** Capture times for nonspherical particle of effective radius of 2 microns.

| material | mass density [ $g/cm^3$ ] | $\beta$ [$10^{-2}$] | resonance | capture times [ $10^3$ years] |
|---|---|---|---|---|
| magnesium-rich silicate | 4.0 | 3.60 | 7/6 | $> 40$ |
| magnesium-rich silicate | 4.0 | 3.60 | 9/8 | $> 50$ |
| iron-rich silicate | 6.0 | 2.43 | 3/2 | $> 40$ |
| iron-rich silicate | 6.0 | 2.43 | 8/7 | $> 50$ |
| iron-rich silicate | 6.0 | 2.43 | 5/3 | $> 160$ |

Fig. 1 illustrates time evolution of semimajor axis and eccentricity for nonspherical iron-rich silicate dust grain. While spherical particle is characterized by a decrease of semimajor axis outside resonance (P-R effect), nonspherical dust grain may exhibit also an increase of semimajor axis outside the resonance; analogous situation holds for eccentricity. The resonance 5/3 corresponds to semimajor axes 41.81 AU $\leqslant a \leqslant$ 42.02 AU, while its theoretical value for $\beta = 0.0243$ equals to $a = 41.908$ AU, according to Eq. (4.1).

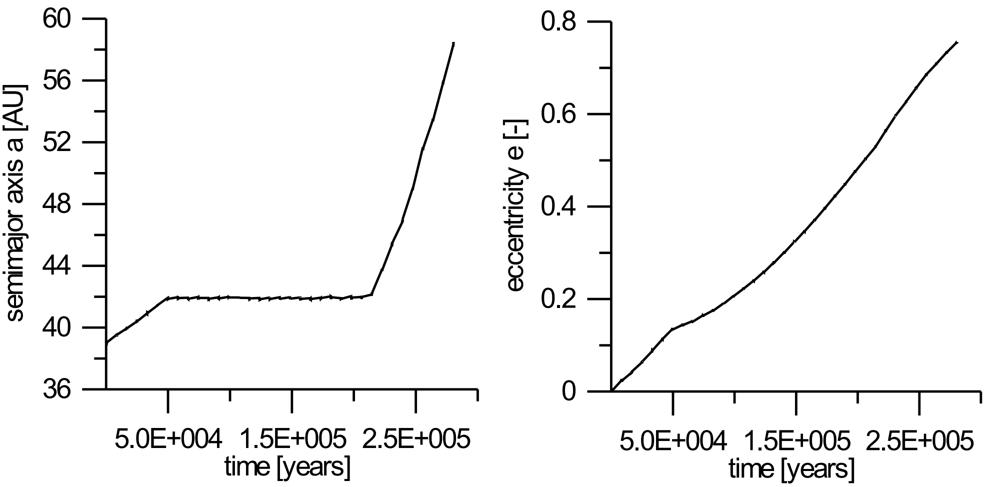


**Figure 1.** Time evolution of semimajor axis and eccentricity for nonspherical iron-rich silicate dust grain of effective radius 2 microns. The plots show exterior resonance 5/3 with Neptune.

## 4. Spherical/nonspherical particles and resonances – comparison

There are important differences in orbital evolution for spherical and nonspherical particles.

While spherical particles can be described by the P-R effect, this may not hold for micron-sized nonspherical particles. While P-R effect exhibits decrease of semi-major axis before trapping in a resonance, interaction of electromagnetic radiation with nonspherical grain may initiate also an increase of semi-major axis close to the resonant regions.

Spherical particles always move in the plane of the planetary orbit, if the initial orbital angular momentum vector of the particle was normal to the plane of the planetary orbit. This does not hold for nonspherical particles – inclination of the particle's orbit is not conserved.

Another important physical difference between the orbital evolution of spherical and nonspherical particles concerns the values of parameter $\beta$. This parameter is a constant of motion for spherical particles. Thus, the third Kepler's law yields

$$\frac{a}{a_P} = (1 - \beta)^{1/3} \left(\frac{T}{T_P}\right)^{2/3} \left(1 + \frac{m_P}{M_\odot}\right)^{-1/3} , \tag{4.1}$$

for the semimajor axes and periods of revolution around the Sun of the particle and the planet. Defining any resonance by the ratio $T/T_P$, Eq. (4.1) yields immediately the ratio $a/a_P$. However, parameter $\beta$ is not conserved during the motion of nonspherical particle. If its relative change is $\varrho_\beta$, then (approximately)

$$\varrho_a = \frac{\beta}{3\ (1 - \beta)}\ \varrho_\beta , \tag{4.2}$$

where $\varrho_a = \sigma_a/a$, $\varrho_\beta = \sigma_\beta/\beta$, $\sigma_a$ and $\sigma_\beta$ are standard deviations for mean values $a$ and $\beta$. The consequence of this equation is, that several first and higher order resonances may overlap for larger values of $\beta$. Moreover, Eq. (4.2) shows that the change of semimajor axis $a$ in orbital resonance caused by the change of parameter $\beta$ may be relatively large. Thus, one should expect that resonant capture times for nonspherical particles are smaller than the capture times for spherical grains.

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## 5. Conclusion

Our numerical simulations have shown that resonant captures of dust grains occur for exterior resonances with the planet Neptune for micron-sized nonspherical particles in circular restricted three-body problem with solar electromagnetic radiation (see Table 1 and Fig. 1). The existence of resonant capture can be influenced by three facts:
i) interaction of electromagnetic radiation with nonspherical particle changes inclination of particle's orbit,
ii) resonances are not defined in a unique way – several resonances may overlap due to the changing value of parameter $\beta$ for its greater values (and, even for spherical grains, capture times are very small for similar values of $T$ and $T_P$), and,
iii) the change of $\beta$ during particle's motion can cause significant change of semimajor axis (see Eq. 4.2), and, thus, also decrease of capture time.

Interaction of electromagnetic radiation with nonspherical grain may initiate an increase of semi-major axis close to the resonant regions. Resonant trapping is not normally expected for diverging orbits such as the trapping presented in Fig. 1.

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