

E.R. Harrison
University of Massachusetts, Amherst, MA 01003, U.S.A.

If the universe is spatially closed, and the simplest cosmological models are valid approximations, then in $\sim 10^{11}$ years the universe will recollapse into an antibang. (A "bang" is explosive, whereas an "antibang" may be considered implosive.) In this note I shall add one or two details to the seminal work of Rees (1969).

In the collapse phase, galaxies merge and tend to vanish when the radiation background has a temperature $T_0 \approx 300$ K. Stars with an initial velocity of $v_0 \approx 10^3$ km s $^{-1}$ now accelerate, and if their motion is free, then $\gamma\beta T_0 = \beta_0 T$, where $\beta = v/c$, $\gamma = (1-\beta^2)^{-1/2}$. As shown by Rees, the stars become relativistic when $T \approx 10^5$ K, and their collisions with one another are negligible.

Let us now take into account the drag force on a spherical body of mass M , radius R , moving through the background radiation of energy density $u = aT^4$. The force is:

$$F = \pi R^2 u \gamma^2 \left\{ \frac{4}{3} + \left(1 + \frac{1}{3} \beta^2\right) \frac{2GM}{R c^2} \ln \frac{D}{R} \right\}, \quad (1)$$

where the second term in curly brackets is the rate of momentum to distant photons due to small-angle deflections, and D is a cutoff length. For bodies large compared with black holes, this second term is negligible. On integrating the equation of motion, we now have:

$$\gamma\beta = \beta_0 \times e^{-\alpha(x^2-1)}, \quad (2)$$

where $x = T/T_0$, and

$$\alpha = 2M_\gamma/M, \quad (3)$$

with $M_\gamma = \pi R^2 L_{0u_0}/c^2$ equal to the photon mass in a cylinder of radius R and the Hubble length at T_0 . For (BD) black dwarfs we have $\alpha \sim 10^{-13}$, for (WD) white dwarfs $\alpha \sim 10^{-17}$, and for (NS) neutron stars $\alpha \sim 10^{-22}$. From equation (2) we find γ has a maximum value:

$$\gamma_m = \beta_o (2\alpha e)^{-1/2} \quad (4)$$

at $T_m = T_o(2\alpha)^{1/2}$, and hence $\gamma_m \sim 10^4, 10^6, 10^9$ for BD, WD, NS, respectively, and the corresponding temperatures are $T_m \sim 10^9, 3 \times 10^{10}, 3 \times 10^{13}$. It is unlikely that these relativistic speeds will be attained because of i) star dissolution, and ii) electron pair production (particularly for WD and NS).

Dissolution due to mass loss is a difficult calculation. In the temperature range of interest, it appears that the incident energy flux due to radiation ($\sim u\gamma^2c$) is greater than that due to gas ($\sim \rho_g\gamma^2c^3$). Avoiding complications such as "limb skimming" and hydrogen burning, and taking account only of atmospheric boiling, the dissolution time scale is:

$$t_{dis} \sim \pi R^2 \gamma^2 u / c \quad (5)$$

In the temperature range of interest ($\beta_o^{-1} < x < \alpha^{-1/2}$),

$$\gamma = \frac{T}{T_o} \beta_o \quad (6)$$

and dissolution occurs therefore on a time scale corresponding to the temperature:

$$T_{dis} \sim T_o \beta_o^{-1/2} \alpha^{-1/4} \quad (7)$$

We find: $T_{dis} \sim 10^7 K, \gamma \sim 10^2$ for BD; $T_{dis} \sim 10^8, \gamma \sim 10^3$ for WD; and $T_{dis} \sim 10^9 K, \gamma \sim 10^4$ for NS.

The increase in specific entropy due to the acceleration of stars is:

$$\frac{\Delta s}{s} \sim \frac{n_b \gamma m_b c^2}{n_\gamma kT} \quad ,$$

and using equation (1), it is seen:

$$\frac{\Delta s}{s} \sim \frac{n_b}{n_\gamma} \frac{\beta_o m_b c^2}{kT_o} \sim 3 \times 10^7 \frac{n_b}{n_\gamma} \quad (8)$$

where n_b/n_γ is the ratio of baryon and photon densities. If the ratio of densities lies in the range $10^{-8} - 10^{-9}$, the entropy increase is significant, as was foreseen by Rees.

REFERENCES

Rees, M.J. 1969, The Observatory, 89, pp. 193-198.

Discussion

Kazanas: I would like to comment that a very simple calculation can show that if the recollapsing phase of the universe is a time-reversed version of its expansion, i.e., the relation between the scale factor and time is that given during the expansion: $R \sim t^{1/2}$, the mass of any black hole diverges at a finite temperature given by $T \sim 10^{12}(M/M_{\odot})^{-1/2}$ K. This is consistent with the view that the time's arrow is in the direction of increasing entropy, since a homogeneous model has a very low gravitational entropy compared with that of a black hole, which is the maximum entropy state for a collection of particles.

I consider this single argument to really impose very serious problems on models of an oscillating universe.