COSMOLOGICAL INFORMATION FROM GALAXIES AND RADIO GALAXIES

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Abstract. An account is given of recent developments in the derivation of the value of the Hubble parameter applicable to regions beyond the local anisotropy. Recent observations relevant to interpretations of the magnitude-redshift diagram for cluster galaxies are discussed, in an attempt to assess uncertainties in the value of the deceleration parameter.

1. Introduction

Some twenty years ago Hubble (1951) in a review of the prospects for observational cosmology outlined what he believed to be the most promising methods of choosing observationally among the available world models. These were, firstly, to determine the local mean density of matter and the local law relating velocity to distance, and, secondly, to determine whether or not there are systematic changes with distance in either of these data. The search for a variation of density with distance through the extension of galaxy counts to successively fainter magnitude limits has not been actively pursued. Robertson's (1955) criticism of the possibilities of the method was followed by Sandage's (1961b) calculations, which indicated that at the limit of the Hale telescope the differences between the predictions of the models would be much less than observational errors and further uncertainties due to the unknown effects of clustering, the broad luminosity function of field galaxies and incalculable evolutionary effects.

The central line of development in classical optical cosmology has been in the more precise formulation of the local velocity-distance relation and the evaluation of the Hubble parameter H_0, together with the search for second-order effects in this relation and estimates of the deceleration parameter q_0. Indeed, except for some preliminary investigations which have been made into the use of the angular diameter-redshift relation applied to galaxies and clusters of galaxies, the redshift-magnitude (m–z) relation is the only cosmological test to have been exploited since Hubble wrote.

Since that time it has of course slipped from its position at the centre of the subject as a whole, partly due to the exploitation of radio-source counts and to the discovery of the isotropic background radiation, both of which offer the possibility of sampling at greater redshifts than are accessible to optical observers, but also partly due to the extreme difficulty of the observation of magnitudes and velocities at large enough distances so that the differences between the interesting models become larger than the observational errors. The main reason for persevering with the m–z relation is that one can get in principle a single conclusion among the possible models, if one can calculate the numerous corrections needed to interpret the observations. Furthermore, these corrections are easier to calculate for the test objects of the m–z relation

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than are, for example, the evolutionary effects associated with the interpretation of the radio-source counts.

This paper does not aim to give an exhaustive account of work in these fields. Critical reviews of methods for H_0 determination have been given recently by Tammann (1969) and van den Bergh (1970a, b), and progress in the photometry of cluster galaxies and radio galaxies has been reported on several times by Sandage (1966a, b, 1968a, b). We have restricted ourselves to a very brief account of recent developments in the derivation of a value of H_0 applicable to regions beyond the local anisotropy, and to a somewhat more extended discussion of recent observations relevant to the interpretation of the m–z diagram for cluster galaxies, in an attempt to assess the uncertainties in the value of the deceleration parameter.

2. Basic Formulae and Their Application

For the zero-pressure Friedman models with zero cosmological constant the relation between the bolometric magnitude of a test object m_{bol}, its absolute magnitude M and the redshift z is (Mattig, 1958)

$$m_{\text{bol}} = 5 \log \frac{c}{H_0 q_0^2} \times \{q_0 z + (q_0 - 1)[\sqrt{(2q_0 z + 1)} - 1]\} + M + 25 \qquad (q_0 > 0) \tag{1}$$

The Hubble parameter H_0 and the deceleration parameter q_0 are defined as

$$H_0 \equiv \dot{R}_0/R_0 \quad \text{and} \quad q_0 \equiv - R_0 \ddot{R}_0/\dot{R}_0^2$$

where R is the time-dependent scale factor in the Robertson-Walker line element, and subscript zero indicates that the quantities are evaluated at the present cosmic time. The deceleration parameter is related to the mean density ϱ_0 by

$$\varrho_0 = \frac{4\pi G}{3H_0^2} q_0 \tag{2}$$

where G is the gravitational constant, and with the spatial curvature k/R_0^2 by

$$kc^2/R_0^2 = H_0^2 (2q_0 - 1) \tag{3}$$

Expanding (1) in powers of z gives the m–z relation derived without use of the field equations by Heckmann (1942) and Robertson (1955)

$$m_{\text{bol}} = 5 \log (cz/H_0) + 1.086 (1 - q_0) z + \cdots + M + 25 \tag{4}$$

For the steady-state model the Hubble parameter is time-independent; thus $R(t) = A \exp(Ht)$ and consequently $q_0 = -1$. The exact m–z relation for this model is

$$m_{\text{bol}} = 5 \log [cz(1 + z)/H_0] + M + 25. \tag{5}$$

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Given a set of test objects with small dispersion in M and a guarantee that M is time-independent one could measure m_{bol} and z and use the relation (1) to derive q_0; the precision of the determination will depend on the dispersion in M and on the redshift interval sampled. Clearly larger redshifts will have higher weight in the solution for the higher order terms. To determine H_0 on the other hand, one must find the modulus of one of the test objects for which z is small enough for the second (q_0-dependent) term in (4) to be negligible, and yet high enough to be not only much larger than the galaxian peculiar velocities of a few hundred km s^{-1}, but also large enough for it to be outside the local anisotropic velocity field, so that it will reflect the motion of the presumably isotropically expanding substratum unperturbed by the gravitational interactions within the local supercluster.

3. The Value of H_0 Outside the Local Supercluster

Since the preliminary mapping of the anisotropy of the local velocity field it has been obvious that it is no longer possible to base a value for the Hubble parameter appropriate at large distances, $H_0(\infty)$, on the modulus and velocity of the Virgo cluster, until the perturbation of the isotropic flow at the Virgo cluster is known. De Vaucouleurs' (1958) model of the anisotropy indicated a reduced expansion rate at Virgo such that $H_0(\infty)/H_0$ (Virgo) $=1.58$; a more recent kinematic model based on more velocities gives 1.35 ± 0.15 (de Vaucouleurs and Peters, 1968). Although the present spread in quoted values for H_0 (from about 50 to 120 km s^{-1} Mpc^{-1}) is larger than the error that would arise from neglecting this anomalous expansion velocity, it is clearly an effect that must be investigated. We describe in this section two methods that have been applied to derive $H_0(\infty)$; the use of the redshift-magnitude plot for brightest cluster galaxies and the luminosity function of rich clusters of galaxies.

Sandage (1968c) has found the modulus of M87 in the Virgo E-cloud by using Racine's (1968) luminosity function for the M87 globular clusters. The brightest of these has $B=21.3$. The brightest globular in M31 is taken to be B282 with $B=15.01$, and the M31 apparent blue modulus is taken as $(m-M)_{AB}=24.84$ from the Cepheid photometry of Baade and Swope (1963), using Sandage and Tammann's (1968) calibration of the Cepheid P–L curve. Then $M_B=-9.83$ for B282 and the apparent modulus for M87 is $(m-M)_{AB}=31.1$, if its brightest globular is of the same absolute magnitude. Now NGC 4472, which according to Sandage's (1970) photometry is the brightest Virgo cluster member, has an apparent magnitude to an isophote of 25 mag. arc sec^{-2} of $B=9.42$. Thus $M_B=-21.68$ for NGC 4472. The m–z curve for brightest cluster members has a dispersion of only $\cong 0.3$ mag. If NGC 4472 is now assumed a typical brightest cluster member, it can be used to calibrate the m–z plot at distances with velocities greater than 4000 km s^{-1} and outside the effects of the Supercluster, and $H_0(\infty)$ can be based on the mean line through the cluster points. This gives $H_0(\infty)=75$ km s^{-1} Mpc^{-1} with 950 km s^{-1} for the E-cloud velocity, whereas H_0(Virgo) $=64$ km s^{-1} Mpc^{-1} as NGC 4472 falls below the mean cluster

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line, presumably due to its anomalous velocity. Thus $H_0(\infty)/H_0(\text{Virgo})=1.17$. Assessing all the errors of the method, excluding possible errors in the distance scale in the Local Group, suggests that these values could be in error by ± 25 km s^{-1} Mpc^{-1}.

De Vaucouleurs (1970) has reexamined this use of brightest globular clusters as distance indicators, and has shown a correlation between the absolute magnitude of the brightest globular cluster M_{gc} and the absolute magnitude of the parent galaxy M_{gal} for seven Local Group members. As the brightest calibrating galaxy is M31 some extrapolation is needed to cover gE galaxies. Assuming his calibration and a measured B magnitude for M87 of 9.0, he uses Sandage's calibration of the brightest globular to find the modulus and absolute magnitude for M87. Treating this as a first approximation he uses the relation between M_{gc} and M_{gal} to find a second approximation to M_{gc} and a revision to the modulus, and this process is repeated until the value of M_{gc} converges. The total effect is to produce a change in M_{gc} for the brightest globular in M87 of 0.5 mag. As he also finds M87 brighter than NGC 4472 by 0.25 mag., following Sandage's procedure in entering the m–z diagram leads to the revised value of $H_0(\infty)=50$ km s^{-1} Mpc and negligible difference between this and $H_0(\text{Virgo})$. Clearly much more work is necessary on the luminosity function of globular cluster populations of galaxies, and on the possible correlation of M_{gc} with the size of this population, before the method becomes more reliable, but it offers the possibility of a modulus to the gE galaxies of the Coma cluster, whose globular clusters will be accessible at about 24 mag.

A different approach has recently been made by Abell and Eastmond (1968), who have used the form of the cumulative luminosity functions of rich clusters of galaxies as a distance indicator. By matching the discontinuity in the luminosity functions of the Coma and Corona Borealis clusters to the Virgo cluster, they found the difference in modulus to be 4.7 mag. and 7.2 mag. respectively. Using Sandage's modulus of M87 of 31.1, this gives moduli of 35.8 for Coma and 38.3 for the Corona cluster. With their mean velocities of 6866, and 21651 km s^{-1} respectively one has $H_0(\infty)=$ 47 km s^{-1} Mpc^{-1}. Once again one has bypassed the Virgo anisotropy.

These two methods indicate some of the inaccuracies in present Hubble parameter determinations and give some indication of the possible spread in values of H_0; an age of the universe, H_0^{-1}, of as much as 20×10^{10} yr or as little as 5×10^{10} yr cannot be excluded by present data.

4. The Determination of q_0. Generalities

As the intrinsic dispersion in M for field galaxies is so great as to mask any cosmological effects over the accessible redshift range, Hubble (1936) suggested the use of cluster galaxies as test objects, because for these it would be possible both to select from a fixed part of the luminosity function, viz., its bright end, and to get better space penetration owing to the high luminosity of brightest cluster galaxies. The first attempt to derive q_0 by this method was by Humason *et al.* (1956) (subsequently referred to as HMS) who measured magnitudes in P and V for the 1st, 3rd, 5th and 10th brightest galaxies in 18 clusters spanning the distance from the Virgo cluster to the Hyades cluster (0855+0321) at $z=0.2$; these magnitudes were then combined to give a synthetic brightest galaxy. This combined magnitude showed a standard deviation of only 0.3 mag when the data were fitted to Equation (4) and gave $q_0=2.5\pm1$. The intrinsic dispersion in the absolute magnitudes is clearly smaller, as part of the observed dispersion must be due to measuring error, patchiness in galactic absorption and possible peculiar velocities of the clusters measured. Baum (1957, 1961a, b) later extended the measurements as far as $z=0.46$ using an eight colour photometric system which obviated the need of a K-correction and gave the redshifts non-spectroscopically from the shift of the continuum through the measuring bands. This work remains of very considerable importance, and Baum's photometry of the brightest members of the clusters 0024+1654, 1448+2617 and 1410+5224 remains the only photometry for cluster galaxies with $z>0.2$. Baum did not analyse his observations in detail, only indicating that a straight line in the m–z plane ($q_0=1$) seemed an adequate description of the data; σ_M for these measures is 0.20 mag., significantly smaller than the HMS result, but perhaps due to the small number of data points. The available data have recently been extended by Sandage's (1970) photometry of about forty cluster galaxies with $z\leqslant0.2$ in B and V.

Radio galaxies share the property of high optical luminosity with brightest cluster galaxies, and the high proportion of emission line objects makes redshift measurement easier. Not only are their luminosities closely similar, but many radio galaxies are brightest members of clusters. There are problems of interpretation connected with them that do not occur in the interpretation of normal ellipticals, namely, the question of the K-correction in the presence of line emission and possible non-thermal radiation, the evolutionary properties over the light travel time, and the observational problem of how to treat the photometry of the so-called 'dumb-bells'. The magnitude dispersion of field radio galaxies is about 0.5 mag., so that over the presently accessible range of z they are of much less weight than brightest cluster galaxies, although possibly avoiding some of the selection effects associated with the latter.

5. The Determination of q_0. Assessment of Systematic Errors

The search for the second-order term is still in a very preliminary stage and it is impossible at present to give a value for q_0 in which any great degree of confidence can be placed. This is due not simply to the comparatively small redshift range of the m–z data but also to the possibility of large systematic errors in these data. This section considers some of the corrections that must be made to the observed magnitudes to free them from effects irrelevant to the problem in hand, and attempts to assess the size of possible systematic errors and their consequential effect on our knowledge of q_0.

A. *K*-CORRECTION

A *K*-correction is applied to the measured apparent heterochromatic magnitudes to

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compensate for the redshift of the energy curve $I(\lambda)$ of the observed galaxy, which sweeps different parts of $I(\lambda)$ measured in the rest frame of the galaxy through the fixed pass-bands of the observer's photometer. Effects due to the decrease in the observed photon energy and the decrease in the photon arrival rate are already incorporated in the m–z relation (1). There are two terms in the K-correction. The first arises from the narrowing of the photometer pass-band in the rest frame of the galaxy by a factor $(1+z)$; this term is wavelength-independent and simply increases the apparent magnitude by 2.5 $\log(1+z)$. The second term is due to the fact that the radiation received by the observer at wavelength λ is that emitted by the galaxy at wavelength $\lambda/(1+z)$; this effect is clearly wavelength-dependent and its sign and magnitude depend on the gradient of $I(\lambda)$ over the wavelength region of interest. We write the two terms of the K-correction as

$$K = 2.5 \log(1+z) + 2.5 \log \frac{\int_0^\infty I(\lambda)\, S(\lambda)\, \mathrm{d}\lambda}{\int_0^\infty I(\lambda/(1+z))\, S(\lambda)\, \mathrm{d}\lambda}. \tag{6}$$

Here $I(\lambda)$ is the flux per wavelength interval and $S(\lambda)$ is the photometer response function. For the evaluation of the second term we need $I(\lambda)$ and an assurance that $I(\lambda)$ is sufficiently homogeneous among giant E and S0 galaxies that we can base a universal K-correction on the detailed observation of $I(\lambda)$ for a few nearby objects. The pass-bands $S(\lambda)$ are typically broad $(\Delta\lambda/\lambda \cong 0.1)$ so that the $I(\lambda)$ required is the continuum intensity distribution.

A first approximation to the K-correction for gE galaxies, apart from Hubble's (1936) assumption that they could be represented by a 6000 K black body, was made by HMS. They used an $I(\lambda)$ based on the 6-colour photometry of M32 by Stebbins and Whitford (1948). With the benefit of hindsight it is now clear that this was unfortunate for two reasons. Firstly, M32 is bluer than a typical gE, being about 0.3 mag. brighter in the ultraviolet. Secondly, as first suggested by de Vaucouleurs (1948) and confirmed by later scanner observations (Code, 1959), the broad bands of the 6-colour system had inadequate spectral resolution to give a sufficiently accurate representation of the sharp drop in the $I(\lambda)$ curve near 3900 Å. Accordingly $I(\lambda)$ in this region appeared too smooth and the K-correction for the P or B bands systematically too small. It was this $I(\lambda)$ and K-correction which gave rise to the Stebbins-Whitford effect; the colours of distant ellipticals appeared redder by as much as 0.3 mag. at $z=0.13$ than would have been predicted from the M32 curve. The presence of the Stebbins-Whitford effect and the possibility of evolutionary effects in colour of this magnitude confused the discussion of the HMS photometry where the problem was left essentially unresolved. A certain confusion still remained when Code's (1959) and Oke's (1962) scans of the Virgo elliptical NGC 4374, which has normal U – B and B – V colours for a gE, were subsequently used by Sandage (1966) to recalculate K_{B} and K_{V}.

There was a small systematic difference between the measured $I(\lambda)$ curves, largely due to the different absolute calibrations used in the spectrophotometric reductions. The Oke curve showed no Stebbins-Whitford effect when the predicted run of B − V colour with z was compared with observed colours out to $z = 0.17$, while there was an excess reddening of 0.2 mag. present with the Code curve.

The situation is now more satisfactory as a result of the work of Oke and Sandage (1968) and Whitford (1970). Oke and Sandage's work is based on scans with an exit slit of about 50 Å of the central 10″ to 12″ of a number of nearby giant E and S0 galaxies. The $I(\lambda)$ curves for these, based on Oke's (1964) calibration of Vega with a small modification in the ultraviolet, appeared identical from 3375 Å to 5840 Å to within the error estimate for each object (± 0.07 mag. for $\lambda > 4000$ Å and ± 0.13 mag. for $\lambda < 4000$ Å), and the mean for all of the observed galaxies was in turn identical with $I(\lambda)$ for the central region of M31, which is known with a standard deviation of only ± 0.02 mag. at all wavelengths. K_B and K_V were calculated using the $I(\lambda)$ for M31 for $\lambda < 5840$ Å and for the less well determined region to the red of 5840 Å from scans of NGC 3379. It was realised that because of the colour gradient known to exist in ellipticals (de Vaucouleurs, 1960; Tifft, 1963) this K-correction would not necessarily be appropriate to photometry with aperture to diameter ratios A/D_0 greater than those used in the spectrophotometry ($\cong 0.08$), but systematic effects due to this were thought to be less than 0.04 mag. in K_V for $z < 0.2$.

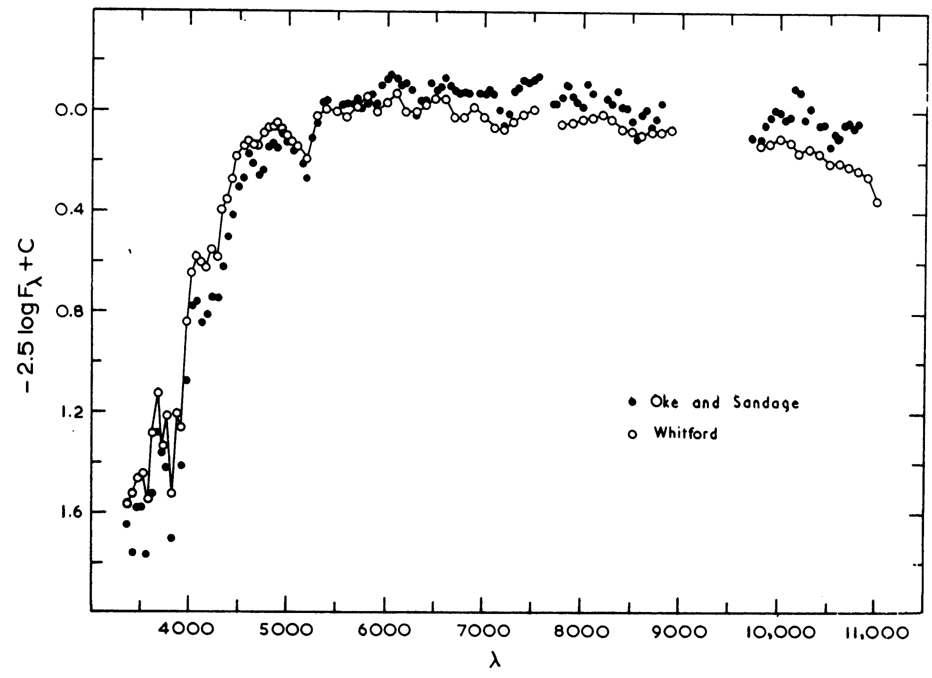

Fig. 1. Intensity distributions in magnitudes per unit wavelength interval for giant ellipticals from the observations of Oke and Sandage (1968) and Whitford (1970). The significantly bluer $I(\lambda)$ with Whitford's larger A/D_0 ratio should be noted.

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Whitford (1970) made similar measurements of E galaxies, but with larger entrance apertures at the same spectral resolution and used Hayes' (1967) calibration of Vega as standard. His two A/D_0 ratios of about 0.2 and 0.6 show aperture sensitive differences in $I(\lambda)$ at all wavelengths. Whitford's $I(\lambda)$ for $A/D_0=0.6$ from the mean of five objects is compared with the Oke and Sandage curve in Figure 1. Whitford attributes the discrepancies to a combination of the aperture-colour effect and the difference in the calibration of the standard stars. In addition there appears to be a real divergence in the photometry in the infrared.

Calculated K_B and K_V from these two investigations are shown in Figure 2.

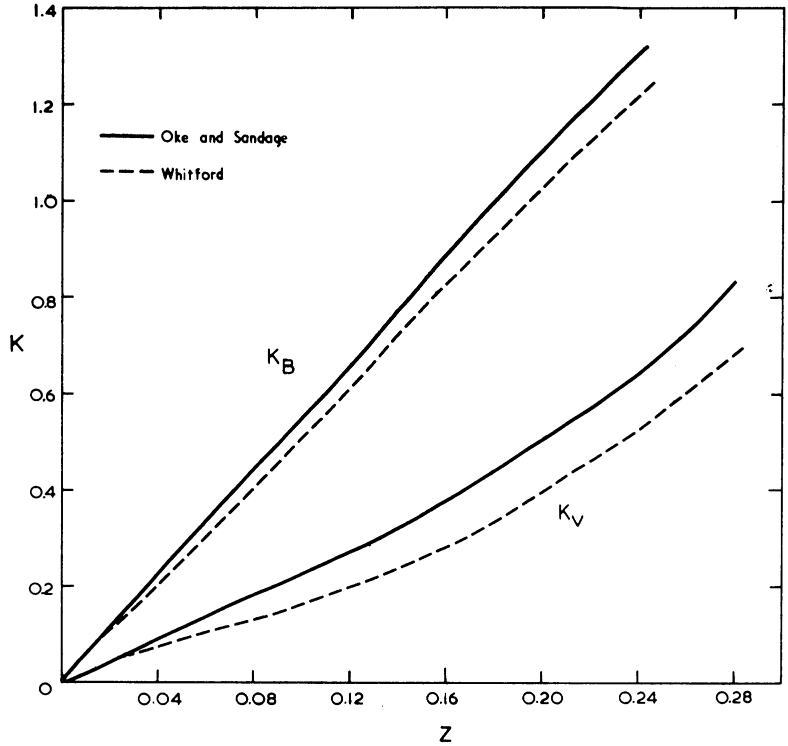

Fig. 2. *K*-corrections in B and V for the $I(\lambda)$ distributions shown in Figure 1.

Whitford's are in both cases smaller, in the case of K_B mainly due to the inclusion of the bluer outer regions, and in the case of K_V to the changed slope of $I(\lambda)$ towards the red. As the latter change is somewhat greater than the former there is a consequential difference in the predicted trend of K_B-K_V or B−V with z. Figure 3 shows the two predictions and the observations. Both fall within the range of error of the broad band colours, but the Whitford curve shows a small $\cong 0.05$ mag. blueing at $z=0.2$ in contrast to the original reddening of the Stebbins-Whitford effect.

Both investigations confirm the high degree of homogeneity of $I(\lambda)$ for giant E and S0 systems expected from the small dispersion of their colours (Stebbins and Whitford,

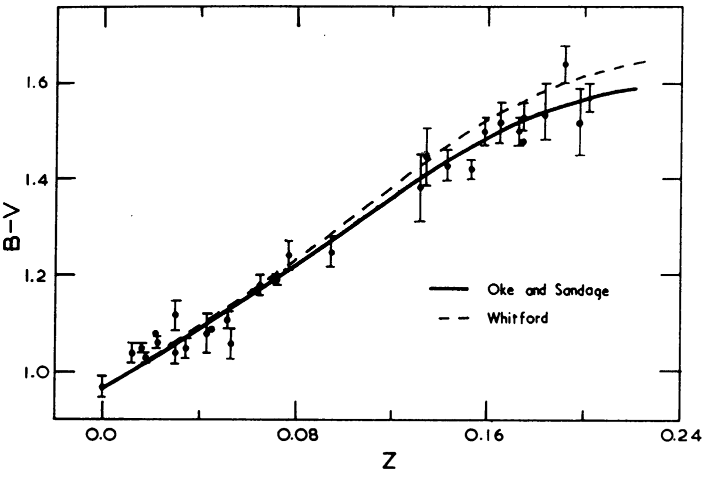

Fig. 3. The predicted run of B − V colour with redshift for the Oke-Sandage and Whitford K-corrections. The observations show slightly better agreement with the former; the error bars are average deviations of the individual measures (Oke and Sandage, 1970).

1952). Further evidence is given by the 8-colour narrow band photometry of 100 E and S0 galaxies with $z<0.02$ by Lasker (1970). E and S0 galaxies considered as separate classes are indistinguishable in any of his colours. There is general confirmation of the colour-magnitude correlation found by Baum (1959) and de Vaucouleurs (1960) for systems with $-21<M_V<-15$, in the sense that less luminous objects are bluer, but no colour variation in the red among giant systems with $-23<M_V<-21$. Lasker has made a heuristic estimate of the uncertainty in the K-correction set by the inhomogeneity permitted by his observations. He shows that to a good approximation the uncertainty in K can be written for small z as

$$\delta K \cong z\delta(\mathrm{d}m/\mathrm{d}\ln\lambda). \tag{7}$$

The uncertainty in $\mathrm{d}m/\mathrm{d}\ln\lambda$ for the region swept out by the B and V bands for $z<0.2$ can be derived from his estimate of the intrinsic dispersion in the index c_{13}, which is based on effective wavelengths of 4220 Å and 5488 Å and has a standard deviation of 0.06 mag. The consequential uncertainty in K from (7) at $z=0.2$ is 0.04 mag. Now it is easy to show from the m–z relation (4) that the uncertainty in q_0 due to an uncertainty δm_{bol} at a redshift z is roughly $\delta m_{\mathrm{bol}}/z$. Thus from $\delta K=0.04$ mag. at $z=0.2$ we find $\delta q_0=0.2$. Using the pass bands further into the red leads to a smaller figure.

It would now appear that uncertainties as to the form of K_V and K_B and possible inhomogeneities in $I(\lambda)$ are among the smaller contributors to errors in q_0, but clearly there are still advantages in switching to a red magnitude system. In this wavelength region the wavelength-dependent second term in the K-correction can be smaller than the first term, and is evaluated from a slowly varying intensity distribution, while at redshifts larger than 0.25, K_B would require knowledge of $I(\lambda)$ in the rocket

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ultraviolet and K_V would depend on the rapidly varying region near 3900 Å. Better still, one could avoid the K-correction problem entirely by using a multicolour photometric system which will sample enough of the spectrum to give a bolometric magnitude directly (Baum, 1961b), or a multichannel scanner (Oke, 1969) which can simultaneously measure the redshift.

B. EVOLUTIONARY CORRECTION

Any use of the m–z relation to derive a value for q_0 demands knowledge of the rate of change of the absolute magnitudes of the test objects during the light travel time. A rough estimate of dM/dt for gE galaxies which has suggested more detailed calculations was made by Sandage (1961a). He assumed on the basis of the evidence on count-brightness ratios (Baum, 1959) and on the evidence for strong-line giants in the spectra of E galaxies (Morgan and Mayall, 1957), that the population of gE galaxies might be similar to old disc Population I systems typified by M67 and NGC 188. The main sequence break-point of NGC 188 comes at $M_V = +4$; assuming that star formation began at the same time in all galaxies and neglecting possible population differences, only stars brighter than this limit can have evolved in a distant galaxy during the light travel time and contributed to any change in the total luminosity of the system. The apparent parallelism of the giant and subgiant branches in the HR diagrams of M67 and NGC 188 suggests that the total luminosity of the evolved stars at any time will be proportional to the luminosity of the stars at the break-point at that time. Assuming homologous evolution, the hydrogen burning age appropriate to this point is

$$t = \text{const.}\ M/L. \tag{8}$$

With a mass-luminosity relation $L = \text{const.}\ M^4$ near $M_V = +4$, the age given by the break-point is proportional to $L^{-3/4}$. The ratio of the luminosities of the break-points at times t_1 and t_0, which will also be the ratio of the total luminosities of the evolved stars, is then given by the relation

$$(L_0/L_1) = \text{const.}\ (t_1/t_0)^{3/4}. \tag{9}$$

As an illustration consider the case of the currently most distant point in the m–z diagram, which is 3C 295 at $z = 0.46$, and assume a model with $q_0 = 1$. If t_0 is the time of the present epoch and t_1 is the time of light emission the ratio $t_1/t_0 = 0.527$. Therefore $L_1/L_0 = 2.36$, or 0.93 mag. Thus if the evolution tracks for the giant branch are parallel, and if the luminosity function between the break-points at t_1 and t_0 is flat, the total change in bolometric magnitude in the light travel time would be 0.93 mag. More plausibly, if 50% of the total light at t_0 comes from the unevolved main sequence, the evolutionary change is reduced to 0.56 mag. This correction will have to be applied to the observed apparent magnitude entered in the m–z diagram; its application requires prior knowledge of q_0 to calculate the light travel time. In practice one can assume a value and proceed by successive approximation (Sandage, 1961; Peach, 1970).

Clearly the evolutionary correction will depend not only on the ratio of the numbers of evolving stars to main sequence dwarfs, but also on the slope of the luminosity function near $M_V = +4$, which was assumed zero in Sandage's calculation. It is now possible to use the work on galaxy population synthesis of Spinrad (1966), Wood (1966), Johnson (1966) and McClure and van den Bergh (1968) to make an informed estimate of this quantity. On the assumptions of the previous paragraph the slope of the main sequence luminosity function required to reduce the evolutionary correction to zero is one which will give populations at the break-points inversely proportional to the luminosities at these points. Taking once again the example of 3C 295, if the ratio of the population at L_0 to the population at L_1 is 2.36 then dM/dt is zero. If the ratio is greater the model galaxy brightens with age. If one assumes that the luminosity function can be approximated from $M_V = +3$ to $+5$ by the expression

$$\log \phi(M_V) = \text{const.} + AM_V \tag{10}$$

values of the constant A can be taken from the published functions and used to recalculate dM/dt in the context of the Sandage model. Table I gives values of A from

TABLE I

The slope A of the luminosity function $\log \phi(M_v) = \text{const} + AM_v$ between $M_v = +3$ and $+5$ and the corresponding evolutionary corrections assuming a 50% dwarf contribution to the total luminosity

Source of $\phi(M_v)$	A	$dM/dt(10^{-9}$ mag. yr$^{-1})$
Spinrad (1966) M31 model	0.55	−0.047
Wood (1966) M31 model	0.60	−0.068
McClure and van den Bergh (1968) M86 model	0.44	−0.013
Upgren (1963) old disc stars	0.46	−0.018
Tinsley (1968) model E2	0.08	+0.133

some recent investigations, with calculated values of the evolutionary correction, with a 50% contribution from unevolved dwarfs to the total light. The extreme sensitivity of the effect to the value of A should be noted. The Spinrad and Wood functions for the nucleus of M31 agree well, while the McClure and van den Bergh value for A from their model for M86 is in better agreement with Upgren's (1963) luminosity function for stars more than 400 pc above the galactic plane, which are presumably a reasonable sample of the old disk population of our galaxy. It should be noticed that all of these values give a galaxy population that brightens with age, and whose absolute rate of change is much less than in the original Sandage model. Chambers and Roeder (1969) have calculated dM/dt for the McClure and van den Bergh model for M86 in more detail and find a dimming of about $+0.02 \times 10^{-9}$ mag. yr^{-1}.

A different approach has been made by Tinsley (1968, 1970) who computed evolutionary changes in a series of model galaxies over a period of 12.10^9 yr, starting from a gas of Population I composition and assuming a stellar birthrate function and evolution of the stars so formed along a series of theoretical and semi-empirical tracks in the $M_{bol} - T_e$ plane; bolometric corrections were assumed to transform to the

$M_V-(B-V)$ plane. From a number of computed models she selected one, E2, as a possible representation of a gE on the basis of the agreement of its predicted colours with the results of Johnson's and Wood's photometry and on the plausibility of its mass to light ratio. This model has a $B-V$ colour change of less than 0.03 mag. in 3.10^9 yr in agreement with the run of $B-V$ of Figure 3. Its bolometric luminosity varies by only 0.05 mag. in the same interval, but it varies much more rapidly in the B and V bands. This apparent anomaly is due to the fact that the evolutionary changes in the model are strongly wavelength-dependent. In her models star formation can continue for as long as 75% of the life of the galaxy; there are thus still upper main sequence stars visible if the light travel time is about 3.10^9 yr. There is a strong evolutionary effect due to these stars in the blue and ultraviolet which at large z can cause high ($\cong 0.1 \times 10^{-9}$ mag. yr^{-1}) evolutionary changes in the visible. The semi-empirical models based on an old disc population tacitly assume that star formation is finished over the time scale of the m–z diagram, so this effect would be absent. Observation of the variation of $U-B$ colour with redshift would be useful in deciding on the presence of these effects as this model predicts a change of $U-B$ of 0.1×10^{-9} mag. yr^{-1}. The rapid evolution of the E2 model is reflected in the very small slope in $\phi(M_V)$ at the top of the unevolved main sequence, $A=0.08$, which is difficult to reconcile with the observations underlying the much higher values in Table I.

If we exclude the large evolutionary corrections suggested by the E2 model on the ground of this discrepancy, then the semi-empirical approach still has uncertainties of $\pm 0.05 \times 10^{-9}$ mag. yr^{-1} due to the variation among the values of A alone; the underlying assumptions of parallelism of the evolutionary tracks, uniform composition from galaxy to galaxy, and the general proportion of dwarfs to giants are doubtless over-simplifications and further contributors to error. It can be shown (Peach, 1970) that the uncertainty in q_0 due to an uncertainty of 0.05×10^{-9} mag. yr^{-1} is about 0.5. This is the largest remaining uncertainty in the use of the m–z plot, and will not be reduced until much more is known about the stellar content of gE galaxies. However, the very large evolutionary corrections of the original Sandage model now look improbable.

C. MAGNITUDE DISPERSION AND SCOTT EFFECT

An implicit assumption underlying the previous discussion has been that brightest cluster galaxies are good cosmological test objects in the sense that their use does not introduce any distance-dependent selection effects into the m–z curve. We shall now consider one possible form of distance-dependent bias which has become known as the Scott effect.

Scott (1957) argued that if the absolute magnitudes of the brightest cluster members are correlated with the population of the cluster, and if the probability of detection of the cluster is also correlated with population, or with the possession of unusually bright brightest members, then the selection of clusters for measurement will bias an m–z plot based on the brightest members. Thus if the brightest members of richer clusters are intrinsically more luminous than those of clusters with fewer members, and if the richer clusters are preferentially recognised and chosen for measurement near the limit of detection, too many galaxies of systematically low apparent magnitude will be entered in the m–z plot at large z and the deduced value of q_0 will be erroneously high. This effect could be very marked. Scott considered a statistical model of the effect in which the probability of choice of a cluster for measurement depended on the number of member galaxies above the plate limit of some uniform survey, and on the apparent magnitude of the brightest member being small enough for measurement. Assuming a luminosity function with a normal distribution with σ_M of 1 or 2 mag. for the cluster galaxies, and with reasonable values for plate limit and so on, based on the experience of the compilers of the HMS catalogue, she evaluated the effect for redshifts up to 0.2. With these assumptions a systematic bias of as much as 0.3 mag. is quite possible at this distance, with a consequential upward change in the derived q_0 of about 1.5.

Without for the moment considering whether the data at our disposal were in fact selected in this way, it is clear that the size of a possible effect will depend critically on the slope of the luminosity function at the bright end and on the degree of correlation of absolute magnitude at the bright end with cluster richness. Due to the work of Abell and others (Abell, 1961, 1970; Rood, 1969) on cluster luminosity functions there is now some information on which to base an evaluation of the effect. If one assumes (Peebles, 1968) that Abell's form of the luminosity function for the Coma cluster is typical of all rich clusters one can construct a probability function $\psi(M)$ common to all clusters, such that the probability that there is a galaxy in a given cluster with absolute magnitude in the range from M to $M+\mathrm{d}M$ is

$$\mathrm{d}P = \psi(M)\,\mathrm{d}M \tag{11}$$

where

$$\psi(M) = \alpha e^{\alpha(M - M_0)} \tag{12}$$

a form consistent with Abell's measurements for the bright end of the Coma function with $\alpha = 1.8$; M_0, which can be derived from the observed mean luminosity of the brightest cluster members, is a measure of the total cluster population. This probability function predicts a distribution of the absolute magnitude M_1 of the brightest member

$$\mathrm{d}P_1 = \alpha \exp\left[\alpha(M_1 - M_0) - e^{\alpha(M_1 - M_0)}\right]\mathrm{d}M_1 \tag{13}$$

and the mean of M_1 is consequently given by

$$\langle M_1 \rangle = M_0 - 0.577/\alpha\,. \tag{14}$$

This distribution is compared with a histogram of absolute V magnitudes for brightest cluster galaxies with $z \leqslant 0.2$ in Figure 4. The distribution calculated on the basis of $\alpha = 1.8$ is about a factor two too wide, and a steepening of the luminosity function is needed to give agreement. Measurements of the luminosity functions of three other clusters, A151, A2065 and A2199, indicate that this steepening is compatible with the observations, which are naturally extremely uncertain because of the poor statistics at the bright end. Bearing in mind that some of the dispersion in the galaxy photometry

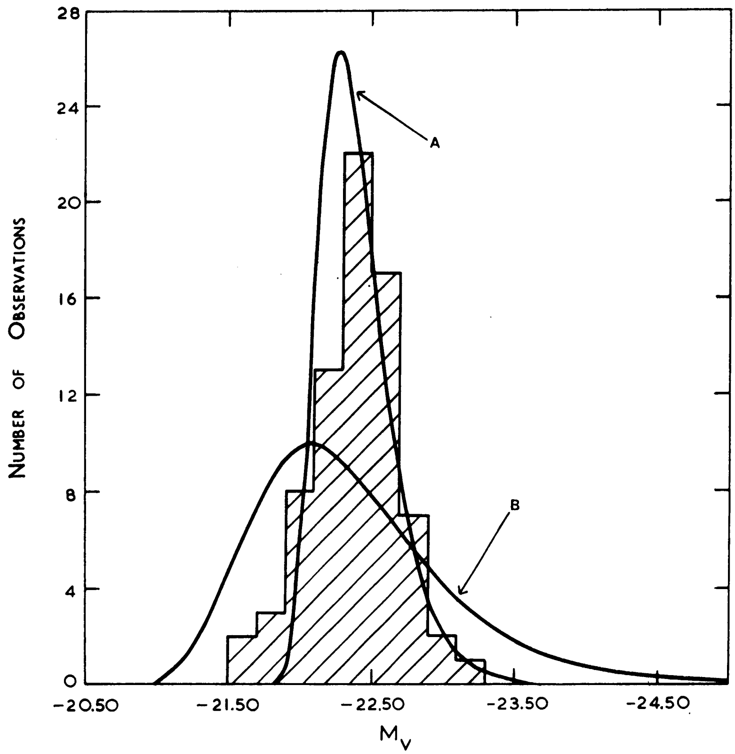

Fig. 4. A histogram of the absolute magnitudes of 75 brightest cluster galaxies measured by Peterson (1970) and Sandage (1970) and normalised to the same magnitude scale. Curve B is the prediction of the statistical theory (Equation (13)) for $\alpha = 1.8$. Curve A has $\alpha = 4.8$ which is too steep to fit the combined data but is a good fit for the Sandage data considered separately (Peach, 1969).

must be due to observational errors and to uncertainty in the galactic absorption it is possible that a plausible statistical model would have to use an even higher slope.

Clearly the higher the slope the less the magnitude of the brightest would be expected to vary with cluster richness. Peach (1969) has calculated the variation of the magnitude of the brightest member as a function of cluster richness as defined by Abell i.e., as a function of the population in the magnitude interval M_3 to $M_3 + 2$, where M_3 is the magnitude of the third brightest member, but holding the total cluster population fixed. Peterson (1969) has extended the calculations to the more realistic case where the total cluster population is allowed to vary. Figure 5 shows the comparison between absolute magnitude and cluster richness as given by these calculations and the absolute magnitudes of the previous figure subdivided among richness classes. Error bars are standard deviations. The predictions of the statistical theory are shown by the continuous line; it can be seen that there is little to choose between the theory and the hypothesis that the absolute magnitudes are independent of cluster richness.

Let us, however, assume that the trend indicated in Figure 5 of about 0.1 mag. brightening per richness class will be substantiated by further work. If all nearby

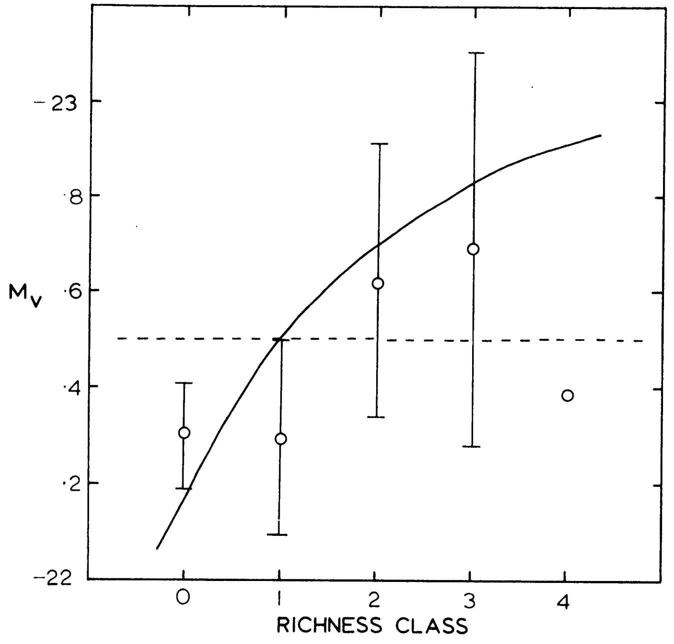

Fig. 5. Absolute magnitude of brightest cluster galaxies as a function of cluster richness. Error bars are standard deviations; there is only one cluster of richness 4. The solid line is the prediction of the statistical theory based on the Coma luminosity function.

clusters chosen for measurement were of richness class zero and all distant clusters chosen were of richness three, the systematic error in magnitude at $z = 0.2$ would be about 0.3 mag. This can reasonably be assumed to be an upper limit on the effect of selection. A more realistic estimate may be made using the statistics of the variation of the number of clusters of different richness at different distances taken from the Abell (1958) catalogue. Just (1959) has shown that there is an apparent excess of richer clusters, or perhaps more plausibly a deficiency in those of smaller population, among the entries in this catalogue. We may reasonably assume that whatever selection effects are incorporated in Scott's analysis have also entered into the compilation of this catalogue, so that we shall treat the whole of the richness trend with distance as due to selection. Once again assuming a change of 0.1 mag. per richness class and assuming that one chooses clusters for m–z observation with the same frequency of richness class per distance interval as in the catalogue, the bias in magnitude is only 0.02 mag. at $z = 0.2$, or a change in the derived q_0 value of 0.1. Of course if one knows the relation between luminosity and richness one can correct for the effect, and if the correlation is as weak as in Figure 5 the correction will be quite simple to apply (cf. Section 6).

It should be pointed out, however, that the statistical theory of the distribution of M_1 is in no sense proved, and it may be doubted if it could be modified to fit the observations if some way could be found to eliminate the observational component in the dispersion. This could well demand a slope for the luminosity function so steep

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that an equally natural explanation of the data would be that M_1 is normally distributed. Against this hypothesis one can point to the very ill-established trend with richness shown by the data of Figure 5, or to the behaviour of the magnitude differences between brightest cluster members shown by the measurements of HMS as interpreted by Peebles (1968). But it is already possible to discount the Scott effect as a permanent barrier to the interpretation of the *m–z* diagram of cluster galaxies.

This discussion is to some extent academic as it is improbable that clusters will be selected for future observation in a way remotely resembling the formalised model of Scott. Among the data at present available, some of the clusters were selected for measurement on the basis of optical identifications of radio sources proving to be brightest cluster members. Of the three distant clusters used by Baum the most distant is that containing 3C 295 (Minkowski, 1960) and was discovered on that account. Current searches for faint clusters are concentrated on the optically empty fields of radio galaxies, and such correlation as exists of brightness with population can in principle be corrected for.

D. GALACTIC ABSORPTION

In the absence of a correlation between redshift and galactic latitude uncertainty in the absorption at the galactic poles, on the assumption of a cosecant law, should have no effect on q_0 determination, although it would of course affect the measurement of H_0. Unfortunately there is a weak correlation between high redshift and high galactic latitude in the present data sample due to a natural selection effect, which implies that an increase in the absorption correction at the poles will reduce q_0 systematically, by artificially brightening nearby galaxies after correction for the effect. This is unfortunate in view of the present confusion surrounding galactic reddening and absorption. Values of $A_B = 0.5$ mag. at the poles have been derived from the galaxy counts of Shane and Wirtanen (1967) and from the rediscussion of Hubble's (1934) counts by de Vaucouleurs and Malik (1969), and both investigations show appreciable local deviations from a cosecant law. On the other hand values for $E(B-V)$ at the poles based on reddening of field RR Lyrae stars by Sturch (1966) $(E(B-V)=0.03$ mag.$)$ and McNamara and Langford (1969) $(E(B-V)=0.01$ mag.$)$ suggest $A_B = 0.08$ mag. using a ratio of total to selective absorption $A_B/E(B-V)=4$. Values for selective absorption derived from integrated colours of globular clusters (van den Bergh, 1967) or from Holmberg's (1958) study of field galaxies give an absorption at the poles of $A_B = 0.24$ mag. Peterson's recent (1970) work on colours and magnitudes of nearby brightest galaxies in clusters has to some extent increased the confusion by giving $A_B = 0.10$ mag. both from the reddening and from the magnitudes direct.

In the analysis of the next section we have used $A_B = 0.24$ mag., which is midway between the extremes and also gives the lowest residuals in the *m–z* plot for brightest cluster galaxies for which Sandage has given the photometry. The complete spread in q_0 solutions with cosecant absorptions A_V varying from 0.05 to 0.35 at the poles is about 0.2; use of the patchy models of either Shane and Wirtanen or de Vaucouleurs and Malik produces no reduction in the magnitude residuals. The uncertainty in q_0

produced by these effects is probably negligible at present, but clearly resolution of the confusion surrounding the true value of galactic absorption is necessary for further advance. Its influence on the value of H_0 is more serious as there is now a spread of values for the absorption in front of the Virgo cluster of the order of 0.4 mag. in B.

E. APERTURE CORRECTION

It is obvious that in measuring a magnitude for a programme galaxy one must devise a measuring procedure that collects a fixed proportion of the emitted radiation and that one avoid any method that will introduce a distance dependent error. HMS effectively measured magnitudes to an angular distance from the galaxy centre equal to $2.5D_s$, where D_s was the estimated diameter of the galaxy on the Sky Survey plates; assuming that the apparent edge of the image occurs always at a position in the galaxy image of the same surface brightness, magnitudes so determined are magnitudes to a given isophote, which was estimated by them to be about 25 mag. arc sec^{-2}. Using a radial intensity profile measured by Dennison for NGC 3379 as standard, a correction function Δm was calculated to give the magnitude contained within the standard isophote at diameter 2.5 D_s as a function of the magnitude as measured through an aperture of diameter D_p, with an appropriate correction for ellipticity.

It was pointed out by Stock and Schücking (1957) that there were systematic errors in this procedure which if left uncorrected would invalidate an attempt to find the magnitude of the second-order term in the m–z relation. There are three sources of error. Firstly, the measured diameter D_s will be a function of galactic latitude as the surface brightness of the estimated edge of the image would be affected by galactic absorption. Secondly, in looking at an isophote of the standard surface brightness in a distant galaxy one would be looking closer to the galaxy centre than in a nearer one because of the K-effect and the intensity gradient across the galaxy image. If one observes at effective wavelength then the observed radiation was emitted with wavelength $\lambda/(1+z)$ in the galaxy's rest frame and the surface brightness measured will be that appropriate to $I(\lambda/(1+z))$ rather than $I(\lambda)$. As the former will generally be larger than the latter the standard isophote will be systematically too close to the galaxy centre for more distant objects. Thirdly, one wishes to measure the light output from a fixed volume of standard radius centred on the emitting galaxy and the HMS procedure assumes that the standard isophote refers to a fixed distance from the galaxy centre measured in the rest frame of the galaxy. Due to the variation of surface brightness with distance, however, the standard isophote will move across the galaxy image towards the centre as the redshift increases. We cannot however correct for this effect to give the relation between the magnitude measured to the standard isophote and the magnitude due to the radiation emitted from a fixed volume unless we know the relation between the metric diameter defining this fixed volume and the isophotal diameter. And as metric diameter varies differently with z for different cosmological models it is necessary to know q_0 before the correction can be made. Fortunately the magnitude change with aperture diameter is small enough to enable one to make the correction with a preliminary estimate of q_0 and to refine the estimate by successive approximations.

As all magnitude estimates for extended objects involve this sort of angular diameter measurement (Hoyle, 1962), to this extent the m–z plot requires a knowledge of the θ–z relation. It might indeed be possible to use the θ–z relation directly for brightest cluster galaxies if their surface brightness distribution proves to be sufficiently homogeneous so that an evaluation of a characteristic metric diameter (say the constant a in the Hubble law $B = B_0(r/a+1)^{-2}$) could be made from a measurement of the brightness gradient or, more simply, the gradient of the curve of magnitude against photometer aperture. The practicability of the method would depend on some way being devised to compensate for the differing effects of seeing on images of different characteristic diameters. Knowledge of the K-correction would not be necessary, if one could discount colour changes across the image; if these were important they could in principle be corrected for, but differential evolutionary effects might prove difficult to estimate.

F. THE POSSIBILITY OF INTERGALACTIC ABSORPTION

If there is appreciable intergalactic absorption much of the discussion in this paper will have to be radically revised. From a comparison of counts of distant clusters which fall behind the Coma cluster and of those in neighbouring regions free from foreground clusters Zwicky (1961) has suggested an absorption in the blue of as much as 0.6 mag. in the centre of this cluster. Karachentsev and Lipovetskii (1968) have analysed the Zwicky catalogue of clusters for this effect and find the mean optical thickness of a cluster to be about 0.2 mag. If the absorbing material they find in clusters were spread out uniformly through space the optical depth in the blue would be $\tau_B = 0.9\,z$ for small z. It is difficult to accept the existence of this amount of intergalactic absorption from this evidence alone. The reddening at $z = 0.2$ would amount to 0.04 mag. with a ratio of blue to visible absorption the same as in the Galaxy ($\frac{4}{3}$), which, while not absolutely excluded by the data of Figure 3, seems rather high. The main reason for scepticism must be with regard to systematic errors in the catalogue; clearly any effect making it more difficult to recognise distant clusters behind nearby ones will produce the same effect as absorption. Until some means of excluding systematic errors of this kind has been devised the presence of large amounts of intergalactic absorbing material must remain in doubt.

Thomson scattering by intergalactic electrons has been considered by Bahcall and May (1968) who have calculated the effects of an intergalactic medium assumed to be completely ionised, and with a density appropriate to the value of q_0 if all the matter in the universe is in an uncondensed form; this is equivalent to assuming that matter in galaxies can be neglected, which is a possible approach if q_0 is large but is less realistic if $q_0 \cong 0$. They calculate the effect of such scattering on the form of the m–z relation and show that the differences between world models no longer increase simply as z increases, but that the maximum discrimination among the models is achieved for $z = 1$ to $z = 5$. The correction for the effect slightly increases the derived value of q_0;

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this correction is 0.2 for $q_0 = 1$ which can probably be neglected at present. For $q_0 = 0$ the effect obviously vanishes.

6. Solutions for q_0 in the $\Lambda = 0$ Cosmologies

Table II gives values for q_0 derived by fitting Equation (1) to some of the groups of available data. The following notation has been used (Peach, 1970) for the different data groups:

(I) Brightest cluster galaxies that are not radio emitters at the level of 9 f.u. at 178 MHz.

(II) Brightest cluster galaxies that are radio emitters at the same level.

(III) Radio galaxies that are not brightest cluster members. This category has been restricted to those having B − V colours greater than 0.70, and $|b^{\mathrm{II}}| > 25°$; N galaxies and double galaxies have been excluded in an attempt to get a homogeneous sample.

(IV) Cluster galaxies for which Baum has given bolometric magnitudes. Solutions have been computed for both Oke and Sandage's and Whitford's K-corrections. The following points emerge immediately from these results; a more detailed analysis of the data is given elsewhere (Peach, 1970; Craven and Peach, 1970). Figure 6 shows

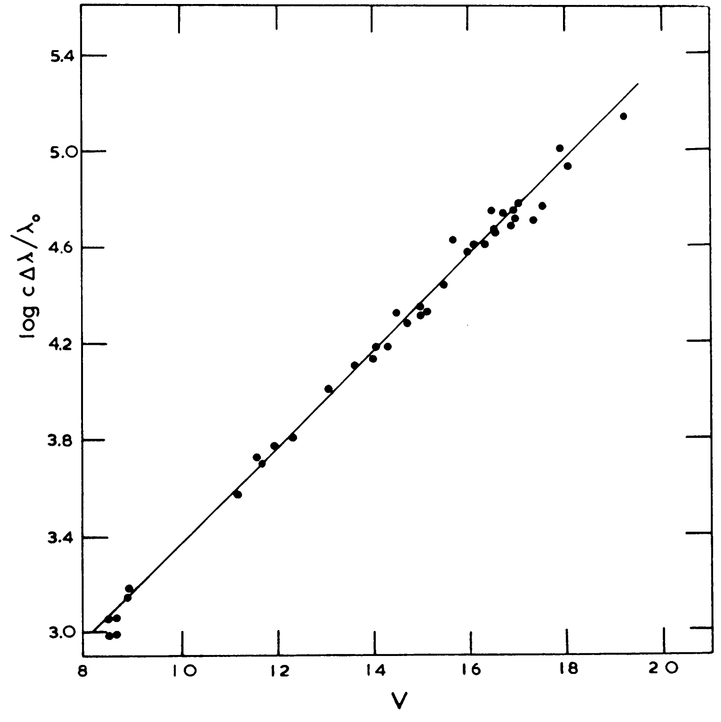

Fig. 6. Redshift-magnitude diagram for brightest cluster galaxies from the photometry of Baum and Sandage. A straight line of slope 5 ($q_0 = 1$) has been drawn in.

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all the brightest cluster galaxies I+II+IV, and Figure 7 shows the radio galaxies, II+III.

The values of q_0 derived from the B and V data for each separate group agree to within their probable errors; this is a reflection of the virtual elimination of Stebbins-

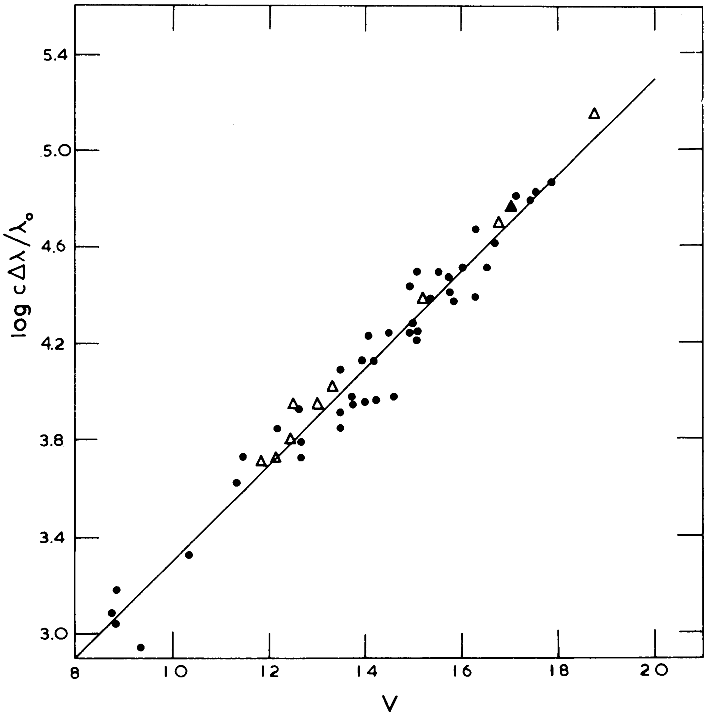

Fig. 7. Redshift-magnitude diagram for radio galaxies. Those which are brightest cluster members are shown as triangles. A straight line of slope 5 ($q_0 = 1$) has been drawn in. It may be noted that the cluster galaxies are all brighter than the mean line.

Whitford effect in both *K*-corrections. The standard deviation of the V magnitudes is less than that of the B magnitudes in each case; this is not unexpected as Lasker's (1970) work indicated a higher degree of homogeneity among *g*E galaxies in the yellow and red than in the blue. All the values, including those from the field radio galaxies, agree to within 2–3 times the probable error of their weighted mean. There is no discrepancy between the results from brightest cluster galaxies and radio galaxies that would seem to disqualify the latter from future use in this application, apart of course from their larger dispersion and fainter ($\cong$0.6 mag.) absolute magnitudes (cf. Figure 7). The distribution of the absolute magnitudes in each data group shows no certain sign of non-normality (but the reservations of Section 5c must be recalled).

The interpretation of the cluster data is entirely dependent on the relative weights assigned to the photometry of Baum and of Sandage. A solution is given in Table II for the two groups combined. It will be seen that the value derived for q_0 is not far

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TABLE II

Solutions for q_0 in the $\Lambda=0$ models for brightest cluster galaxies and radio galaxies using the Oke and Sandage (OS) and Whitford (W) K-corrections

Data group	Number of data points	q_0		Probable error in q_0	Standard deviation in M
		OS	W		
I B	28	0.00	(−0.36)	0.53	0.31
I V	28	0.23	(−0.31)	0.50	0.29
II B	9	1.65	1.19	1.36	0.31
II V	10	1.63	0.85	0.83	0.22
III B	24	6.10	5.41	3.37	0.52
III V	25	3.90	3.58	1.90	0.51
IV	8	2.44		0.62	0.20
(I + II) B	37	0.34	(−0.01)	0.47	0.31
(I + II) V	38	0.64	0.03	0.44	0.27
(I + II) V + IV	46	1.54	0.90	0.39	0.27

from a mean of the two independent determinations; this is a reflection of the high weight of Baum's three points with $z>0.2$, which carry similar weight in the solution to Sandage's 38 points with $z<0.2$. We consider it futile to speculate on the relative value of the two investigations, but note that Baum was considering "further readjustments and corrections" to his photometry (Baum, 1961b). We shall simply explore the consequences of the new photometry.

There is no doubt that the new value of $q_0=0.03\pm0.4$ from the V magnitudes for all cluster galaxies has certain attractions. With $H_0=75$ km s^{-1} Mpc^{-1} it corresponds through Equation (2) to a mean density $\varrho_0=6.10^{-31}$ g cm^{-3}, which may be compared with the well-known direct determinations by Oort (1958) of 3.10^{-31} g cm^{-3} or van den Bergh (1961) of 7.10^{-31} g cm^{-3}; it is also comfortably under the upper limits of 4.10^{-30} g cm^{-3} for ionised intergalactic hydrogen (Henry *et al.*, 1968) and 4.10^{-30} g cm^{-3} for intergalactic neutral hydrogen (Penzias and Wilson, 1969). But quite clearly the uncertainty in this value, due to the random errors in the observations alone, would be sufficient to allow a density 10 times higher at the 50% level of probability. Large evolutionary corrections with $\mathrm{d}M/\mathrm{d}t$ positive (i.e., a dimming of the galaxies with time) would force q_0 negative. If we accept the observations as excluding values of q_0 as high as 0.8 (the 80% confidence level), then evolutionary corrections of more than +0.2 mag. are excluded at $z=0.2$, that is $\mathrm{d}M/\mathrm{d}t<+0.07\times10^{-9}$ mag. yr^{-1} on the basis that they would give negative densities. This is not a very useful upper limit. The more plausible evolutionary corrections considered in Section 5b with $\mathrm{d}M/\mathrm{d}t$ negative raise the deduced value of q_0. Taking $\mathrm{d}M/\mathrm{d}t=-0.05(\pm0.02)\times10^{-9}$ mag. yr^{-1} as a reasonable mean value gives $q_0=0.5$ with an uncertainty of at least ±0.2 due to this correction alone.

An obvious further advantage of $q_0\cong0$ lies in the time scale since the Friedman singularity, which would be $\cong H_0^{-1}$ or 13×10^9 yr. In the light of the present uncertainty of the ages of the globular clusters (Iben and Faulkner, 1968), and Dicke's

(1969) calculation of the uranium decay age of the Galaxy as 7×10^9 yr, this would be conveniently long, but clearly depends on further revisions of H_0; with $H_0 = 50$ km s^{-1} Mpc^{-1} it is further increased to about 20×10^9 yr.

These conclusions all depend on the assumption that the cluster galaxies have absolute magnitudes which are normally distributed and independent of cluster richness. We can test the effect of this assumption by representing the trend in Figure 5 by the linear approximation

$$M_V = -22.50 - 0.1\,(N - 1.5),$$

where N is the richness class as defined by Abell, and we have taken the slope as 0.1 mag. per richness class. If now we add to each observed apparent magnitude the factor 0.1 $(N-1.5)$, we will reduce them all to the absolute magnitude system of clusters with equivalent richness 1.5, that is, to $M_V=-22.50$. This reduces the value $q_0=0.03$ to $q_0=-0.40\pm0.29$ with $\sigma_M=0.25$ mag., as there is a small positive correlation of redshift with richness. This large change is not in contradiction with the predictions of Section 5c, which were made for a sample with the same distribution of richness with distance as the Abell catalogue. Carrying through this calculation more rigorously, by solving for q_0 and for the coefficients of a linear relation between M_V and N which minimises the squares of the magnitude residuals, gives $q_0=-0.44$ and a slope dM_V/dN of 0.12. The reduction in σ_M and the probable error in q_0 reflects the removal of another contributor to the magnitude dispersion.

This value for q_0 lies almost midway between the lowest density Friedman model and the $q_0=-1$ of the steady-state theory and formally these would seem about equally probable. But whereas the steady-state picture will not admit an evolutionary correction, the value of $dM/dt=-0.05\times10^{-9}$ mag. yr^{-1} which we used above would pull q_0 back to 0.1 for the Friedman models. We do not wish to lay any great weight on these calculations, and it should be noted that the value $q_0=-1$ would imply that the magnitude of 3C 295 at $z=0.46$ from Baum's photometry would lie 0.8 mag. above the m–z curve. We simply present them as indicative of the effects of the various corrections, which will one day presumably be applied to more purpose when a larger data sample at larger redshift is available.

7. Solutions for q_0 in the $\Lambda\neq0$ Cosmologies

The previous discussion has been limited to the Friedman models and the steady-state theory which both have zero cosmological constant Λ. When high values of q_0 were thought reasonable and the ages of the globular cluster stars were thought to be possibly as long as 20×10^9 yr the time scale difficulty of the Friedman models directed attention to the $\Lambda\neq0$ cosmologies. The potential usefulness of the extra free parameter in resolving time scale and density anomalies has been extensively discussed (Tomita and Hayashi, 1963; McVittie, 1962; Petrosian and Salpeter, 1968). In view of the very large random and systematic errors to be expected in the values of H_0 and q_0 which gave rise to the anomalies, it may be doubted whether it was worth complicating the analysis for this reason alone. However, the omission of the Λ-term from the field equations appears arbitrary (Heckmann, 1961) and ideally the use of the $\Lambda=0$ cosmologies should be based on some observational demonstration that the cosmological constant is indeed zero. We have seen the large errors inherent in the determination of q_0 from a limited data sample at small z in the last section. The introduction of a further parameter into the analysis can only be made at the cost of a considerable increase in these errors. Peach (1970) has devised a computational scheme for the determination of q_0 and Λ from the exact form of the m–z relation in these cosmologies, and has used it to analyse Sandage's and Baum's data.

Let us write $\Lambda=3H_0^2(\sigma_0-q_0)$ where $\sigma_0=4\,G\pi\varrho_0/3H_0^2$ is the density parameter. For Baum's clusters $q_0=2.3$ and $\sigma_0=2.5$, and $\Lambda=0$ to a good approximation. Sandage's clusters (I+II) V give $q_0=2.3$ and $\sigma_0=-3.0$ with the Oke and Sandage K-correction, and $q_0=1.6$ and $\sigma_0=-3.8$ with the Whitford K-correction. While the negative density is clearly absurd the error in σ_0 is so large that it could formally rise to $+3$ even at the 50% confidence level. Combining Baum's and Sandage's data gives a final result $q_0=-0.4$ and $\sigma_0=3.8$. The non-zero value for Λ is not very significant; we have interpreted the observations as setting rough limits ± 10 on σ_0-q_0, or

$$+2\times 10^{-55}\geqslant \Lambda \geqslant -2\times 10^{-55}\ \mathrm{cm}^{-2}.$$

The characteristic distance of the Λ-interaction is then greater than $\Lambda^{-1/2}$ or 700 Mpc, which is about the distance covered by the data to $z=0.2$. It is clear that there is little point in this kind of analysis until there are more and better data; while there may be applications of the more flexible models with $\Lambda\neq 0$ in other problems of cosmology there is little more that the present m–z data can say about them.

8. Prospects

It should now be clear that while the extension of the observations to greater z is obviously necessary, there is still a great deal of uncertainty surrounding the interpretation of the present m–z diagram. There remains a discordance in the K-correction and considerable vagueness as to possible evolutionary corrections. Progress in both of these problems will be expedited by the use of multichannel scanners which can both avoid the former, and provide constraints on the latter by testing the temporal evolution of $I(\lambda)$.

A further pressing need is for an independent method of q_0 determination. A number of distance indicators are currently being exploited to measure H_0, but we still rely entirely on the m–z method for q_0. Any information from the θ–z test applied to individual galaxies or clusters of galaxies would be of importance. It is, however, comforting to note that the present interpretation of the m–z diagram for cluster galaxies and radio galaxies is remarkably consistent with time scales and densities determined independently; although unfortunately it would not be entirely cynical to remark that this is because the present results are ambiguous enough to be consistent with almost anything.

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Discussion

Wolfe: Could you explain why the negative values of q_0, which were shown in the last slide, were contained in parentheses?

Peach: Negative q_0 values are meaningless in the context of the Friedman models for which results are presented in Table II. The negative values were derived by fitting the data to the Heckmann *m–z* relation (4) and can be interpreted in terms of the relative probabilities of the zero-density Friedman model ($q_0 = 0$) and the steady-state model ($q_0 = -1$).

McVittie: The evolutionary effect in the determination of q_0 seems to depend on the person to whom one speaks. In conversation with Sandage a month or two ago I obtained the impression that he thought this effect unimportant. As to the Scott selection effect, I would say that the reasons originally given for its existence are surely no longer very strong because clusters are now chosen by such accidental circumstances as the presence of a radio galaxy, for example.

Peach: The value given recently by Sandage for the evolutionary correction is -0.044 mag. per 10^9 yr (*Physics Today*, 1970) which agrees with the value shown in Table I for the Spinrad model and is significantly non-zero. As important as the absolute value of the effect is the uncertainty in dM/dt, which I base on the scatter in the slope of the luminosity function, and which is large enough to make it at present a considerable barrier to the interpretation of the *m–z* diagram.

My illustration of selection effects was based on the data sample at present available. Further distant clusters will not be selected in the way considered in the Scott model, but they will be selected in some way, probably through searches of radio source fields. It will have to be shown that these identifications are not biased in absolute magnitude.

Abell: The 'Scott effect' may be entirely masked by individual differences among clusters of the absolute magnitudes of their brightest members. Some clusters contain giant *c*D galaxies that stand out a magnitude or more above the other cluster members; others do not. The cluster around NGC 6166 (cluster A2199) is an example of the former; the Corona Borealis cluster (A2065) is an example of the latter. Both of these clusters have very similar luminosity functions which, at the bright end, can be approximated by an exponential function. But the first ranked galaxy does not necessarily fit any such interpolation formula, and such a formula is not, therefore, particularly useful for predicting the absolute magnitude of the first-ranked galaxy.

The Virgo and Fornax clusters, incidentally, are not in my catalogue, but would be of richness class 0, not 1 as shown in one of your slides.

de Vaucouleurs: To the sources of error you have discussed should be added the still very serious problems involved in measuring consistent magnitudes, V_T or V_{25} or whatever, for similar galaxies near and far; I do not believe that this question has been correctly solved so far and the recent discovery of the large exponential envelopes of NGC 4486 (Virgo), and NGC 4889 (Coma), increasing the total luminosities by nearly 0.4–0.5 mag., shows that the question is not closed.

In addition possible inter- or intra-cluster absorptions, as discussed by Zwicky, may complicate matters even more.

Baum: My data, particularly for the clusters having redshifts greater than 0.2, were mentioned in several connections and I should like to clarify three points: (1) My photoelectric observations were made at six to eight wavelengths so that all parts of the spectral energy distribution curves were represented, and the *K*-correction was thereby avoided altogether. In my opinion, the involvement with *K*-corrections is an unnecessary headache that it is wise to avoid or minimize by the choice of photometric technique; (2) In the original discussion of my photoelectric redshift-magnitude data, I estimated a value of q_0 slightly greater than $+1.0$, but not so large as the value of $+2.4$ mentioned by Dr Peach. The latter evidently includes an evolutionary correction larger than I would favor. From my own estimates of stellar populations in elliptical galaxies based on multicolor photometry in the 1950's, I estimated the evolutionary correction to be so small that it should best be omitted until better known; (3) My multicolor photoelectric observations of ellipticals in various clusters (IAU Symposium, Santa Barbara, 1961) yield $E(\lambda)$ curves that are more similar to Whitford's recent curve than that of Oke and Sandage. I suspect that Whitford and I probably used larger, more inclusive diaphragms.

Mrs Burbidge: What was done for *K*-corrections of the N-galaxies in the slide you showed? – there is likely to be excess radiation in the B and UV, and this must make the correction smaller. When the data are published, I wish the raw measures could be given as well as the *K*-corrected ones.

Peach: N-galaxies were not used in the analysis.

Oke: Schild and I have recently done spectrophotometry of two giant elliptical galaxies in the Virgo cluster using an aperture of 7 arcmin. Our results confirm the energy distribution obtained by Whitford.

Solheim: A recalculation of Tinsley's model E2 (*Astrophys. J.* **151**, 547, 1968) shows that the dimming is reduced to $0^m.12$ per 10^9 yr, and at the same time a calculation on Spinrad and Peimbert's M31 disk population model (preprint of Chapter II of Stars and Stellar Systems, Vol. 9, in press) gives about $0^m.04$ per 10^9 yr dimming, if one looks at light emitted in the U band, which is redshifted into V at $z = 0.55$. The redshift for a given lookback time depends on the cosmological model; the table shows the redshift at which the B and U filter bands are redshifted into V, and the evolutionary correction if this redshift corresponds to 3×10^9 yr. (Calculation by Tinsley, private communication, 1970.)

Evolutionary corrections to V magnitudes in 3×10^9 yr [a]

Wavelength band	V	B	U
Shifted into V at $z =$	0.0	0.25	0.55
Spinrad & Peimbert's M31 disk model	-0.07	0.00	$+0.13$
Tinsley's model "E2"	$+0.09$	$+0.14$	$+0.36$

[a] Sign positive if model was brighter in the past.

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Scott: Firstly, I want to comment on a point in the talk by Dr Peach. Differences in richness are not the same as differences in the unknown total number of galaxies in the cluster, which is one cause of the selection effect under discussion (if a cluster has many members, there is more chance for it to have an extremely bright member; this is without any special assumptions about correlation of increasing luminosity function with the number of members which only makes matters worse). Richness depends on the total number but also on the distance; as a cluster with many members is moved out from the observer, its richness class will change towards less rich. The so-called Scott effects are classical. Observers working at the limit of observations will always be bothered by these selection effects. Thus, to the point raised by Dr McVittie – that astronomers no longer select clusters for observation as in my paper. So far as the numerical computations are concerned, he is correct. However the selection effects will still be present. When the paper referred to was published, about 1957, the clusters were those of Humason *et al.* and so the computations of the numerical size of the effects were based on their methods of observing. If the method of selection for observation were changed, the numerical value of the effects of selection might well change but the effect would still be there. If the selection were based on searching among radio sources, we first need the radio source bright enough to be observed and then its galaxy bright enough to be observed. I do not think it realistic that a real-life astronomer will be able to select at random.

Heidmann: Concerning the value of the distance of the Virgo cluster, I obtained this year a new determination using the diameter-luminosity relation for spirals. Incorporating elaborate corrections for inclination effects on the diameters and magnitudes of galaxies, resulting from work done with my wife and de Vaucouleurs, a very strong correlation results between diameter and luminosity, with a 0.97 correlation coefficient over a range of five magnitudes. Then, using as calibrators M31, M33 and four spirals in the M81 group taken from van den Bergh's review, the distance modulus of the Virgo cluster spirals turns out to be 30.7 ± 0.5.

Barnothy: The Friedman models you have listed have cosmological constants equal to zero. As Z. Horak (*Bull. Astron. Inst. Czech.* 1970, **21**, 96) has shown, with $\Lambda = 0$ the general theory of relativity contradicts Mach's principle. Could you state your position in this matter?

Peach: The solutions I have calculated with cosmological constant taken as a free parameter show that the possibility that $\Lambda = 0$ cannot be excluded to within the limits considered in the paper. At present the Friedman models give an adequate description of the observations and there is no evidence for a non-zero value of Λ.