4.5 NAṢĪR AD-DĪN ON DETERMINATION OF THE DECLINATION FUNCTION

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Introduction

A commentary on the spherical astronomy in the Zīj-i Īlkhānī [1], written by the famous Naṣīr ad-Dīn aṭ-Ṭūsī (c. 1270) would be an important contribution to the history of medieval astronomy. But we postpone this and attempt here to describe the contents of the third and fourth sections of Treatise III of this zīj. We hope that it will serve to elucidate a small part of this work, which has remained unexploited. I have made a preliminary English translation of the entire Zīj-i Īlkhānī, which awaits final revision and preparation for publication.

The rules explained verbally and without proof by Naṣīr ad-Dīn in the sections under consideration are similar to the ones described by Kāshī in his Zīj-i Khāqānī and analysed by E. S. Kennedy in [2]. Therefore, we do not present our proofs of the rules, but refer the reader to [2].

The microfilm copy of Manuscript 1418/2 of the Central Library of Tehran University has been used as the main text. It seems to be a careful job, as compared with the other copies available to us: MS 684 of Sipah Salar (now Muṭahari) Library, Tehran, and MSS 5331 and 5332 of the Shrine (Āstān Quds) Library, Mashhad.

Section 2 below presents a description of the contents of the third section of Treatise III in modern notation. It contains the definitions of the declination functions for points on the ecliptic and some trigonometric methods for their determination, Section 3 of this paper describes the material in the fourth section of the text which defines the declination function for arbitrary points on the celestial sphere and gives two formulas for its determination based on spherical trigonometry. We note that the term declination (mayl) was restricted to points on the ecliptic by the medieval astronomers, and they used the word distance (buᶜd) for arbitrary points [2].

The symbols used in the present paper are the standard astronomical symbols, where applicable, and defined as they are encountered. There are no figures in the original Persian text, but we have made them in order to understand the material. To indicate the folios and the corresponding lines of the microfilm copy, we have used numbers inside parentheses. For example, (194r: 2-20) means folio 194r, lines 2-20 of the manuscript.

The medieval trigonometric functions were not identical with their modern

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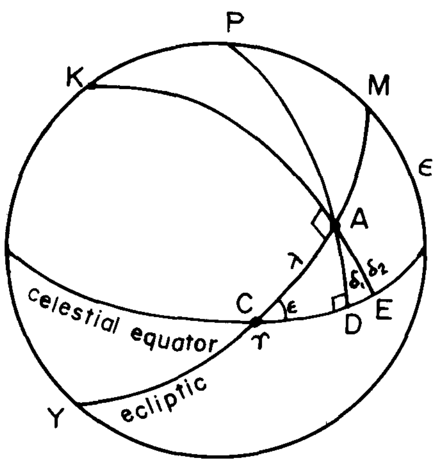
counterparts, and are written here with initial capitals to distinguish them from the latter. The relation between the two is, e.g., $\mathrm{Sin}\,\theta = R \sin\theta$, where R is the radius of the defining circle, usually 60, not unity. This explains the appearance of the R in equations below.

Declinations of the Ecliptic Points (194r: 2-20)

The obliquity of the ecliptic (ϵ, ghayat-i mayl) is defined to be an arc of the great circle that passes through the four poles. These are the north and south poles of the ecliptic and those of the celestial equator. The arc between the summer solstice point M and the equator is northern, and the one between the winter solstice point Y and the equator is southern. (See Figure 1 below).

Figure 1



The obliquity of the ecliptic ϵ is determined by observation (line 4). Also, according to our text, Ptolemy's value is $23;51^{\circ}$; most Islamic astronomers give $\epsilon = 23;35^{\circ}$, and some of their successors have given $\epsilon = 23;33^{\circ}$. Naṣīr ad-Dīn prefers his own value, given as $\epsilon = 23;30^{\circ}$, which he adopts.

Partial declinations are defined for an arbitrary point A on the ecliptic, other than the solstitial points (lines 6-10). The first declination (δ_1, mayl-i awwal) is defined as the great circle arc issuing from A and perpendicular to the celestial equator (Fig. 1). This circle is called the declination circle and it passes through the north pole P.

The second declination (δ_2, mayl-i thānī) of the ecliptic point A is the great circle arc indicated as such on Figure 1, issuing from A, as before, but now passing through the poles of the ecliptic. This circle is called the latitude circle, since the celestial latitude (ᶜarḍ) of a star on the celestial sphere is defined as the distance between it and the ecliptic along such a circle.

According to our text (lines 10-11), some medieval astronomers called the

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first declination the absolute declination, and the second declination the latitude of the ecliptic point A.

In our notation, the text says (lines 11-13) that

$$\delta(\lambda) = \delta(180^0 - \lambda) = -\delta(\lambda + 180^0) = -\delta(-\lambda).$$

As examples, the beginnings of Taurus, Pisces, Virgo, and Scorpio have the same declinations except for a plus or minus sign. The declinations of Taurus and Virgo are northern; those of the other two are southern.

We note that the above equations suffice to determine only the declinations of the points of the first quadrant of the ecliptic which is pointed out in line 13 of our text.

With known λ and ϵ, Naṣīr ad-Dīn uses the right spherical triangle ADC to obtain δ_1. The formula he gives (lines 14-15), in our notation, is the following:

$$\text{Sin}\,\lambda \cdot \frac{\text{Sin}\,\epsilon}{R} = \text{Sin}\,\delta_1, \qquad (1)$$

or

$$\delta_1 = \text{Sin}^{-1}[\text{Sin}\,\lambda \cdot (\text{Sin}\,\epsilon)/R]$$

which can be read from the sine table of the zīj.

Given the same values, Naṣīr ad-Dīn uses the right spherical triangle EAC and the following formula (line 16)

$$\text{Sin}\,\lambda \cdot \frac{\text{Tan}\,\epsilon}{R} = \text{Tan}\,\delta_2 \qquad (2)$$

to obtain δ_2 as

$$\delta_2 = \text{Tan}^{-1}[\text{Sin}\,\lambda \cdot (\text{Tan}\,\epsilon)/R].$$

Then, using the same triangle, he first obtains the angle $\widehat{AEC}$ (line 17-18) from

$$\text{Cos}\,\lambda \cdot \frac{\text{Sin}\,\epsilon}{R} = \text{Cos}\,\widehat{AEC}$$

and, having calculated Cos $\widehat{AEC}$, he uses it in the following formula to obtain Cos δ_2 as

$$\text{Cos}\,\delta_2 = \frac{\text{Cos}\,\widehat{AEC}}{(\text{Cos}\,\epsilon)/R}.$$

Then

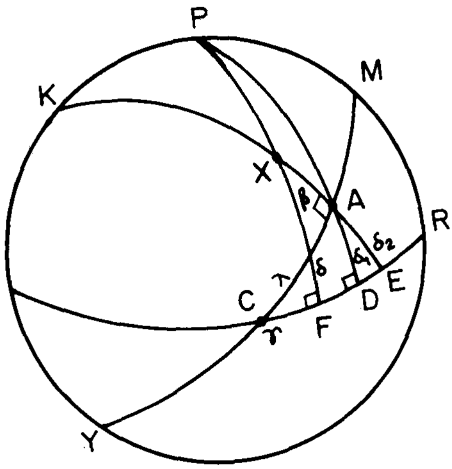
$$\delta_2 = \text{Cos}^{-1}[(R\ \text{Cos}\,\widehat{AEC})/\text{Cos}\,\epsilon].$$

The last two lines of the third section of our text state that if a table is available for the right ascensions of an ecliptic point λ, which we denote by $A_0(\lambda)$, to be defined later in the text, then λ is found for each value of $A_0(\lambda)$ from this table. Then δ_1 and δ_2 are determined by the above rules. To facilitate the determination of the declinations of the ecliptic arc and the corresponding inverse operations, Naṣīr ad-Dīn has tabulated the tables of both declinations δ_1 and δ_2 for arcs from 0^0 to 90^0 in his zīj.

Declination of Other Points (194r:21-27)

The distance (buᶜd) or the present day declination δ of a point X on the celestial sphere from the celestial equator is defined to be an arc of the declination circle, issuing from X and perpendicular to the celestial equator (Figure 2).

Figure 2



To determine this distance δ = XF, Naṣīr ad-Dīn says (lines 22-24) find the algebraic sum of β, the celestial latitude, and δ_2, the second declination of its celestial longitude λ. This sum, $\beta + \delta_2$, is called the argument (ḥiṣṣah) of the declination, its direction being that of the algebraic sum. Then he determines the first declination δ_1 corresponding to λ, and calls it the inverse declination of the point X. Finally, to determine δ, he prescribes the following formula (line 25);

$$\mathrm{Sin}(\beta+\delta_2)\cdot\frac{\mathrm{Cos}\,\delta_1}{R} = \mathrm{Sin}\,\delta ,$$

from which δ can be determined using the inverse sine function. Alternatively, (lines 26-27) in our symbols, he suggests the following formula:

$$\frac{\mathrm{Sin}\,(\beta+\delta_2)\,\mathrm{Cos}\,\varepsilon}{\mathrm{Cos}\,\delta_2} = \mathrm{Sin}\,\delta ,$$

from which again δ can be determined.

References

[1] Naṣīr ad-Dīn aṭ-Ṭūsī, Zīj-i Īlkhānī. Microfilm 1418/2, Tehran University Central Library. (Other copies consulted are: MS684, Mutahari Library, Tehran, and MSS 5331 and 5332 of the Shrine Library, Mashhad.)

[2] Kennedy, E. S. (1985) Spherical Astronomy in Kāshī's Khāqānī Zīj. Zeitschrift für Geschichte der Arabisch-Islamischen Wissenschaften, 2, 1-46.

DISCUSSION

A.K.Bag : It is referred that Zīj-i-Khāgāni of al-Kāshi is a work on spherical trigonometry. In Indian tradition kha means heaven or celestial sphere, gāni is similar to jyāni which means arcs (may be sine or cosine arcs of) celestial sphere. The rule of three and four were known to India from beginning of early times. This suggests that Khāgāni that is somewhat similar to Khajyāni which means arcs of the celestial sphere. Since both sine and cosine functions were known in India from Āryabhaṭa's time, was it that the al-kāshi was influenced by knowledges from India.

Javed Hamadani Zadeh : I do not think that the name khāgāni in any way related to the traditional words in Sanskrit or Indian language.

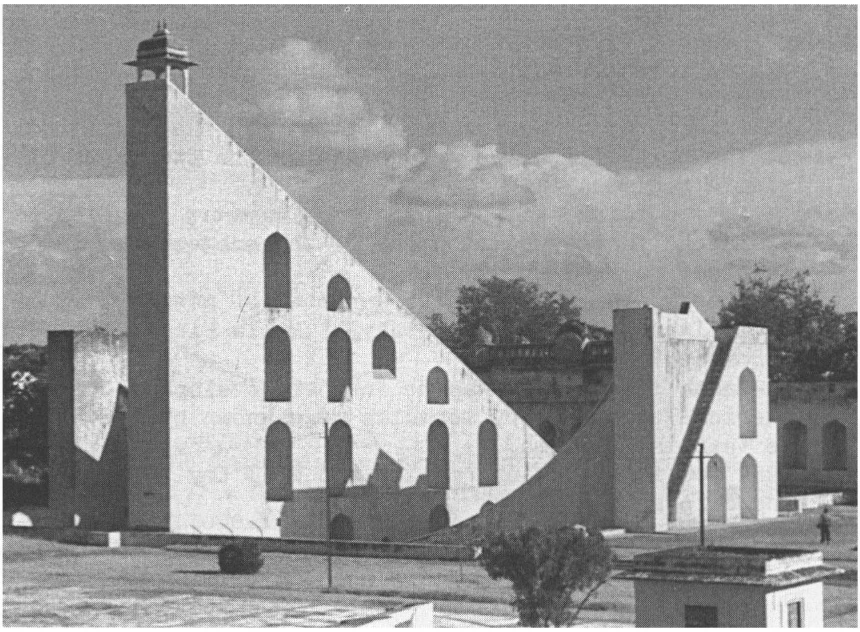
The sine function was perhaps taken from the Indian sources, but the rule of four and the sine law are the works of the Islamic period.

S.M.R.Ansari : Do you think that al-Ṭusi's trigonometry as given in his Zīj is the further development of the subject as given in his other works ?

May I draw your attention to a number of manuscripts of Zīj-i-Ilkhanī with a commentary at Mulla Firoze collection at Bombay ?

Javed Hamadani-Zadeh : No, there was no further development. The spherical trigonometry formulas were known by the readers of the text in his time.

Thank you for your information and will try to locate these manuscripts.



The Great Samrat Yantra. A 23 m high equinoctial sun dial designed to measure local time, and the declination and right ascension of celestial objects.