Theoretical

Global Oscillations in Late-Type Stars (Invited Paper)

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Abstract: The advent of new techniques to measure the global oscillation spectrum of the Sun has provided a new and powerful tool to investigate solar structure. One of the most challenging and potentially rewarding problems in contemporary astronomy is to devise techniques which will allow similar studies of other stars. This paper outlines the theory of global oscillations of late-type main-sequence stars, and summarises some of the achievements of global oscillation studies of the Sun. It then reviews the very few successful attempts that have been made to study stellar oscillations, and briefly discusses several apparently promising lines for further instrumental development.

Introduction

For many decades, astronomers have used observations of oscillations (or pulsations) of certain classes of stars (Cepheids, RR Lyrae, δ Scuti, β Cephei, ZZ Ceti, and so on) to probe the internal structure of these objects. For most of these classes the oscillations consist of only a small number of modes, but nevertheless a great deal has been learned in this way about stellar masses, radii and structure. Some of this information, such as the internal distribution of density and temperature, cannot be obtained by any other observational technique.

Recent discoveries regarding the nature of small-amplitude oscillations on the Sun imply that a large number of late-type stars may exhibit a rich spectrum of oscillations, opening up the possibility of a major new technique to study stellar structure. Although the solar 'five-minute' oscillations were discovered in the late 1950s (see Leighton *et al.* 1962), it was not until 1970 that Ulrich (1970) and, independently, Leibacher and Stein (1971) suggested that the waves observed in the photosphere could be the surface manifestation of a phenomenon rooted in the convection zone. Deubner (1975) carried out painstaking observations which covered sufficient space and time to reveal the existence of the modes proposed by the theoreticians, and the subdiscipline of *helioseismology* was born.

While most observations of the Sun are made with sufficient spatial resolution to reveal modes of many orders, some of the most sensitive studies have concerned the detection of solar oscillations in light from the unresolved Sun (Brookes *et al.* 1978; Claverie *et al.* 1979; Fossat 1981; Woodard and Hudson 1983; Grec *et al.* 1983). Remarkably complex power spectra have been obtained, and the theoretical interpretation of the frequencies and relative amplitudes of the spectral peaks is well advanced (e.g. Deubner and Gough 1984).

The detection and subsequent detailed study of analogues of the solar global oscillations in other stars is one of the most challenging and potentially rewarding observational problems in astrophysics today. Success will require long time-series (at least 8 hours) of measurements made at high rates (one per minute) with extremely demanding precision (Doppler shifts down to ~ 10 cm s^{-1}; relative intensity variations to $\sim 10^{-6}$). The reward will be new data which can be used to determine fundamental stellar parameters (such as mass, radius, temperature, and angular velocity) and to study such basic processes as convection and magnetic cycles.

This paper reviews work on oscillations detected in light from the unresolved Sun and from other stars, and addresses some of the challenging instrumental problems which need to be overcome for the field of *asteroseismology* to advance. For fuller details, the reader may refer to the recent monograph on the subject (Gough 1986).

Theory

The theory of small-amplitude, non-radial oscillations in a compressible, self-gravitating sphere has been discussed extensively by several authors (e.g. Ledoux and Walraven 1957; Unno *et al.* 1979; Cox 1980). It is sufficient for our purposes to briefly summarise the techniques employed and then to concentrate on asymptotic results pertaining to modes of high degree.

The theory begins with the time and (3D) space dependent equations of motion for a compressible gas. These equations are linearized by assuming small-amplitude perturbations. While frequencies and eigenfunctions can be quite acurately determined by using the adiabatic assumption (e.g. Vorontsov and Zharkov 1981), questions regarding the growth rate and excitation mechanism require a more complete treatment of the energy equation (e.g. Unno *et al.* 1979). Solutions of the resulting system of perturbation equations are expanded as a sequence of spherical harmonics in the form

$$F_n(r)P_l^m(\cos\theta)\sin m\phi e^{i\omega t} \quad \ldots (1)$$

Here, t is the time and ω the wave frequency; (r,θ,ϕ) are spherical polar coordinates, and P_l^m is the associated Legendre function. We call l the *degree* of the mode and m the *azimuthal order*: l counts the number of nodal circles on the surface of the sphere, while m $(-l \leq m \leq l)$ counts the number of these which pass through the poles. In the absence of rotation or other asymmetric processes, the eigenfunctions are degenerate with respect to m. For each value of (l,m), there are an infinite number of radial eigenfunctions $F_n(r)$ which satisfy the differential equations and the boundary conditions at the centre and surface. These different eigenfunctions are classified by the *order* number n, which is roughly equivalent to the number of nodal surfaces along the radial direction. For each value of (l,m,n) there are generally two associated eigenfrequencies ω, one corresponding to relatively high frequency oscillations in which pressure forces dominate, and the other to relatively low frequency oscillations in which bouyancy forces dominate (more complicated situations do exist—see Vorontsov and Zharkov 1981). In this discussion we are concerned with the pressure-dominated modes.

Evidently, modes with large l will not be readily observable in integrated star- (or sun-) light, because of cancellation between almost equal areas of opposite phases of the surface disturbance. However, because very low degree modes $(l=0,1,2,3,4)$ are only partially cancelled (Christensen-Dalsgaard and Gough 1982), we might hope to measure low degree modes for a number of orders n. For modes with $l/n << 1$, Tassoul (1980) has shown that the frequencies are given by the asymptotic expression

$$\omega(n,l) = 2\pi(n+l/2+\epsilon)\Delta\nu_o + \delta(n,l), \quad \ldots (2)$$

where ϵ and δ are parameters related to the equilibrium structure,

$$\Delta\nu_o = [\,2 \int_0^R \frac{dr}{c}\,]^{-1}, \quad \ldots (3)$$

and $c(r)$ is the sound speed. Equation (2) predicts that for fixed l, modes of differing n should be separated by the (approximately) regular interval $2\pi\Delta\nu_o$, and that for given n, the modes of even or odd l should be separated by the same interval. The interval between modes of equal n and adjacent values of l should be approximately $\pi\Delta\nu_o$. From equation (3) it is clear that the determination of the frequency difference between modes provides important information on the internal structure and stellar radius.

The simple theory described above can be refined to take account of rotation [which splits the spectrum by an amount of order $(\text{Period})^{-1}$] or other perturbations such as convective cells or magnetic structures. Moreover, the theory can be extended to consider the amplitudes and quality $(Q = \Delta\omega/\omega)$ of the modes, and perhaps their phases: these quantities will reflect the mechanisms responsible for excitation and damping. At present, these mechanisms are understood very poorly, although some consideration has been given to overstability similar to that which excites Cepheids (e.g. Ando and Osaki 1975), or to stochastic excitation by convection (Goldreich and Keeley 1977).

Helioseismology

Two quite different techniques have been used successfully to study solar oscillations in integrated sunlight. The *photometric* method relies on the analysis of a time-series of measurements of the absolute broad-band irradiance, made by very stable radiometers carried on spacecraft (e.g. Woodard and Hudson 1983). The *Doppler-shift* method relies on time-series observations of intensity variations in integrated sunlight made in the wings of Fraunhofer lines. To date, most successful studies have used resonance scattering from atomic vapour cells to maintain the necessary wavelength stability. While spectroscopic techniques can be used successfully from the ground (being more immune to atmospheric effects than photometric methods), the diurnal cycle experienced at ground-based observatories (except near the poles) imposes a data window with unacceptable sidebands for the most accurate work. Consequently, some novel campaigns have been mounted to obtain long, unbroken time-series observations, involving either observing seasons at the South Pole (Grec *et al.* 1983; Pomerantz 1986) or coordinated observations at widely spaced longitudes (Isaak 1985).

The observed power spectra exhibit the regularity which would be expected on the basis of the recursive properties of the asymptotic $\omega(l,n)$ relation (Equation 2). Once the fundamental interval $\Delta\nu_o$ has been identified (its value is 136 μHz for the Sun), the pattern of peaks associated with the various degrees l of a given order n can be used to identify specific values of (l,n) associated with each peak. The accuracy of the available data is certainly adequate to determine the model-dependent parameters $\Delta\nu_o$, δ and ϵ in equation (2), and there has been a considerable effort made to explain discrepancies between theory and observation (e.g. Grec *et al.* 1983; Isaak 1980).

The success of helioseismology carried out in integrated sunlight provides a sound basis for the design of experiments to measure analogous oscillations on other stars. It has been shown that solar modes with degrees $l=0,1,2$ and 3 can be detected, with amplitudes of the order 10^{-6} in relative intensity and 10 cm s^{-1} in velocity. Oscillatory power is detected over the range 2-5 mHz, and the line width of single modes is about 1 μHz. Of course, there have also been intensive studies of oscillations on the spatially resolved Sun (e.g. Pomerantz 1986) and these have supported the development of a very strong theoretical basis for the analysis of spatially unresolved data.

Asteroseismology

Predictions of the properties of high-order oscillations in several kinds of stars are now available (Ando 1976; Christensen-Dalsgaard and Frandsen 1982; Christensen-Dalsgaard 1984). At the most basic level, the theories suggest that the frequency at which oscillatory power is concentrated will vary roughly as the acoustic cut-off frequency in the stellar photosphere, so that the oscillatory period will scale approximately as

$$\text{Period} \sim T_{eff}^{0.5} g^{-1}, \quad \ldots (4)$$

where T_{eff} is the effective stellar temperature and g the surface gravity. We thus expect periods in the range 2-10 minutes for main-sequence stars, while late-type giants will have periods of a few hours, and late-type supergiants periods of a few days.

Although the present paper concentrates on main-sequence stars, there are indications that giants and supergiants may be well worth study. For example, it is known that such stars have high levels of spectroscopic 'macroturbulence', and there are several reports of small-amplitude spectrum variability in stars such as Arcturus. Furthermore, it is likely that convective energy transport is rather inefficient in the low-density envelopes of such stars, and the resulting superadiabatic gradients could be a potent source of oscillatory power (Christy 1962). Although it is difficult to study stars having many modes with periods ranging from days to weeks, Lucy's (1976) work on the early supergiant α Cyg (A2 Ia) shows that there is the potential for substantial rewards in such investigations.

For main-sequence stars, the parameters $\Delta\nu_o$, ϵ and δ (equation 2) are quite sensitive to the structure of the stars. For example, Christensen-Dalsgaard (1984) has shown that for a sequence of zero-age main-sequence (ZAMS) models, $\Delta\nu_o$ falls from about 250 μHz at $M=0.6\ M_\odot$ to about 90 μHz at $M=1.5\ M_\odot$. For a star of $1M_\odot$, $\Delta\nu_o$ falls from about 160 μHz at the ZAMS stage to about 100 μHz at an age of 10^{10} years. It is clear that fundamental stellar parameters can be deduced if individual peaks in oscillation spectra can be observed.

To date, there have been only a small number of successful observations of global oscillations on late-type main sequence stars. Noyes *et al.* (1984) found some evidence of oscillations in ϵ Eri (K2 V) by measuring the variation of the relative intensity in 1 Å bands centred on the Ca II and K lines. The detected period was about 10 minutes, and the frequency spacing $\Delta\nu_o$ was about 170 μHz, in good agreement with the expected value. However, the amplitude of the oscillation was substantially larger (100x) than the corresponding value in the Sun, and the oscillation was not detected in all of their observing sessions.

Gelly *et al.* (1986) used a sodium atomic vapour resonance scattering cell to study oscillations on Procyon (F5 IV) and α Cen A (G2 V). Following a very careful and involved data analysis procedure, they deduced the results summarized in Table 1. The results for Procyon are consistent with our best understanding of that star, but for α Cen it is expected that the spacing $\Delta\nu_o$ should be slightly less than 142 μHz [since the mass (1.09 $M_\odot$) and radius (1.23 $R_\odot$) are presumably well known], much smaller than the measured value of 165.5 μHz. It is thus already clear that asteroseismology has the potential to yield strong tests of fundamentally important astrophysical concepts.

Prospects

It is a very challenging task to measure stellar oscillations with amplitudes of ~ 10 cm s^{-1}, periods of ~ 5 minutes, and modal splittings of ~ 100 μHz. Although there are many technical advantages in making such observations from space (Mageney and Praderie 1984), it is likely that substantial initial progress can be made at much lower cost from the ground.

Harvey (1987) has reviewed many of the proposed techniques for observational asteroseismology. It is clear that broad-band photometry is extremely difficult in view of the effects of atmospheric transparency fluctuations and scintillation. There are better prospects with spectrophotometric instruments designed to measure intensity oscillations in different spectral regions which have different amplitudes or phases (such as the Ca II-line technique of Noyes *et al.* 1984). Perhaps the best prospects for photometric techniques lie in proposals (Harvey 1987) to use an array detector in a coudé spectrometer to compare intensity oscillations in line cores with oscillations in the adjacent continuum.

Doppler methods appear to be potentially more sensitive to stellar oscillations, but there are significant problems to be overcome in relation to (1) photon counting statistics and (2) instrumental wavelength stability. An instrument operating on a single spectral line (such as an atomic resonance scattering cell) accepts only a small fraction of the potentially useful photons, although the work of Fossat and his coworkers (Gelly *et al.* 1986) shows that useful results can be obtained as a result of the extreme stability that is available. Fourier transform spectrometers (FTS) can achieve the required wavelength stability over a wide spectral range, but the need to accumulate interferograms with a cadence of a minute or so requires a form of multiplexing which is not available in current astronomical FTS instruments.

Grating spectrographs perhaps offer the greatest potential sensitivity in the near future. Apart from the relatively poor optical efficiency of a telescope-spectrograph combination, the greatest challenge in using such systems appears to be the need for a sufficiently stable wavelength reference. Smith (1983) has shown that atmospheric winds preclude the use of telluric lines. The superposition of a reference spectrum by passing the starlight through an appropriate gas (I_2, HF, N_2O, etc.) is one promising line of attack (e.g. Campbell 1983). Alternatively, it is possible to impose a reference pattern on the detected spectrum by reflecting the starlight from an interferometer, either a Michelson or a Fabry-Perot. Of course, the interferometer itself must have very stable fringe positions: this may be done either by controlling the geometrical properties accurately (e.g. Winter 1984; Butcher and Hicks 1986) or by actively locking a controllable interferometer to a stable wavelength reference (e.g. Burton *et al.* 1987).

The actively-stabilized solid etalon developed by the Johns Hopkins Applied Physics Laboratory and the CSIRO Division of Applied Physics (Burton *et al.* 1987) could be well suited to providing a stable reference pattern for spectroscopic observations of stellar oscillations. A scheme for applying this system to study stellar oscillations is sketched in Figure 1: perhaps the greatest technical obstacle to this device is the need to provide a close control over the temperature of the etalon (for typical etalon characteristics, a temperature change of 1 K produces the same fringe shift as a voltage change of 50 V).

TABLE 1

Properties of stellar oscillations
(from Gelly, Grec and Fossat 1986)

Star:	Procyon (F5 IV)	α Cen A (G2 V)
Period range:	10-25 min	3.5-85 min
$\Delta\nu_o$:	79.4 μHz	165.5 μHz
deduced radius:	1.87 $R_\odot$	0.93 $R_\odot$
amplitude:	70 cm s^{-1}	150 cm s^{-1}

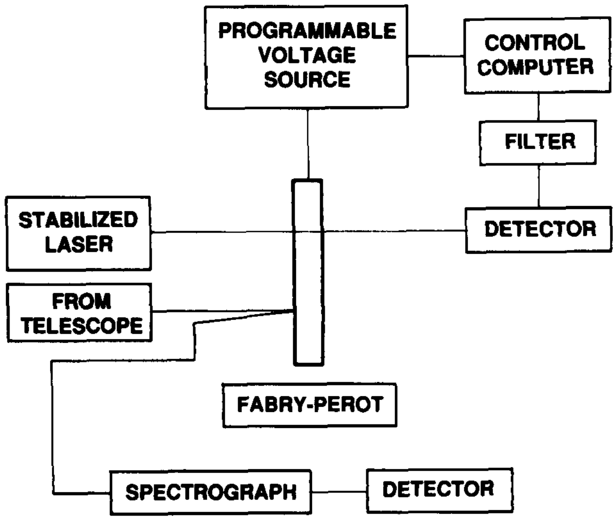

Figure 1 — Schematic layout of a system which uses a laser-stabilized, voltage-tunable Fabry-Perot interferometer to impose stable reference lines on a stellar spectrum observed with a conventional high-dispersion spectrograph. The pass-band of the Fabry-Perot is dithered under computer control to modulate the transmission of light from a frequency stabilized laser. The modulation phase is used to lock the Fabry-Perot fringe to the laser frequency. Starlight from a telescope is reflected from the etalon, and detected after being dispersed by a spectrograph. The observed spectrum consists of the stellar spectrum disected by a regular pattern of etalon fringes at a precisely known wavelength.

There are other significant problems to be solved before observational asteroseismology can flourish. For example, solar workers have demonstrated the detrimental effects of the sidelobes produced by the data window imposed by the diurnal cycle. While useful work can be done with ~8-12 hour data strings, ultimately it will be necessary to undertake coordinated observing campaigns at widely separated longitudes. It appears that relatively large telescopes/apertures are needed, at least with current sensitivities, and the need for long, unbroken time series will conflict with the requirements of other users of major facilities. However, it may be possible to alleviate this problem by undertaking studies of bright stars during daylight hours.

We are only just beginning an exciting and potentially very rewarding branch of observational astrophysics. It will probably take several decades of work before we can accumulate a set of high-quality oscillation power spectra of, let us say, a sample of several hundred stars of different internal structure. The payoff in terms of increased understanding of the stars is likely to be very significant.

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