BERNDT, BRUCE C., *Ramanujan's Notebooks*, Part II (Springer-Verlag, Berlin–Heidelberg–New York, 1989), 359 pp., 3 540 96794 X, £55.

The first part of Professor Berndt's edition of Ramanujan's Notebooks appeared in 1985 and was reviewed on pp. 284–285 of volume 29 of these *Proceedings*. It covered Chapters 1–9 of the second notebook, and included a description of Ramanujan's Quarterly Reports to the University of Madras. At that time it was thought that the complete work would occupy three volumes, but it is now known that four will be required. The book under review follows on from Part I and covers Chapters 10–15. These six chapters are among the most interesting chapters in the notebooks. In all they include 605 results, which, as usual, are stated without proof. When a result is known the author gives a reference and, if not known, he provides, where possible, a proof. It is only in a very few instances, where Ramanujan's intent is not clear, that no proof is offered. Professor Berndt emphasizes that many (perhaps most) of the proofs are undoubtedly different from those found by Ramanujan, since mathematical theories with which he was unfamiliar have had to be employed.

Ramanujan rediscovered most of the classical formulae in the theory of hypergeometric series, but Chapters 10 and 11 contain numerous new, useful and highly interesting results on the subject. Chapter 12 contains a large number of new results on continued fractions, such as continued fraction expansions of products and quotients of gamma functions. Ramanujan was undoubtedly the greatest exponent of this subject at a technical level, and it is therefore of interest that, if none of his notebooks and manuscripts had survived, his work in this area would be scarcely known, since only one continued fraction occurs among his published papers.

Chapters 13 and 14 contain many fascinating theorems on integrals, series and asymptotic expansions. Of all the chapters in the volume Chapter 15 is the most unorganized and is devoted to a number of disparate topics including asymptotic expansions and modular forms, in particular Eisenstein series. Berndt remarks that to obtain his asymptotic expansions Ramanujan appears to have used the Euler–Maclaurin sum formula, but in a non-rigorous way, which occasionally led him to make minor errors. However, it is truly astonishing that among such a vast collection of formulae so few errors occur. Part III, which is due to appear shortly, will cover Chapters 16–21. The remaining chapters in the second notebook and the short third notebook will be dealt with in the final fourth part.

The contents of the book have appeared separately in seven papers written by Professor Berndt, who all along has been the main guiding spirit of the enterprise, in collaboration with R. L. Lamphere, R. J. Evans and B. M. Wilson. This last name pays tribute to the man who, together with G. N. Watson, left extensive notes as part of their labours on the notebooks in the 1930s. In addition, acknowledgement is made to other individuals who have made valuable contributions.

R. A. RANKIN

FAUVEL, J., FLOOD, R., SHORTLAND, M. and WILSON, R., *Let Newton be!* (Clarendon Press, Oxford, 1989), 288 pp., cloth 0 19 853 924 X, £17.50; paper 0 19 853937 1, £8.95.

What is a full-page photograph of Harpo Marx doing in a book about Newton? To find out you will have to get hold of this book, something which I recommend for more substantial reasons as well. But clearly this book is no narrow account of Newton's mathematics and physics, though these topics are covered. The twelve authors between them have as much to say about the more neglected aspects of Newton's output, in fields such as alchemy and history, and much is said about the influence of Newton and Newtonianism on his contemporaries and on later generations.

Mention of Harpo Marx draws attention to one gap: nothing is said about Newton's sense of humour. According to Gjertsen (*The Newton Handbook*, London 1986, pp. 263–264) there is not

https://doi.org/10.1017/S0013091500028996 Published online by Cambridge University Press

much to tell. When Newton once loaned someone a copy of Euclid's Elements, the recipient asked what possible use the study of Euclid could be to anyone, a remark which Newton apparently found very funny.

This joke would be lost on Jon Pepper, who contributes the chapter on Newton's mathematical work in this book. Or at least he might take it the wrong way, since he appears to believe that "Euclidean geometry was never much of an attraction to Newton, nor did it influence his work very much" (p. 63). He also believes (p. 66) that a curve turns quickly when its curvature is small, and (p. 73) that finding derivatives and determining tangents are inverse processes.

Fortunately most of the chapters are written by masters of their subject, and even if your interest in Newton centres primarily on the mathematics, you will find that the chapters cross-fertilize each other in unexpected ways. For example the chapter on alchemy, which had a tradition of looking at itself as a body of secret knowledge, helps us to understand one reason why Newton was so secretive also about his more familiar discoveries. One of the most satisfying chapters, by Casper Hakfoort on Newton's optical work, analyses the ways in which Newton's scientific methods were at variance with those of some of his immediate predecessors and contemporaries, and helps to illuminate that strange and famous phrase "I frame no hypotheses" at the very end of the *Principia*. Even Newton's theological researches and religious beliefs are brought into the picture, as John Brooke in his essay shows how these underlay, in part at least, Newton's concept of absolute space-time, which itself lies at the foundation of the Principia. Indeed Newton himself appears to have believed that, in his dynamical work, he was fundamentally rediscovering facts which were known to "the ancients" and subsequently lost or distorted. It is little wonder that he was so irritated by Hooke's claim to have invented the inverse square law when he himself believed it was known to Pythagoras!

This cross-fertilization should not surprise us, as these essays are all struggling to comprehend just one thing—the life and work of one man. But the fact that they succeed so well is remarkable evidence of how far Newton studies have progressed in the last few decades. As Jan Golinski recalls, it is not so very long ago that Cambridge University declined to accept a large collection of Newton's chemical writings, on the grounds that they were of no scientific interest. And no one can now assent to Voltaire's apologetic assurance that Newton's theological researches were "done to amuse himself after the fatigue of severer studies".

And what does Newton mean to us at the present day? Maureen McNeill raises a number of possibilities, including even (but not endorsing) the suggestion that Newtonian mechanics was a product of a particular stage in the development of capitalism. (No, this is not where Harpo Marx comes in.) Her own answer is that, to us in the late twentieth century, Newton is primarily a symbol of "Englishness", a conclusion that may raise contemptuous eyebrows even north of the border, not to mention abroad.

My conclusion, for the tolerably educated part of the population at least, would be that the image of Newton is essentially the one which Voltaire had in mind, i.e. Newton the rationalist. All the authors who have contributed to this book are engaged, deliberately or not, in the worthy task of showing that such a simple view fails to do Newton justice. While this is a constructive enterprise, it cannot be done entirely without a certain amount of debunking. On the other hand I felt that one or two of the contributors take this too far, using faintly pejorative language whose purpose, I guess, is simply to try to cut Newton down to size. Jan Golinski, in particular, sometimes seems less concerned with the content of Newton's chemical theories than with the way in which Newton "manoeuvred [his] disciples into important positions" in order to advance both their careers and his own "doctrines".

If this is late twentieth-century anti-Newtonianism, it is part of a long and distinguished tradition, as the essay by Geoffrey Cantor shows. He reminds us, for example, of the view of S. T. Coleridge, who believed that "the souls of five hundred Sir Isaac Newtons would go to the making up of a Shakespeare or a Milton".

One of the casualties of the process of debunking is, of course, the story, here confidently dismissed as a myth, of the apple falling in Newton's garden at Woolsthorpe. Ironically enough, the apple appears as a logo at the head of every chapter! And while one author is busy

https://doi.org/10.1017/S0013091500028996 Published online by Cambridge University Press

demolishing this myth it is amusing to find another author propagating a different myth—the story that Newton's sole contribution to a parliamentary debate was to ask for one of the windows to be closed. At least, if this is not a myth, it is a story I have heard told about lack-lustre MPs in our own day.

To be fair, such faults as this book has seem minor in comparison with how greatly it enriches one's appreciation of the real Newton. It digests for the general reader a great range of recent scholarly studies, and is handsomely illustrated. Moreover, as with Newton's output itself, the whole is greater than the sum of its parts.

D. C. HEGGIE

HIGMAN, G. and SCOTT, E., *Existentially closed groups* (London Mathematical Society Monographs (New series) No. 3 Oxford University Press, 1988), 170 pp., 0 19 853543 0, £25.

Let G be a group. A system of equations and inequalities, involving elements of G and a collection of variables, is said to be soluble *in* G if they become true statements on replacing the variables by suitable elements of G. It is said to be soluble *over* G if it is soluble in some group H that contains G as a subgroup. A group G is said to be *existentially closed* if every finite system of equations and inequalities that is soluble over G is actually soluble in G.

This fairly innocuous definition opens up a rich and fascinating branch of group theory, closely related to logic in the form of model theory and recursive functions. The subject was first studied in the early 1950s by B. H. Neumann and W. R. Scott, who looked at the slightly weaker notion of *algebraically closed group* (the same definition as above, but without the inequalities: surprisingly, it turns out that the trivial group is the only algebraically closed group that is not existentially closed). Perhaps the most striking result in the subject is the theorem of Neumann, Simmons and Macintyre that a finitely generated group has soluble word problem if and only if it can be embedded as a subgroup of every existentially closed group. The subject developed further in the 1970s and 1980s through work of Hickin, Macintyre, Ziegler and others, and is still an area of current research activity.

The book under review is a welcome introduction to the subject, based on a course of advanced lectures given by Higman. It is aimed at group-theorists of at least research student level. No prior knowledge of logic is assumed, although it would be an advantage to have some. Certainly a fair amount of mathematical sophistication is required of the reader. Within these constraints, the book succeeds in making accessible a sizeable piece of mathematics, much of it highly non-trivial, that was previously available only in the form of research papers.

The first four chapters are fairly gentle. Various group-theoretic properties of an existentially closed group G are considered. For example, G is simple and infinitely generated, and there are strong restrictions on the centralizers and normalizers of subgroups of G. The existence of any existentially closed group at all is a non-trivial fact, which is not proved until Chapter 3. There follows a chapter on recursion theory. Again the treatment is fairly gentle, and does not assume a prior knowledge of logic.

The pace is stepped up in the second half of the book, which is largely based on work of Ziegler, together with some results of Hickin and Macintyre. Recursion theory, together with a subgroup theorem of Higman, are used to prove a number of results about existentially closed groups, including one half of the Neumann–Simmons–Macintyre result mentioned above. To prove the other half, along with a number of further results, the notion of "games" is introduced. This powerful device is used to show, among other things, that the set of isomorphism types of countable existentially closed groups has the cardinality of the continuum. The book ends with a substantial chapter on the first-order theory of existentially closed groups.

The material presented in this book is probably too specialized to be used for teaching purposes, except possibly for a very advanced graduate course. Its main use will be for specialists in group theory who are looking for a research topic, or merely interested in learning some new mathematics. To anyone in this category I recommend it as an interesting and informative source.

JAMES HOWIE

https://doi.org/10.1017/S0013091500028996 Published online by Cambridge University Press