EVOLUTION OF MAGNETIC CATACLYSMIC BINARIES

D.Q. LAMB and F. MELIA
Department of Astronomy and Astrophysics, Enrico Fermi Institute, University of Chicago, Illinois, U.S.A.

I. INTRODUCTION

More than a third of all cataclysmic variables (CV's) with established binary periods contain strongly magnetic degenerate dwarfs (Ritter 1985). The history of the manner in which the magnetic character of these systems was established suggests that many more CV's—perhaps most—are magnetic. By comparison, only $\approx 2\%$ of isolated degenerate dwarfs are magnetic (Angel, Borra, and Landstreet 1981; Schmidt and Liebert 1986). The magnetic CV's divide naturally into two subclasses: the DQ Herculis stars (recently reviewed by Warner 1983, 1985) and the AM Herculis stars (recently reviewed by Liebert and Stockman 1985). Each subclass currently has about a dozen members.

In magnetic CV's, the magnetic field of the degenerate dwarf funnels the accretion flow near the star, producing an accretion shock at the stellar surface, and pulsed optical and X-ray emission. The known DQ Her systems typically have binary orbital periods $P_b \gtrsim 3$ hours. They show evidence of an accretion disk (Warner 1983, 1985) and the degenerate dwarf is believed to have been spun up to a short rotation period P by the accretion torque (see, *e.g.*, Lamb and Patterson 1983; Hameury, King, and Lasota 1986). The optical light from these systems shows little or no optical polarization (< 1 %; Warner 1983, 1985;

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TABLE 1

PROPERTIES OF MAGNETIC CATACLYSMIC VARIABLES

DQ Her stars	AM Her stars
• $P \ll P_b$	• $P = P_b$
• $P_b \gtrsim 3$ hours	• $P_b \lesssim 3$ hours
• Little or no optical polarization (< 1 %)	• Strong optical polarization ($\gtrsim 10$ %)
• Optical modulated at P, P_b, and P_{side}	• Optical, UV, and X-rays modulated at $P = P_b$
• UV(?) and X-rays modulated at P, P_b	
• Accretion disk present	• No accretion disk
• $\mu_1 \sim 10^{31} - 10^{33}$ G cm^3 ($B_1 \sim 10^5 - 10^7$ G)	• $\mu_1 \sim 10^{33} - 10^{34}$ G cm^3 ($B_1 \approx 2 - 3 \times 10^7$ G)

Penning, Schmidt, and Liebert 1986). The observed modulation of the optical light at P is due partly to emission directly from the degenerate dwarf and partly to reprocessing of the pulsed X-ray emission, while that at the sideband $P_{side} = (P^{-1} - P_b^{-1})^{-1}$ is due to reprocessing of the pulsed X-ray emission from a point at rest in the binary system (see, *e.g.*, Warner 1985). These properties are summarized in Table 1.

In contrast, the known AM Her systems typically have $P_b \lesssim 3$ hours. The degenerate dwarf in them has a magnetic moment μ_1 sufficient to prevent the formation of a disk. Its magnetic moment is also sufficient to couple the degenerate dwarf to the companion star and synchronize its rotation period P with the orbital period P_b, in spite of the accretion torque (see, *e.g.*, the review by Lamb 1985). The optical light from these systems is strongly polarized ($\gtrsim 10$ %; Liebert and Stockman 1985). These properties are summarized in Table 1.

While the above properties adequately characterize the known DQ Her and AM Her systems, theory indicates that systems can exist which have some properties of each. Therefore, we shall henceforth define DQ Her stars to be magnetic CV's with *asynchronously* rotating degenerate dwarfs and AM Her stars to be those with *synchronously* rotating [$\equiv (P_b - P)/P_b \ll 1$] degenerate dwarfs.

The possibility that DQ Her stars evolve into AM Her stars has recently received considerable attention (Chanmugam and Ray 1984; King, Frank, and Ritter 1985; King 1985; Lamb 1985). According to this hypothesis, DQ Her and AM Her stars are intrinsically similar but are observed at different evolutionary epochs. This picture is attractive for several reasons. First, as a close binary evolves, its orbital period P_b decreases (to a minimum period $P_{b,min}$). Thus, the known DQ Her systems with periods $P_b \gtrsim 3$ hours will eventually have periods $P_b \lesssim 3$ hours, like those of the known AM Her stars. Second, the Roche lobe of the degenerate dwarf shrinks as P_b decreases, and the radial extent of the disk therefore also decreases. At the same time, the Alfvén radius $r_A^{(d)}$ for disk accretion increases because the mass transfer rate decreases. Eventually, these two radii cross, and the disk must disappear. Third, magnetic coupling (of almost any kind) between the magnetic degenerate dwarf and the (magnetic) secondary increases rapidly as the binary separation a and the mass transfer rate decreases with decreasing P_b, so that synchronization becomes more likely.

In this talk, we explore this picture using the physics of magnetic cataclysmic binary evolution. We describe the results of recent calculations (Lamb and Melia 1986), in which the transition between the DQ Her and AM Her states is determined by balancing the accretion torque against the MHD synchronization torque (Lamb, Aly, Cook, and Lamb 1983). Previous workers (Chanmugam and Ray 1984; King, Frank, and Ritter 1985) used a heuristic approach and assumed that synchronization occurs when $r_A^{(0)} \approx a$, where $r_A^{(0)}$ is the Alfvén radius for spherical accretion (Ghosh and Lamb 1979) and a is the orbital separation of the binary.

We identify four regimes: (1) When $\mu_1 \lesssim 10^{31}$ G cm^3, the magnetic field of the degenerate dwarf is unable to funnel the accretion flow; these systems are not DQ Her stars and may show little or no evidence of a magnetic field. (2) Systems with 10^{31} G cm$^3 \lesssim \mu_1 \lesssim 10^{33}$ G cm^3 are always DQ Her stars. (3) Systems with 10^{33} G cm$^3 \lesssim \mu_1 \lesssim 10^{35}$ G cm^3 are DQ Her stars which evolve into AM Her stars, assuming μ_1 is constant throughout their evolution (an assumption which may not be valid). These three regimes agree qualitatively with those of King, Frank, and Ritter (1985). (4) Systems with 10^{35} G cm$^3 \lesssim \mu_1$ are always AM Her stars.

We confirm that AM Her stars do not have accretion disks, while DQ Her stars may or may not have them, depending on the value of μ_1, as pointed out by King (1985) and

Hameury, King, and Lasota (1986). The ranges of μ_1 for the known DQ Her and AM Her stars allowed by observation are very large, due partly to uncertainties in the theory of disk and stream accretion (Lamb and Patterson 1983; Hameury, King, and Lasota 1986) and partly to the dependence of the ranges on the mass M_1 of the degenerate dwarf. Nevertheless, it is likely that the μ_1's of the known DQ Her stars (including those with longer rotation periods) are significantly less than those of the known AM Her stars, unless all known DQ Her and AM Her stars have $M_1 \gtrsim 1\ M_\odot$. This differs from the conclusion reached by King (1985) and King, Frank, and Ritter (1985). The criteria for synchronization and for the presence or absence of a disk in DQ Her systems are also uncertain, due to uncertainties in the theory of the MHD synchronization torque and to the dependence of the criteria on M_1. Nevertheless, it appears that the known DQ Her stars will not evolve into AM Her stars, assuming that μ_1 is constant throughout their evolution. This differs from the conclusion reached by Chamugam and Ray (1984), King (1985), and King, Frank, and Ritter (1985).

The fact that the strong magnetic field of the degenerate dwarf dramatically alters the optical, UV, and X-ray appearance of magnetic CV's has come to be appreciated (see the reviews by King 1983, 1985; and Lamb 1983, 1985). That it can also alter the evolution of the binary itself is only now becoming recognized. Spin-up and spin-down of the magnetic degenerate dwarf temporarily speeds up and slows down the binary evolution of DQ Her stars (Ritter 1985; Lamb and Melia 1986). Co-operative magnetic braking may speed up the binary evolution of AM Her stars when the degenerate dwarf has a particularly strong ($B_1 \gtrsim 5 \times 10^7$ G) magnetic field (Schmidt, Stockman, and Grandi 1986). Here we point out that synchronization of the magnetic degenerate dwarf injects angular momentum into the binary, driving the system apart and producing a *synchronization-induced period gap* (Lamb and Melia 1986). This process yields an entirely new way of producing ultra-short period binaries. The secondaries in these binaries are hydrogen-rich rather than hydrogen-depleted or pure helium (*cf.* Nelson, Rappaport, and Joss 1985; Lamb *et al.* 1986).

II. PHYSICS OF MAGNETIC CV EVOLUTION

Figure 1 shows the distribution of binary periods P_b for the magnetic CV's and for all CV's. Several features are evident. First, the two distributions are similar. Second, both have a cutoff at a minimum period $P_{b,min} \approx 80$ minutes. Third, both show the well-known "period gap" between 2 and 3 hours. Fourth, most of the DQ Her stars have $P_b \gtrsim 3$ hours, while most of the AM Her stars have $P_b \lesssim 3$ hours, as already noted.

Figure 2 shows the observed mass loss rate $|\dot{M}_2|$ from the secondary as a function of orbital period for the magnetic CV's (Patterson 1984). The rates shown assume that $|\dot{M}_2|$ is approximately the same as the mass transfer rate into the disk in the case of DQ Her stars and that $|\dot{M}_2|$ is approximately the same as the mass accretion rate $\dot{M}_1$ onto the degenerate dwarf in the case of AM Her stars, which have no disk. Here and in subsequent figures, we use a double-valued axis for the orbital period, so that systems continue to evolve from right to left after minimum period (denoted by the vertical dotted line at $P_b = 1.2$ hours). Data for the DQ Her stars are shown as filled squares and data for the AM Her stars as filled circles; no error bars are shown, but the uncertainties in $\dot{M}_2$ are a factor ≈ 3 (Patterson 1984). From this figure, it is evident that $|\dot{M}_2|$ is smaller and shows less scatter at shorter orbital periods. The distribution of observed $|\dot{M}_2|$'s for the set of all CV's is similar.

Substantial progress has been made in recent years in understanding the qualitative features of close binary evolution evident in Figures 1 and 2. Stable mass transfer is possible only if the mass ratio $q \equiv M_1/M_2 \lesssim 1$, *i.e.* the less massive star transfers mass to the more massive one. This implies that the secondary stars in CV's have masses $M_2 \lesssim 1.2\ M_\odot$. For binary periods $P_b \gtrsim 10$ hours, such low-mass secondaries must be significantly evolved in order to fill their Roche lobe. It is therefore believed that such binaries are driven by nuclear evolution of the secondary (Whyte and Eggleton 1980; Webbink, Rappaport, and Savonije 1983; Taam 1983b). Assuming conservative mass transfer ($\dot{M}_1 = -\dot{M}_2$), $\dot{P}_b > 0$ in these systems since $q > 1$. AE Aqr, with $P_b = 9.9$ hours, and GK Per, with $P_b = 46$ hours, are thought to be evolving due to this mechanism (*cf.* Whyte and Eggleton 1980, Patterson 1984).

a) Angular Momentum Loss

Binaries with orbital periods $P_b \lesssim 10$ hours contain secondary stars whose nuclear evolution proceeds too slowly to drive mass transfer at the rates observed. This is consistent with observational evidence that these secondary stars are not significantly evolved (Patterson 1984). These binaries are therefore believed to evolve due to the loss of orbital angular momentum. The rate of change of the angular momentum of the system can be written as

$$\dot{J} = \dot{J}_1 + \dot{J}_2 + \dot{J}_b \, , \tag{1}$$

where $\dot{J}_1$ and $\dot{J}_2$ are the rate of change of the spin angular momenta of the degenerate dwarf and the secondary, respectively, and $\dot{J}_b$ is the rate of change of the orbital angular momentum of the binary.

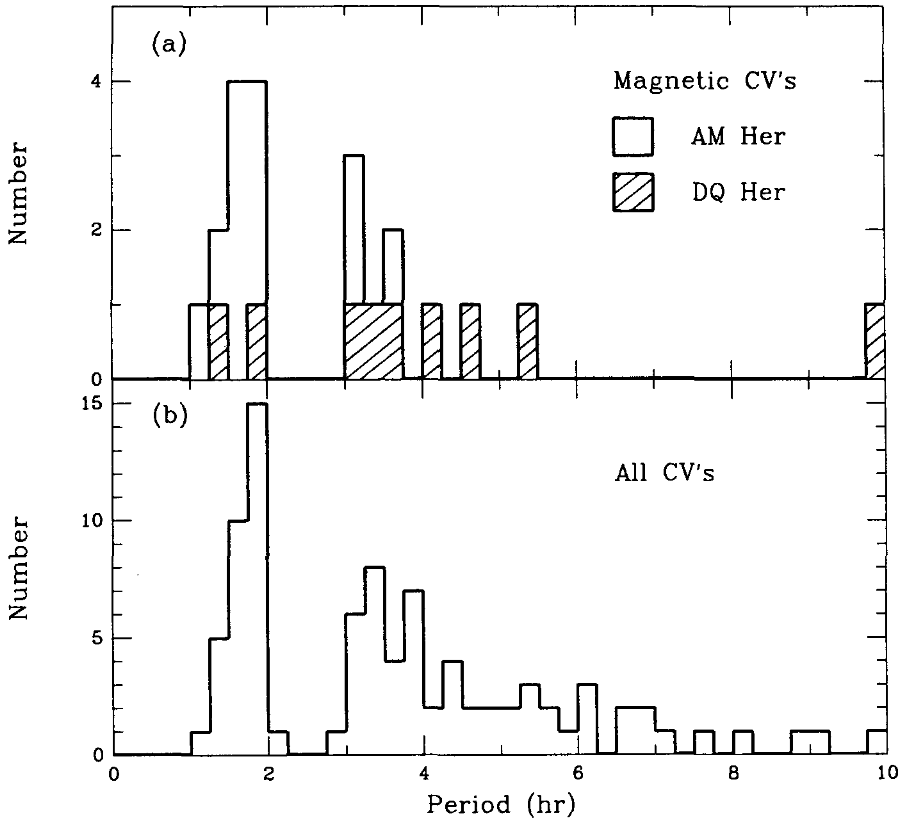

Fig. 1.–Orbital period distributions of CV's. a) Magnetic CV's. The DQ Her stars are shown as shaded boxes. b) All CV's. Note the "gap" in both distributions between 2 and 3 hours, and the fact that the DQ Her stars have $P_b \gtrsim 3$ hours, while the AM Her stars have $P_b \lesssim 3$ hours.

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Gravitational radiation (GR) drives the evolution of the binary by extracting orbital angular momentum, $\dot{J}_{GR} = \dot{J}_b$ (Kraft, Mathews, and Greenstein 1962; Paczyński 1967; Faulkner 1971). This mechanism gives a minimum orbital period of about 1.2 hours, corresponding to the transition of the secondary from the main sequence to the degenerate sequence (Paczyński and Sienkowicz 1981; Rappaport, Joss, and Webbink 1982), and therefore successfully accounts for the observed cutoff seen at $P_b \approx 1.3$ hours (see Figure 1). However, evolution due to GR gives mass transfer rates as large as those observed only for $P_b \lesssim 3$ hours.

Consequently, other mechanisms which could drive binary evolution effectively at intermediate binary periods (3 hours $\lesssim P_b \lesssim$ 10 hours) have been investigated. One possibility that has recently received a great deal of attention is magnetic braking (Verbunt and Zwaan 1981; Taam 1983a; Spruit and Ritter 1983; Rappaport, Verbunt, and Joss 1983; Patterson 1984; Verbunt 1984). According to this idea, the secondary star has a strong magnetic field which couples it to its stellar wind and extracts spin angular momentum from the star, $\dot{J}_{MB} = \dot{J}_2$. Since the secondary is largely convective, tidal coupling between the secondary and the binary system is very effective ($P_{tidal} \sim 100$ years; Zahn 1977). This keeps the secondary synchronized, so that the angular momentum is lost from the binary system itself ($\dot{J}_{MB} = \dot{J}_b$). For large but conceivable values of the companion's magnetic field ($B_2 \sim 300 - 1000$ G) and stellar wind ($\dot{M}_{wind} \sim 10^{-10} M_\odot/yr^{-1}$), magnetic braking can produce mass transfer rates as large as those observed for $P_b \lesssim 10$ hours.

b) The Period Gap

The gap in the orbital period distribution shown in Figure 1 may be the result of very low mass transfer rates (the systems are too faint to be seen), a brief episode of very high mass loss from the secondary at $P_b \approx 3$ hours (the system loses mass too quickly to be seen), or the fact that the systems do not evolve through the gap at all (they avoid it somehow).

Spruit and Ritter (1983) have shown that a gap of the first kind can be produced by an angular momentum loss mechanism $\dot{J}_{add}$ acting in addition to GR, if it is sufficiently strong and ceases suddenly at $P_b \approx 3$ hours. In this picture, a gap occurs because the radius of the secondary star becomes larger than its main sequence value when the star is driven out of thermal equilibrium, *i.e.* $\tau_{EVOL} < \tau_{KH}$, where τ_{KH} is the Kelvin-Helmholz thermal time scale, and τ_{EVOL} ($\approx \tau_{add}$) is the evolutionary time scale. When the additional angular momentum loss mechanism ceases ($\dot{J}_{add} \to 0$), the secondary star shrinks to its main sequence radius and comes out of contact, ending mass transfer. The system continues to evolve to shorter binary periods and smaller separations, but now on the much longer time scale τ_{GR} ($>> \tau_{add}$). The secondary comes back into contact when the radius of its Roche lobe equals the main sequence radius of the star, and mass transfer is re-established, but at the much lower rate given by evolution due to GR.

Robinson *et al.* (1981) noted that the secondary becomes fully convective at $P_b \approx 3$ hours. Subsequently, Spruit and Ritter (1983), and Rappaport, Verbunt, and Joss (1983) suggested that magnetic braking is disrupted when the secondary becomes fully convective, possibly because the magnetic field of the secondary disappears, and that this produces the period gap in the way just described (with $\dot{J}_{add} = \dot{J}_{MB}$). The latter authors parametrize the magnetic braking torque by

$$N_{MB} = -3.8 \times 10^{-30} f M_2 R_\odot^4 \left(\frac{R_2}{R_\odot}\right)^\eta \Omega_b^3 \text{ dyne cm} , \tag{2}$$

where M_2 and R_2 are the mass and radius of the companion star, respectively, $\Omega_b = 2\pi/P_b$ is the binary orbital frequency, and f is a constant of order unity.

Figure 2 compares the $\dot{M}_2(P_b)$ relation given by disrupted magnetic braking models and the observed $\dot{M}_2$'s for magnetic CV's. The curves labeled DMB2, DMB2.5, and DMB4 correspond to Equation (2) with indices $\eta = 2$, 2.5, and 4, and $f = 1$. This figure shows that disrupted magnetic braking provides an elegant explanation of the qualitative features of CV evolution.

However, more detailed comparison shows model DMB4 gives a period gap that is much too narrow. Models DMB2 and DMB2.5 give period gaps that are about right, but mass loss rates that are much too large. Furthermore, these curves are lower bounds to the mass transfer rate given by magnetic braking, since they assume $M_1 = 1.2\ M_\odot$ (lower mass degenerate dwarfs give rates that are as much as a factor of ten larger). The possibility that non-conservative mass transfer could explain the discrepancy has been noted by Rappaport, Verbunt, and Joss (1983); however, this would require mass loss from the system at $\gtrsim 10$ times the rate of accretion onto the degenerate dwarf, a situation we regard as unlikely.

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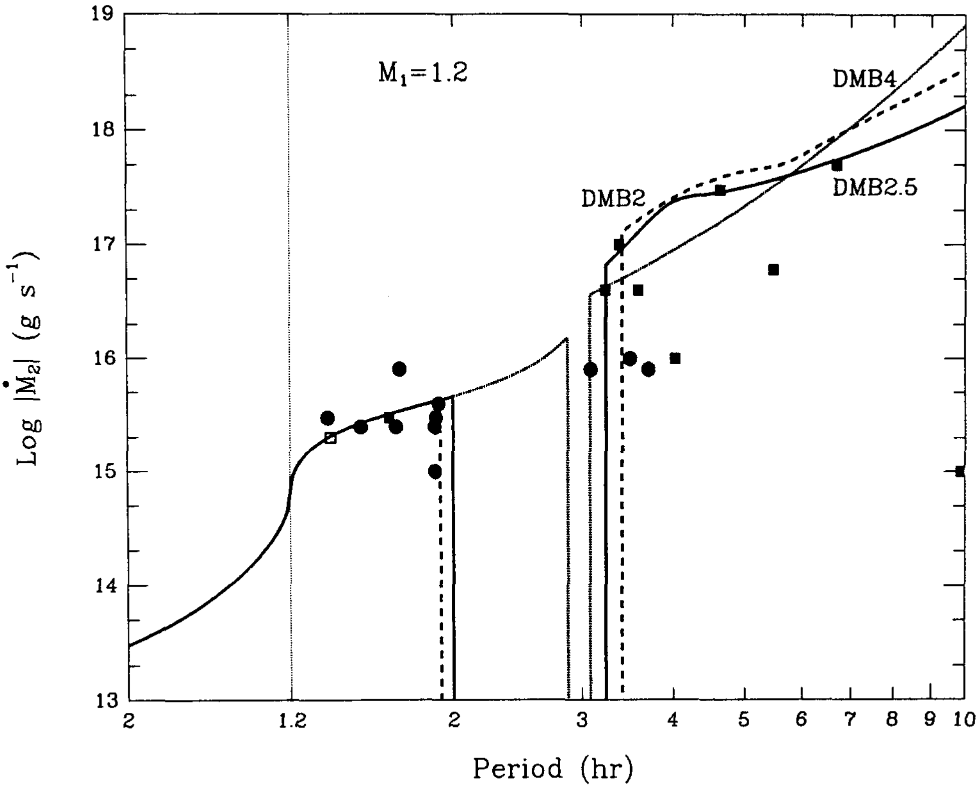

Fig. 2.–Mass transfer rate as a function of orbital period for several disrupted magnetic braking models. DMB2 and DMB2.5 give period gaps similar to that observed, but mass transfer rates longward of the gap which are about a factor of ten larger than those inferred from observation. DMB4 gives a period gap that is too short, but a mass transfer rate that is closer to that inferred from observation. Shortward of the gap, the evolution is driven only by GR and is therefore the same in all three models. The resulting mass transfer rate is similar to that observed.

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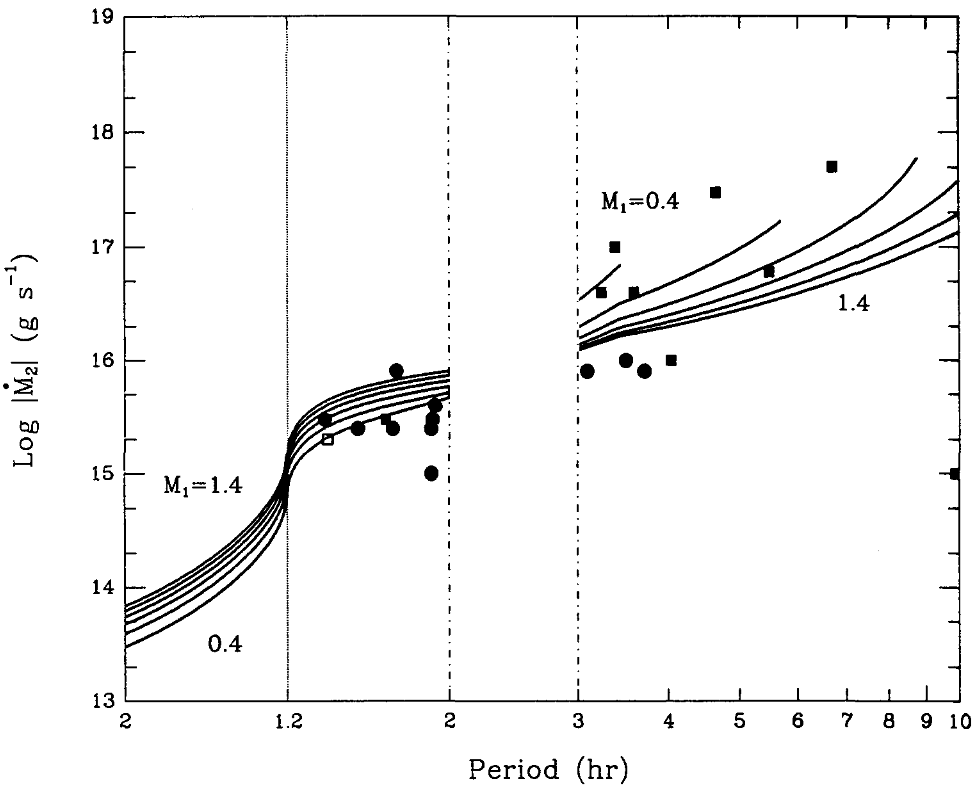

Fig. 3.–Mass transfer rate as a function of orbital period, assuming a rate longward of the period gap equal to that given by the magnetic braking law (2) with $f = 0.15$ and $\eta = 4$, and a rate shortward of the period gap equal to that given by gravitational radiation.

There is thus an inherent difficulty in the disrupted magnetic braking model for the period gap: The choices of f and η that give period gaps of about the right size give $\dot{M}_2$'s that are much too large, while those that give about the right $\dot{M}_2$'s give period gaps that are much too narrow. Therefore, it would seem that other explanations of the period gap should be pursued.

The discrepancy between the mass transfer rates longward of the period gap predicted by the disrupted magnetic braking model and those observed poses a dire problem when

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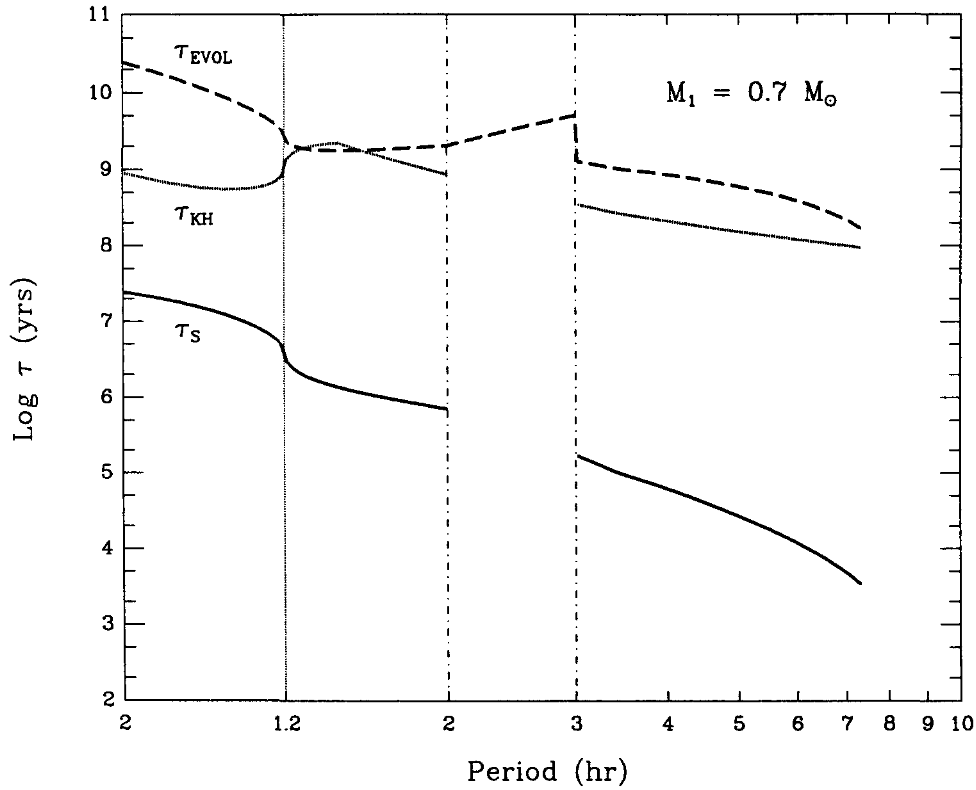

Fig. 4.–The evolutionary time scale τ_{EVOL} of the binary, the thermal time scale τ_{KH} of the secondary star, and the synchronization (or spin-up) time scale τ_S *at synchronization* (they are equal then) as functions of orbital period P_b, assuming $M_1 = 0.7\ M_\odot$ and $\dot{M}_2$ as shown in Figure 3.

investigating the subtle questions which interest us here, i.e whether the magnetic field of the degenerate dwarf channels the accretion flow, whether an accretion disk is present or absent, and whether or not synchronization occurs. We handle the problem by adopting the following phenomenological model of CV evolution. Longward of the period gap, we take the braking law (3) with $f = 0.15$ and $\eta = 4$, which fits the observed mass transfer rates reasonably well (see Figure 3). At $P_b = 3$ hours, we abruptly reduce the mass of the secondary to $0.18M_\odot$, a mass which brings the secondary back into contact at $P_b = 2$ hours. Shortward of $P_b = 3$ hours, the evolution is driven by GR alone. This model gives a period gap of the second kind defined earlier; we shall henceforth refer to it as DMB4*.

Figure 4 shows the evolutionary time scale τ_{EVOL} and the thermal time scale τ_{KH} for model DMB4* as functions of the orbital period P_b. The evolutionary time scale increases at $P_b = 3$ hours, since the magnetic braking torque ceases then and $\tau_{GR} > \tau_{add} = \tau_{MB}$. The jump is not as large as that in models DMB2, DMB2.5 and DMB4, which have $\dot{J}_{MB} \gg \dot{J}_{GR}$. For model DMB4*, $\tau_{KH} \lesssim \tau_{EVOL}$, except between $P_b \approx 1.7$ hours and $P_{b,min}$ (≈ 1.2 hours). The time scale τ_{KH} decreases after $P_{b,min}$ because the secondary becomes increasingly degenerate.

c) Existence of an Accretion Disk

When mass transfer first begins, the trajectory of the stream from the secondary star passes by the degenerate dwarf at a radius ϖ_{min}, swings around, and intersects itself at a somewhat larger radius (Lubow and Shu 1975). The stream then circularizes and forms a ring. The radius of the ring is

$$\varpi_{max} \approx X^2 \, r_{L1} \ , \tag{3}$$

where r_{L1} is the radius of the inner Lagrangian point, and $X \approx 0.3 - 0.4$ is a factor that takes into account the ejection angle of the stream at r_{L1} (Lubow and Shu 1975). Eventually the ring spreads, forming a disk. Figure 5 shows R_1, ϖ_{min}, ϖ_{max}, and r_{L1} (measured from the center of the degenerate dwarf) relative to the binary separation a as a function of P_b, assuming $M_1 = 0.7M_\odot$.

Whether or not a disk *forms* in a magnetic CV depends on the size of the Alfvén radius $r_A^{(d)}$ for disk accretion relative to ϖ_{min}. If $r_A^{(d)} < \varpi_{min}$, the answer is yes; if $r_A^{(d)} > \varpi_{min}$, what happens depends on a more detailed discussion of the physics involved (*cf.* Hameury, King, and Lasota 1986). Whether or not a disk *persists* in a magnetic CV depends on the size of $r_A^{(d)}$ relative to ϖ_{max}. If $r_A^{(d)} < \varpi_{max}$, the answer is yes; if $r_A^{(d)} > \varpi_{max}$, the answer is no. Thus the presence or absence of a disk depends not only on the present physical parameters of the binary ($P_b, \dot{M}_1, \mu_1$, etc), but also on its history.

Consider what happens if most CV's come into contact at long orbital periods and

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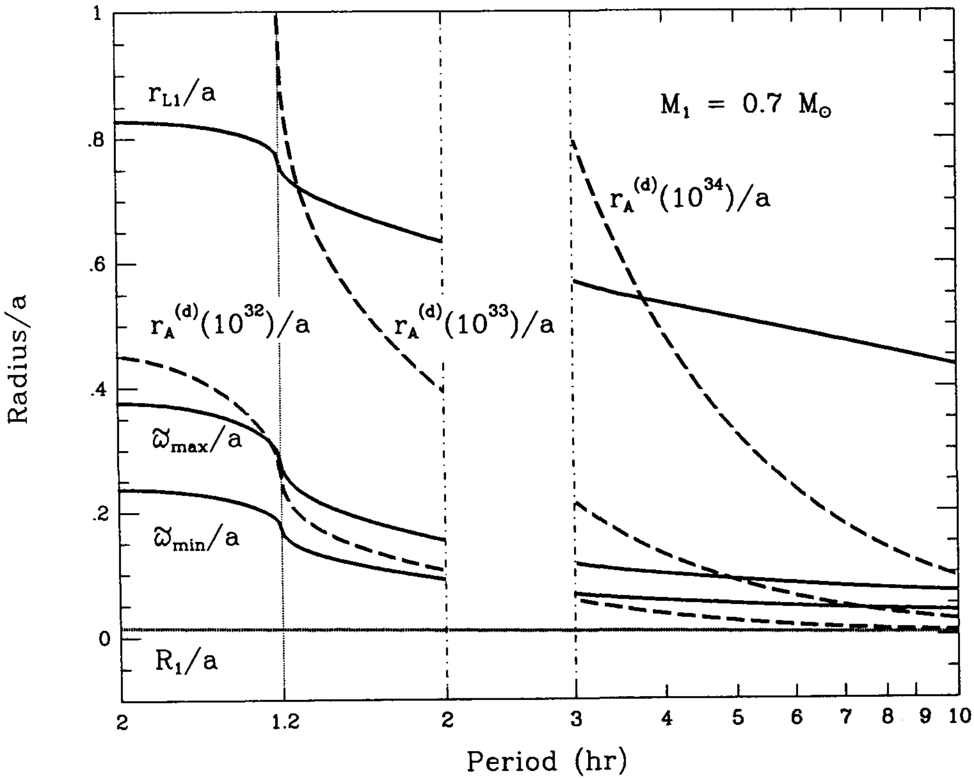

Fig. 5.–Various radii relative to the binary orbital separation a as functions of P_b, assuming $M_1 = 0.7\ M_\odot$ and $\dot{M}_2$ as given in Figure 3. Shown are the radius R_1 of the degenerate dwarf, the radius ϖ_{min} of minimum approach of the stream from the secondary star when mass transfer first begins, the circularization radius ϖ_{max}, the Alfvén radius $r_A^{(d)}$ for disk accretion, and the radius r_{L1} of the inner Lagrangian point (measured from the center of the degenerate dwarf).

$\mu_1 \lesssim 10^{33}$ G cm^3. The Roche lobe of the primary is large and typically $r_A^{(d)} < \varpi_{min}$ (see Figure 5). A disk therefore forms. As long as mass transfer continues, the radius ϖ_{min} is irrelevant and the disk persists until $\varpi_{max} < r_A^{(d)}$. [This differs from King (1985) and Hameury, King, and Lasota (1986), who conclude that disks disappear when $r_A^{(d)} > \varpi_{min}$.] Thus we expect disks to exist longward of the period gap in most systems with $\mu_1 \lesssim 10^{33}$ G cm^3.

Assuming that mass transfer ceases when a system enters the period gap, any disk present quickly disappears. When the system leaves the gap and mass transfer resumes, a disk certainly forms if $r_A^{(d)} < \varpi_{min}$ and does not form if $\varpi_{max} < r_A^{(d)}$. When $r_A^{(d)}$ lies in the narrow range $\varpi_{min} < r_A^{(d)} < \varpi_{max}$, the answer is not clear cut; what happens depends on a more detailed consideration of the physics involved (see, e.g., Hameury, King, and Lasota 1986). Such a discussion lies beyond the scope of the present talk. However, we make the following remarks. The Alfvén radius $r_A^{(s)}$ for accretion from a stream is smaller than that for accretion from a disk because the velocity of the matter is similar in both cases ($\approx$ free fall) but the density in the stream is higher. In fact, an estimate of the minimum radius r_{min} at which the accretion stream is disrupted by the magnetic field of the degenerate dwarf gives $r_{min} < \varpi_{min}$ [*cf.* Lamb 1985; Equation (20)]. The stream will be disrupted by the time it reaches this radius, but its trajectory may not be affected if the stream is Rayleigh-Taylor unstable, as seems likely. Moreover, when a system comes back into contact at $P_b \simeq 3$ hours, a brief stage of very high mass transfer ensues because the mass loss time scale for the envelope of the secondary is shorter than the thermal time scale τ_{KH} for the star as a whole (Webbink 1976, 1985). Thus, for a brief period $r_A^{(s)}$ is much smaller than its subsequent equilibrium value. We therefore expect that the stream will intersect itself and establish a disk in the case when the equilibrium value of $r_A^{(d)}$ lies in the narrow range (factor of ≈ 1.5) between ϖ_{min} and ϖ_{max}. Motivated by these considerations, we assume a disk always forms unless $\varpi_{max} < r_A^{(d)}$.

d) Accretion Torque

The accretion torque arises from the coupling of accreting matter to the magnetic field of the degenerate dwarf. It can be written as

$$N_{acc} \approx n(\omega_s)\,\dot{M}_1\,l_{acc}\;, \tag{4}$$

where $\dot{M}_1$ is the mass accretion rate onto the degenerate dwarf and l_{acc} is the angular momentum per unit mass of the accreting matter. The function $n(\omega_s)$ is a generalization of the dimensionless torque for disk accretion defined by Ghosh and Lamb (1979) and $\omega_s \equiv \Omega/\Omega_K(r_A^{(d)})$ is the fastness parameter, where $\Omega \equiv 2\pi/P$ is the angular frequency of rotation of the degnerate dwarf, and $\Omega_K(r_A^{(d)})$ is the Keplerian angular frequency at the inner edge of the accretion disk. When a disk is present, we use the $n(\omega_s)$ given by Ghosh and Lamb [1979: Equation (10)]; when a disk is absent, we use $n(\omega_s) = \Theta(1-\omega_s)$, where Θ is the Heaviside function and now $\omega_s = \Omega/\Omega_K(r_A^{(s)})$.

In magnetic CV's, the value of l_{acc} varies as the size of the Alfvén radius for disk accretion changes relative to the sizes of R_1 and ϖ_{max}, where R_1 is the stellar radius of the degenerate dwarf. When $r_A^{(d)} < R_1$,

$$l_{acc} \approx (GM_1R_1)^{1/2} . \tag{5}$$

When $R_1 < r_A^{(d)} < \varpi_{max}$,

$$l_{acc} \approx (GM_1 r_A^{(d)})^{1/2}. \tag{6}$$

In the absence of a disk ($\varpi_{max} < r_A^{(d)}$),

$$l_{acc} \approx (GM_1\varpi_{max})^{1/2} , \tag{7}$$

independent of the radius at which matter couples onto the magnetic field of the degenerate dwarf, since the angular momentum per unit mass of the accreting matter is nearly the same as what it had when it left the secondary at r_{L1} (Coriolis forces modify it, but the change is not large for the component of angular momentum perpendicular to the orbital plane—see Campbell 1986).

The accretion torque is quite insensitive to the presence or absence of a disk:

$$\frac{N_{acc}(r_{L1})}{N_{acc}(R_1)} \lesssim \left(\frac{\varpi_{max}}{R_1}\right)^{1/2} = \left(\frac{10\times 10^9\text{cm}}{0.7\times 10^9\text{cm}}\right)^{1/2} \approx 4 \, . \tag{8}$$

Thus, N_{acc} lies in a narrow band whose width is a factor of ≈ 4 at long orbital periods and a factor of $\lesssim 1.5$ near period minimum. This is illustrated in Figure 6, which shows the accretion torque as a function of binary orbital period for a 0.7 $M_\odot$ primary, assuming μ_1 in the range $10^{31} - 10^{34}$ G cm^3.

e) Synchronization Torque

Several synchronization torques based on magnetic coupling of the degenerate dwarf to the secondary have been proposed (Joss, Katz, and Rappaport 1979; Campbell 1983; Lamb, Aly, Cook, and Lamb 1983; for a critical review, see Lamb 1985). Here we adopt the MHD synchronization torque (Lamb, Aly, Cook, and Lamb 1983), which is given by

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$$N_{MHD} \approx \begin{cases} \alpha\gamma a R_2^2 \left(\frac{\mu_1}{a^3}\right)^2 & (\mu_2 = 0) \\ \gamma\frac{\mu_1\mu_2}{a^3} & (\mu_2 > \mu_1 > 0) \end{cases}, \tag{9}$$

where the quantity α is the fractional area of the secondary threaded by the magnetic field of the degenerate dwarf (here assumed equal to the cross-sectional area of the convective region of the secondary), and γ is the pitch angle of the degenerate dwarf's dipole magnetic field at the secondary. In the MHD torque picture, synchronization is achieved by the coupling between the degenerate dwarf and the secondary star due to the field lines that thread the secondary. These field lines are not the same as those along which accretion occurs; thus the condition for synchronization is not related to the heuristic condition $r_A^{(0)} = a$.

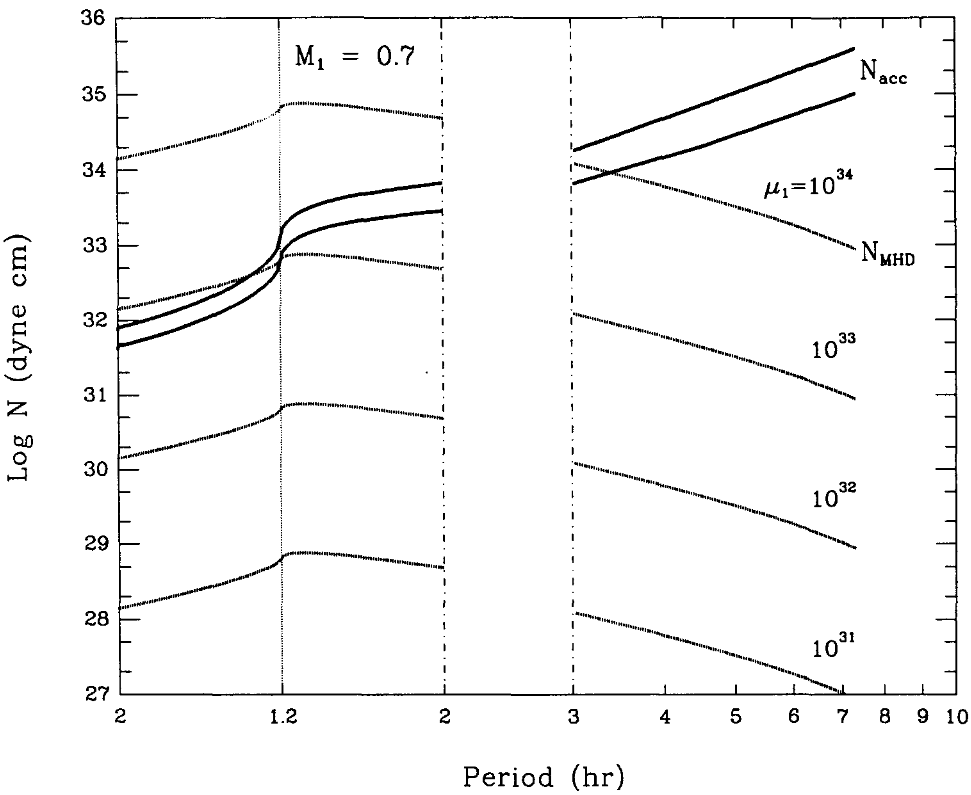

Fig. 6.–Accretion torque N_{acc} and MHD synchronization torque N_{MHD} as functions of P_b, assuming $M_1 = 0.7\ M_\odot$, $\mu_1 = 10^{31} - 10^{34}$ G cm^3, and $\dot{M}_2$ as shown in Figure 3.

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Synchronization rather requires that two conditions be met (Lamb 1985). First, the synchronization torque must exceed the accretion torque if it is to spin down the degenerate dwarf to synchronous rotation; i.e.

$$N_{MHD} > N_{acc} \ . \tag{10}$$

Equation (10) tells us that when synchronization occurs, the synchronization and spin-up times scales are equal $[\tau_{syn} = (I_1\Omega_1)/N_{MHD} = (I_1\Omega_1)/N_{acc} = \tau_{spin}]$. Second, the synchronization torque must be able to bring about synchronous rotation faster than the binary is evolving; *i.e.*

$$\tau_S < \tau_{EVOL} \ , \tag{11}$$

where we have set $\tau_S \equiv \tau_{syn} = \tau_{spin}$. Otherwise, the rotation period of the degenerate dwarf will not be able to "catch up" to that of the binary. Figure 4 shows the time scale τ_S at the epoch of synchronization. We see that condition (11) is always well-satisfied, except in the period gap. In the gap, Eq.(11) gives a lower bound on the μ_1 for which the degenerate dwarf synchronizes.

III. RESULTS

We have carried out calculations for a variety of cases. Here we describe the results of calculations which assume (1) disrupted magnetic braking model DMB4*, as defined in §II, (2) conservative mass transfer between novae outbursts (Taam, Flannery, and Faulkner 1980), (3) secondary magnetic moment $\mu_2 = 0$, (4) degenerate dwarf rotation period $P \approx P_{eq}$, and (5) aligned rotation of the degenerate dwarf. Occasionally, we shall refer to the results of our other calculations for comparison.

a) Criteria for Funneling of the Accretion Flow

The magnetic field of the degenerate dwarf funnels the flow of accreting matter near the stellar surface and produces pulsed optical, UV, and X-ray emission if $r_A^{(d)} \gtrsim 3R_1$. We

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find that this occurs for $\mu_1 \gtrsim 10^{31}$ G cm^3. Systems with smaller values of μ_1 are not DQ Her stars, and may exhibit little or no observational evidence of magnetic fields. These results agree qualitatively with those of King, Frank, and Ritter (1985).

b) Criteria for Existence of an Accretion Disk

Figure 7 shows the values of the magnetic moment μ_1 above which an accretion disk is absent as a function of orbital period P_b. Curves are shown for degenerate dwarf masses $M_1 = 0.4 - 1.4\ M_\odot$. We terminate the curves at long orbital periods P_b when the condition for stable mass transfer is violated; the solid curves correspond to conservative [*i.e.*, $\beta = 1$ in the notation of Rappaport, Joss, and Webbink (1982)] mass transfer, while the dashed extensions correspond to fully non-conservative ($\beta = 0$) mass transfer. The former are relevant to observation, since we assume that the mass transfer is conservative between nova outbursts. Note that *the stability condition constrains the masses of degenerate dwarfs to be high in DQ Her systems with long orbital periods.* The disk criteria curves slope downward toward shorter orbital periods and through minimum period because the decreasing mass transfer rate $|\dot{M}_2|$ (see Figure 3) leads to an increasing Alfvén radius $r_A^{(d)}$ (see Figure 5). Whether a disk is present or absent during the period gap depends on whether the mass transfer rate is very high or very low then.

Figure 8 shows the μ_1 criteria for the existence of an accretion disk *divided by* $|\dot{M}_2|^{1/2}$. The solid and dashed portions of the curves have the same meaning as in Figure 7. Although the required μ_1-values cannot be read directly from this plot, it has the advantage that the curves are universal, in the sense that they are *independent* of the mass transfer rate. They are therefore independent of the evolutionary scenario and of the variations in $\dot{M}_2$ at a given P_b observed among different systems (see Figures 2 and 3).

Uncertainties in the theory of disk accretion lead to an uncertainty of a factor of 2 - 3 in the μ_1 criteria. Figures 7 and 8 show that the P_b at which a disk disappears is very sensitive to the value of μ_1 and M_1; the uncertainty in the μ_1 criteria and in the mass of the degenerate dwarf therefore leads to a much larger uncertainty in P_b.

c) Criteria for Synchronization

Figure 9 shows the magnetic moment μ_1 required for synchronization as a function

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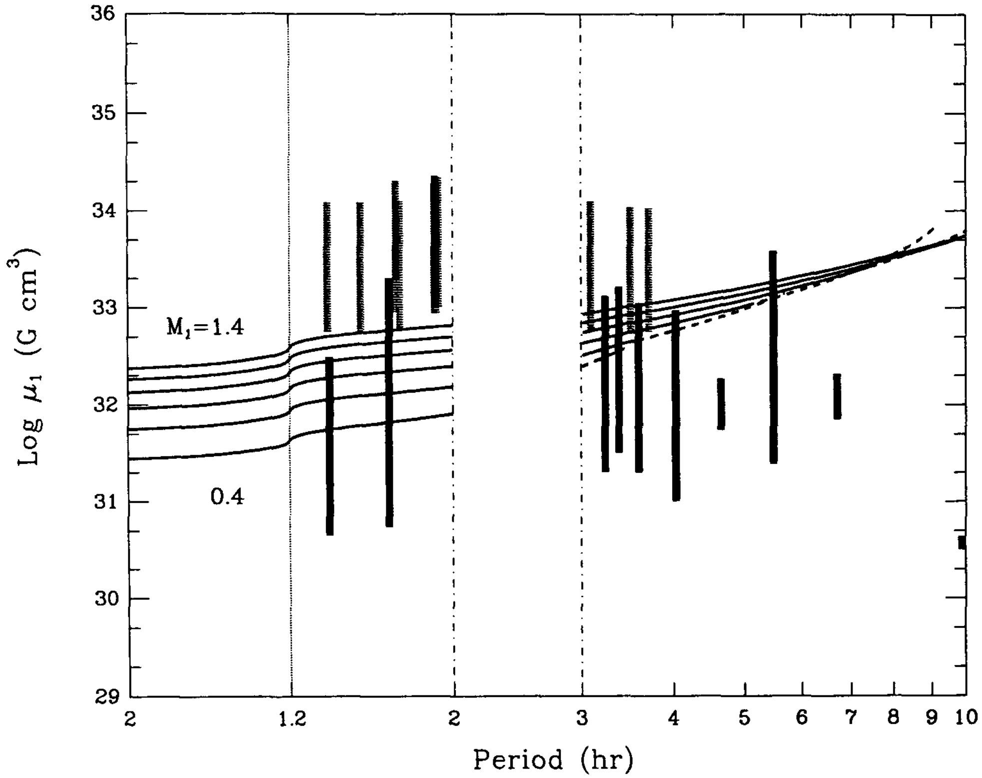

Fig. 7.–Maximum magnetic moment μ_1 for which a disk can exist as a function of P_b, assuming $M_1 = 0.4 - 1.4\ M_\odot$. The ranges of μ_1 for the DQ Her stars inferred from observation are shown as solid vertical bars while those for the AM Her stars are shown as shaded vertical bars.

of binary period P_b, while Figure 10 shows the μ_1 criteria for synchronization *divided by* $|\dot{M}_2|^{1/2}$. Curves are shown for degenerate dwarf masses $M_1 = 0.4 - 1.4\ M_\odot$, assuming $\mu_2 = 0$. When $\mu_2 \neq 0$, the μ_1 criteria become smaller longward of the period gap but are little changed shortward of the gap. The solid and dashed portions of the curves have the same meaning as in Figures 7 and 8. The curves again slope downward toward shorter orbital periods and through $P_{b,min}$ because of the decreasing mass transfer rate $|\dot{M}_2|$. The sharply lower value of $\dot{M}_2$ shortward of the period gap, which arises because the evolution is due only to GR, enhances the probability that systems undergo the transition from DQ

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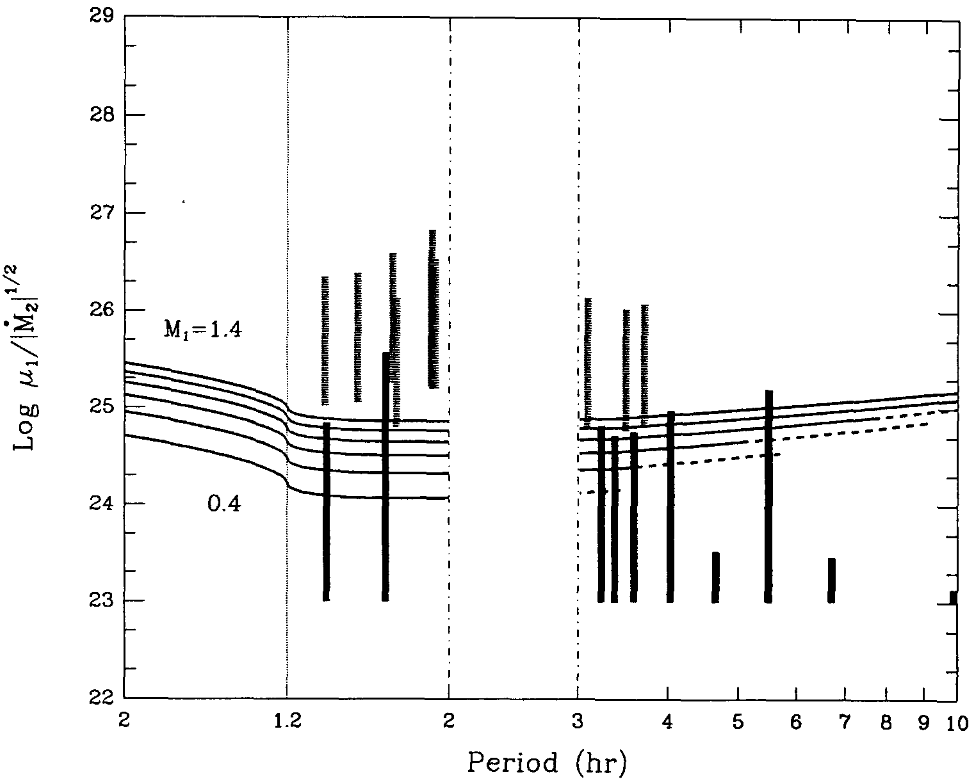

Fig. 8.–Maximum magnetic moment μ_1 for which a disk can exist *divided* by $|\dot{M}_2|^{1/2}$, assuming $M_1 = 0.4 - 1.4\ M_\odot$. The curves are universal in the sense that they are independent of $\dot{M}_2$. The notation for the ranges of μ_1 inferred from observation is the same as in Fig. 7.

Her star to AM Her star at $P_b \approx 2$ hours (Chanmugam and Ray 1984; King, Frank, and Ritter 1985; Lamb 1985).

Comparison of Figures 7 and 9, or 8 and 10, shows that no accretion disk is present when synchronization occurs. Since the accretion torque is then given by Equation (7), the sychronization criteria are independent of uncertainties in the theory of accretion from disks and streams. However, *uncertainties in the theory of the MHD synchronization* torque again lead to an uncertainty of a factor of 2 - 3 in the μ_1 criteria. Figures 9 and

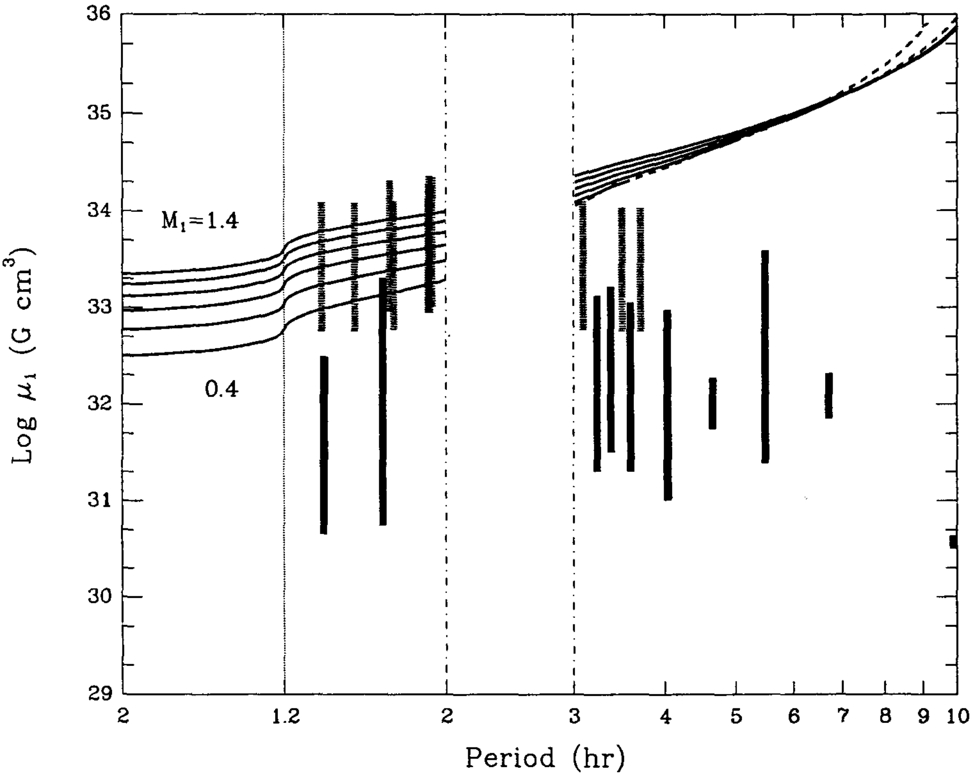

Fig. 9.–Minimum magnetic moment μ_1 required for synchronization as a function of P_b, assuming $M_1 = 0.4 - 1.4\ M_\odot$. The notation for the ranges of μ_1 inferred from observation is the same as in Fig. 7.

10 show that the P_b at which the transition from DQ Her star to AM Her star occurs is also very sensitive to μ_1 and M_1. The uncertainty in the μ_1 criteria and in the mass of the degenerate dwarf therefore again leads to a much larger uncertainty in P_b.

Our results show that the μ_1 criterion given by the heuristic condition $r_A^{(0)} = a$ lies only about a factor of 3 lower than that given by the physical argument of balancing the accretion torque and the MHD synchronization torque, even though the two approaches lead to criteria with very different dependences on the physical variables. This agreement,

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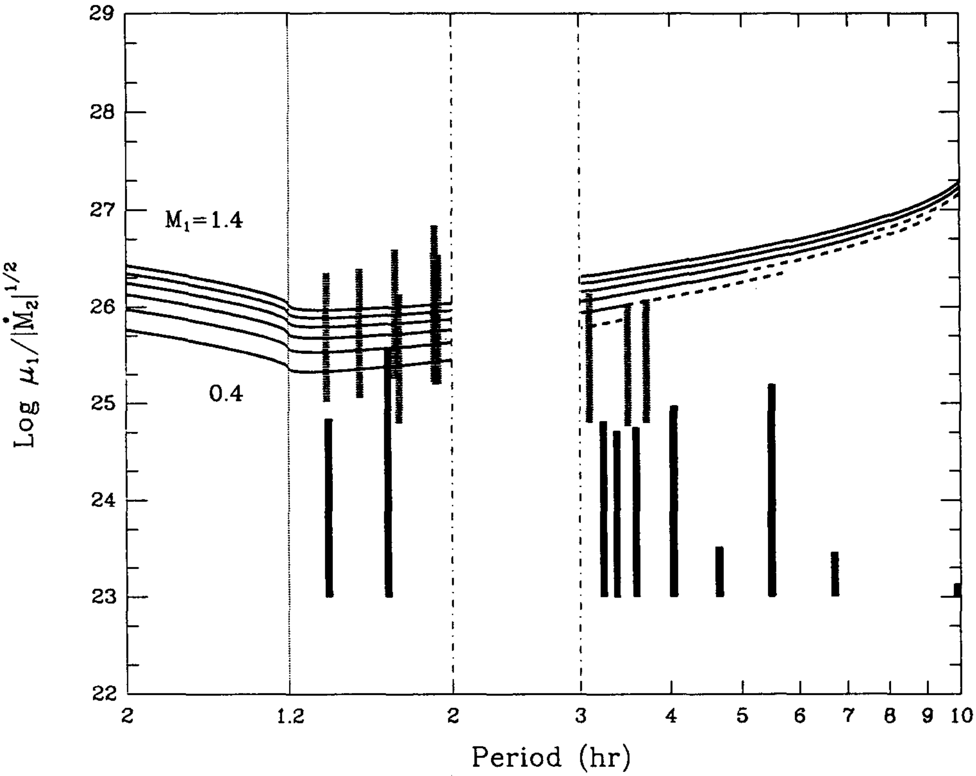

Fig. 10.–Minimum μ_1 required for synchronization *divided* by $|\dot{M}_2|^{1/2}$ as a function of P_b, assuming $M_1 = 0.4 - 1.4\ M_\odot$. These curves are universal in the sense that they are independent of $\dot{M}_2$. The notation for the ranges of μ_1 inferred from observations is the same as in Fig. 7.

which is comparable to the uncertainties in the theory, is due partly to coincidence and partly to the fact that the length scales in the problem (e.g., ϖ_{max}, R_2, and a) are all of similar size.

Comparison of Figures 7 and 9 (or 8 and 10) allows us to identify four regimes: (1) In systems which have $\mu_1 \lesssim 10^{31}$ G cm^3, the magnetic field of the degenerate dwarf is unable to funnel the accretion flow near its surface. These systems therefore are not DQ Her stars, and may show little or no evidence of a magnetic field. (2) Systems which lie above this but below the disk criterion curve (10^{31} G cm$^3 \lesssim \mu_1 10^{33}$ G cm^3) are DQ Her

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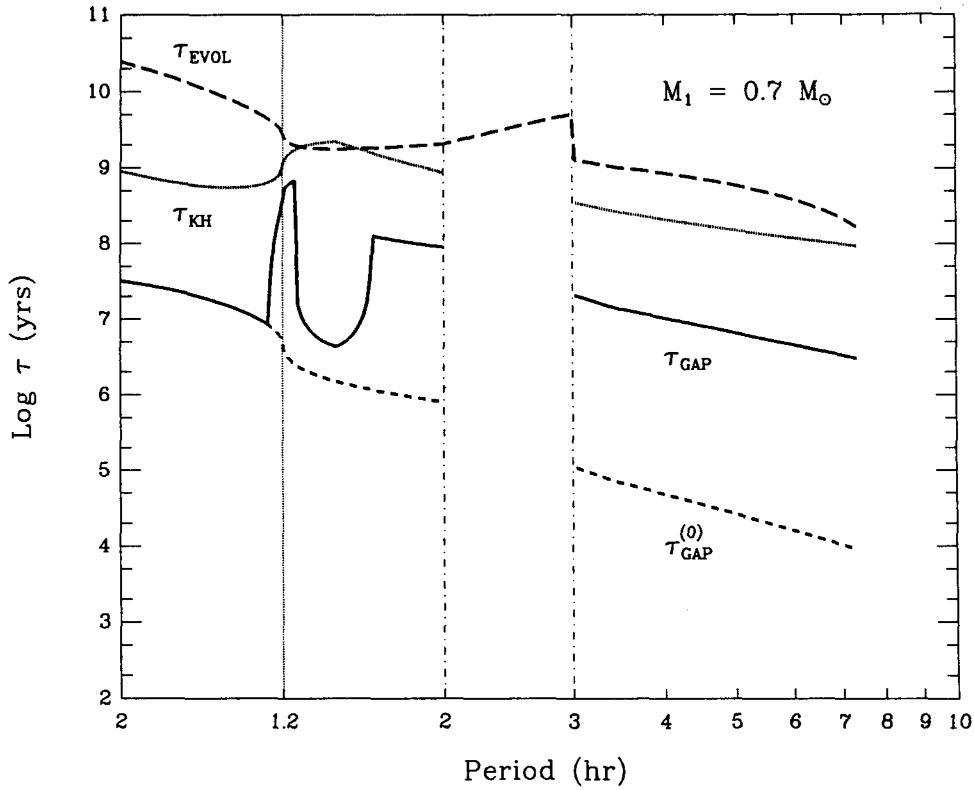

Fig. 11.–Duration τ_{GAP} of the synchronization-induced period gap compared with the evolutionary time scale τ_{EVOL}, the Kelvin-Helmholtz thermal time scale τ_{KH}, and the duration $\tau_{GAP}^{(0)}$ that would result from the injection of J_1 alone [*cf.* Eq. (12)], for a system containing a 0.7 $M_\odot$ degenerate dwarf.

stars with disks, while systems which lie above this but below the synchronization criterion curve are DQ Her stars without disks (King 1985; Hameury, King, and Lasota 1986). (3) Systems which lie in the band traversed by the synchronization criterion curve (10^{33} G cm^3 $\lesssim 10^{35}$ G cm^3) are DQ Her stars which evolve into AM Her stars (Chanmugam and Ray 1984; King, Frank, and Ritter 1985; Lamb 1985), assuming μ_1 is constant throughout their evolution (an assumption which may not be valid). (4) Systems which lie above this band are AM Her stars.

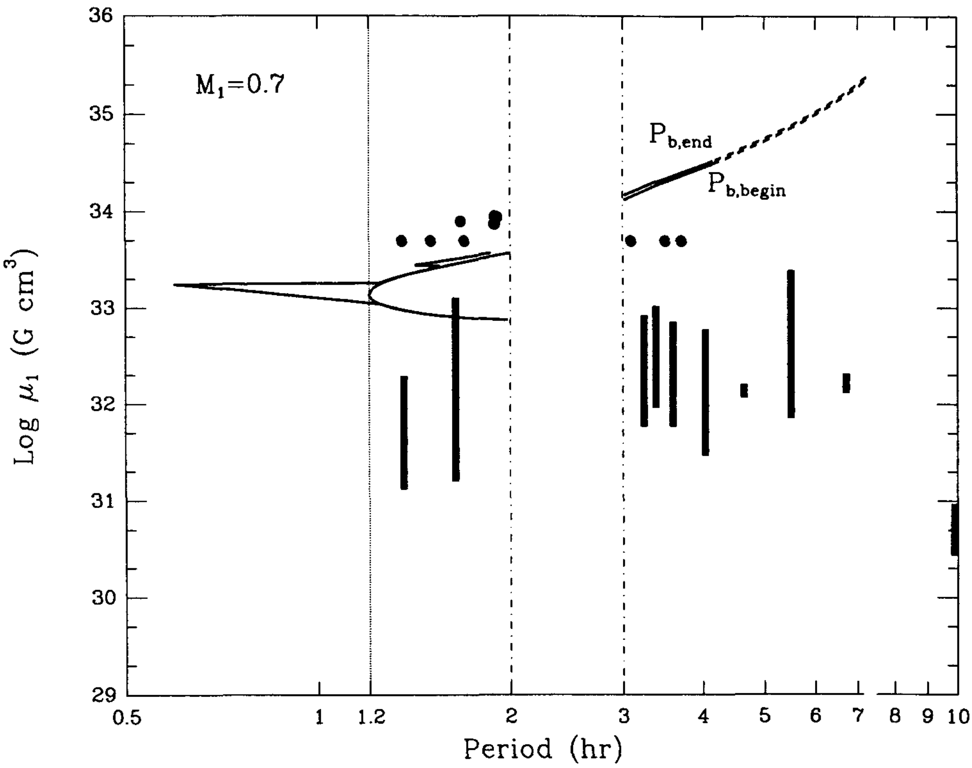

Fig. 12.–Synchronization-induced period gap as a function of magnetic moment μ_1 for a 0.7 $M_\odot$ primary. The rightward curve shows the binary orbital period $P_{b,begin}$ at which the system goes out of contact (this curve is identical to the one for the minimum magnetic moment required for synchronization; see Figure 10), and the leftward curve shows the orbital period $P_{b,end}$ at which the system comes back into contact. The large gap near minimum period arises because the secondary star lies below the main sequence and contracts all the way to the degenerate sequence when synchronization brings it out of contact.

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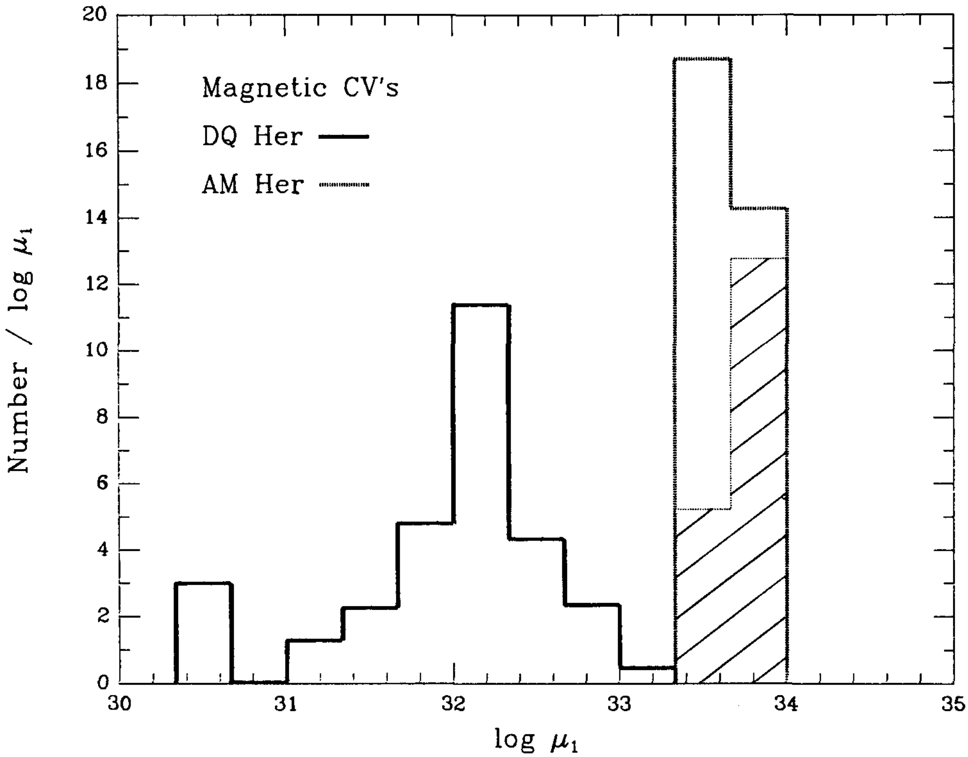

Fig. 13.–Distribution of the magnetic moments μ_1 of the 12 known AM Her stars and of 10 of the known DQ Her stars, assuming disks exist in the latter. The vertical axis gives the number per unit $\log \mu_1$, so that the total number of binaries within a given interval of $\log \mu_1$ is simply the area within that interval. The AM Her stars with directly measured magnetic field values are shown shaded.

d) Synchronization-Induced Period Gap

Synchronization in magnetic CV's leads to a new kind of period gap and gives an entirely new way of producing ultra-short period binaries. A gap arises because the angular momentum J_1 residing in the degenerate dwarf is injected directly into the binary system by the MHD torque when synchronization occurs. This drives the system apart, bringing the secondary out of contact and ending mass transfer. The radius R_2 of the secondary then shrinks back toward its main sequence value $R_{2,MS}$. The secondary is brought back

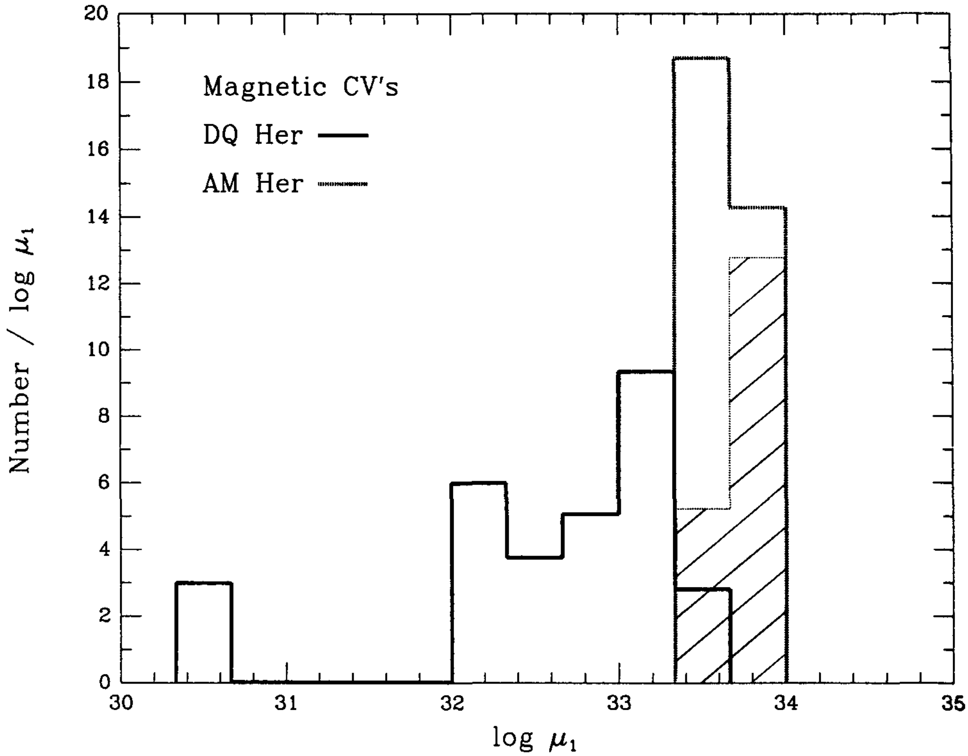

Fig. 14.–Distribution of the magnetic moments μ_1 of the 12 known AM Her stars and of 10 of the known DQ Her stars, assuming no disks exist in the latter (except for AE Aqr, DQ Her, and V533 Her, for which no self-consistent solutions without a disk exist). The AM Her stars with directly measured magnetic field values are shown shaded. The units are the same as in Figure 13.

into contact by angular momentum loss due to GR. Since R_2 can be 20 - 50 % larger than $R_{2,MS}$ for some evolutionary scenarios (see, e.g., Verbunt 1984), the resulting period gap can be far larger than the gap $\tau_{GAP}^{(0)}$ which would result from the injection of J_1 alone:

$$\tau_{GAP}^{(0)} \equiv \frac{J_1}{\dot{J}_b} \approx \frac{I_1 \Omega_{eq}}{\dot{J}_b} \,. \qquad (12)$$

It is for this reason that we use the term "synchronization-induced period gap." For the particular evolutionary model discussed here (DMB4*), $\tau_{KH} \approx \tau_{EVOL}$ and the secondary

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star is only modestly out of thermal equilibrium. Therefore the actual duration τ_{GAP} of the gap is only somewhat greater than $\tau_{GAP}^{(0)}$.

If synchronization occurs during the usual 2 – 3 hour period gap, as may often happen (see Figures 9, 10 and 13), it only lengthens the duration of the gap. When $P_b \approx 1.5$ hours, the mass of the secondary is approximately $0.086 M_\odot$. As P_b decreases further, the secondary leaves the main sequence (no equilibrium configuration balancing thermal energy generated by nuclear burning and gravitational energy exists). Just after minimum period (≈ 1.2 hours), the secondary reaches the degenerate sequence. If synchronization occurs between these two orbital periods, the secondary shrinks all the way to $R_{2,DS} << R_2$, where $R_{2,DS}$ is its radius on the degenerate sequence, before contact is resumed. This produces a synchronization-induced period gap whose duration can approach the evolutionary time scale τ_{EVOL} for the system. This possibility is realized if $\tau_{KH} < \tau_{EVOL}$; otherwise, the secondary shrinks little, and $\tau_{GAP} \approx \tau_{GAP}^{(0)}$. For the particular evolutionary model discussed here (DMB4*), $\tau_{KH} > \tau_{EVOL}$ from $P_b \approx 1.2 - 1.7$ hours and the duration of the synchronization-induced gap is suppressed in most of this interval except near $P_{b,min}$. These features are shown in Figure 11, which compares τ_{GAP}, τ_{EVOL}, and τ_{KH}, respectively.

These same features are evident in Figure 12, which shows the orbital period $P_{b,begin}$ at the beginning and the orbital period $P_{b,end}$ at the end of the synchronization-induced period gap. In this figure, we use a single-valued orbital period axis in order to show $P_{b,end}$; systems thus evolve toward the left prior to period minimum and toward the right afterward. The curve of $P_{b,begin}$ is identical to those in Figure 9, but is now shown for a $0.7\ M_\odot$ primary.

Before the period gap and after period minimum, the secondary is very close to thermal equilibrium and $P_{b,end}$ lies close to $P_{b,begin}$. Between $P_b \approx 1.5$ hours and $P_{b,min} \approx 1.2$ hours, the secondary lies off the main sequence and has not yet reached the degenerate sequence. If synchronization occurs during this interval, there is the possibility that $P_{b,end} << P_{b,begin}$. In the particular evolutionary scenario DMB4*, much of this behavior is suppressed because $\tau_{KH} > \tau_{EVOL}$ then, as discussed above. Nevertheless, if $\mu_1 \approx 1 - 4 \times 10^{33}$ G cm^3, synchronization leads to orbital periods as short as $P_b \approx 40$ minutes. This process is an entirely new way of producing ultra-short period binaries. The secondaries in these binaries are hydrogen-rich rather than hydrogen-depleted or pure helium (*cf.* Nelson, Rappaport, and Joss 1985; Lamb *et al.* 1986).

IV. DISCUSSION

a) Magnetic Moments μ_1 from Observation

i) DQ Her stars

The optical spectra of DQ Her stars are dominated by continuum and line emission from the disk, and to date no absorption lines have been detected from the photosphere of the degenerate dwarf in these systems. The optical light shows little or no polarization (Kemp, Swedlund, and Wolstencroft 1974; Penning, Schmidt, and Liebert 1985) and no evidence of emission or absorption at cyclotron harmonics. Thus the magnetic field of the degenerate dwarf must be inferred by indirect means.

Lamb and Patterson (1983) used the spin behavior of the known DQ Her stars to place bounds on the μ_1's of these stars, assuming the existence of a disk. They used the spin-up rate $\dot{P}$ of these stars to estimate μ_1. Unfortunately, measurements of $\dot{P}$ are available for only a few DQ Her stars (although the situation is changing), and we do not use this method here.

Accretion is not expected to occur if the centrifugal force exceeds the gravitational force at the Alfvén radius (Lamb, Pethick, and Pines 1973). This leads to the condition (Ghosh and Lamb 1979)

$$\omega_s \equiv \Omega/\Omega_K(r_A^{(d)}) \lesssim 0.35 \ . \tag{13}$$

Inequality (13) gives an estimate of μ_1 only if $P \approx P_{eq}$. DQ Her stars are often assumed to satisfy this approximate equality (Chanmugam and Ray 1984; King, Frank, and Ritter 1985; King 1985; Hameury, King, and Lasota 1986) because the spin-up time scale is far shorter than the evolutionary time scale, *i.e.* $\tau_{spin} << \tau_{EVOL}$ (see Figure 3). However, P may be $> P_{eq}$ for the following reason. The accretion torque $N_{acc} \approx N_o$ when $\omega_s << 1$, where $N_o = \dot{M}_1(GM_1R_1)^{1/2}$ is the nominal value due to the matter stresses tending to spin up the star (Lamb, Pethick, and Pines 1973). However, when $\omega_s \approx 1$, the magnetic stresses tending to spin down the star exceed the matter stresses and the accretion torque becomes large and negative, $N_{acc} \approx -20N_o$ (Ghosh and Lamb 1979). Because of the disparity between the magnitudes of N_{acc} above and below the equilibrium spin period $P_{eq} \equiv 2\pi/\Omega_K(r_A^{(d)})$, P can exceed P_{eq} significantly if the accretion rate $\dot{M}_1$

varies appreciably and the variation lasts a time comparable to τ_{spin} (*cf.* Elsner, Ghosh, and Lamb 1980). The larger the variations in $\dot{M}_1$ and the longer they last, the more pronounced the discrepancy can be.

When $P > P_{eq}$, the inequality (13) gives only an *upper bound* on μ_1, as emphasized by Lamb and Patterson (1983). This upper bound is accurate only to within a factor 2 – 3 because of uncertainties in the theory of disk accretion. Lamb and Patterson (1983) used $\omega_s = 0.35$ (Ghosh and Lamb 1979) and assumed a mass $M_1 = 1\ M_\odot$. These values yield a conservative upper bound $\mu_1^{(max)}$, since lower masses give lower values of $\mu_1^{(max)}$ ($\sim M_1^{5/6}$). Here we adopt $\omega_s = 1$ and assume a mass $M_1 = 1.2\ M_\odot$, which gives a very conservative upper bound. The resulting estimates of $\mu_1^{(max)}$ for the known DQ Her stars range from 3×10^{32} to 2×10^{33} G cm^3, depending on the value of $\dot{M}_2$ and P.

Lamb and Patterson (1983) also derived a *lower bound* on μ_1 using the fact that the DQ Her stars show pulsed optical and X-ray emission, indicating that the magnetic field of the degenerate dwarf is strong enough to channel the flow of accreting matter near the stellar surface. This requires

$$r_A^{(d)} \gtrsim 3R_1 \ . \tag{14}$$

The resulting lower bound $\mu_1^{(min)}$ depends only weakly on the mass of the degenerate dwarf ($\sim M_1^{1/4} R_1^{7/4} \sim M_1^{-1/3}$), but inequality (14) is only a dimensional argument, and is therefore uncertain by a factor of 2 – 3. The values of $\mu_1^{(min)}$ for the known DQ Her stars range from 3×10^{31} to 2×10^{32} G cm^3, depending on the value of $\dot{M}_2$.

Recently, King (1985) and Hameury, King, and Lasota (1986) have raised the intriguing possibility that disks are absent in the known DQ Her stars with $P_b \lesssim 5$ hours. The observational bounds on μ_1 allow a self-consistent picture to be constructed for either case, given present uncertainties in the theory of accretion from a disk or stream onto a magnetic star. Thus the issue must be settled by other observations. There is, in fact, other observational evidence that accretion disks exist in these systems (see below). However, if no disk is present, $\mu_1^{(max)}$ could be a factor $\lesssim 2$ larger than the value we have adopted (see below).

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ii) AM Her stars

The magnetic fields of seven of the twelve known AM Her stars have been determined directly, either by measuring the Zeeman splitting of absorption lines from the photosphere of the degenerate dwarf during a low state or by measuring the frequencies of cyclotron emission features and deducing the frequency of the cyclotron fundamental (Schmidt, Stockman, and Grandi 1986; Schmidt and Liebert 1986). The magnetic fields all lie in the range $\approx 1 - 3 \times 10^7$ G.

Because μ_1 is the natural physical variable in the problems of interest to us here, we convert the AM Her magnetic field determinations to μ_1 determinations. Because observational estimates of the degenerate dwarf mass in AM Her stars are insecure, we take a range $\approx 0.4 - 1.2\ M_\odot$ for each. This leads to magnetic moments of the AM Her stars in the range $\mu_1 \approx 1 \times 10^{33} - 1 \times 10^{34}$ G cm^3.

b) Comparison of Magnetic Moments

i) DQ Her and AM Her stars

Figures 13 and 14 compare the distribution of magnetic moments μ_1 of the 12 known AM Her stars and of the 10 known DQ Her stars, assuming $0.7\ M_\odot \lesssim M_1 \lesssim 0.9\ M_\odot$.

Figure 13 assumes that disks exist in the DQ Her systems, and uses the $\mu_1^{(min)}$ and $\mu_1^{(max)}$ derived from inequalities (13) and (14) with $r_A^{(d)}$. According to the figure, the μ_1 distributions of the DQ Her and AM Her stars differ significantly. The centroid of the DQ Her distribution lies at $\sim 10^{32}$ G cm^3, while the centroid of the AM Her distribution lies at $\sim 4 \times 10^{33}$ G cm^3. There is no overlap in the distributions.

Figure 14 assumes that no disks exist in the DQ Her systems (with the exceptions of AE Aqr, DQ Her, and V533 Her, for which no self-consistent solution without a disk exists), and uses the bounds derived from inequalities (13) and (14) with $r_A^{(s)}$ [Lamb 1985, Equation (20); see also Hameury, King, and Lasota (1986)]. According to the figure, the μ_1 distributions of the two subclasses still differ. The centroid of the DQ Her distribution lies at $\sim 6 \times 10^{32}$ G cm^3, while the centroid of the AM Her distribution is the same as before. There is hardly any overlap in the distributions, but the DQ Her distribution now nestles up against the AM Her distribution.

Since $\mu_1^{(min)} \sim M_1^{-1/3}$ and $\mu_1^{(max)} \sim M_1^{5/6}$, the range in μ_1 for individual DQ Her stars is determined solely by the highest assumed mass. The higher the mass, the broader the range in μ_1. Conversely, the higher the mass assumed for the AM Her stars, the lower the μ_1, since $\mu_1 \sim R_1^3 \sim M_1^{-1}$. Thus the two distributions are more disjoint for smaller values of M_1, and less so for larger values. We find that the two distributions overlap significantly only if $M_1 \gtrsim 1.2\ M_\odot$, assuming disks exist in the DQ Her systems, and only if $M_1 \gtrsim 1.0\ M_\odot$, assuming no disks exist (with the exceptions of AE Aqr, DQ Her, and V533 Her). Thus it is likely that the μ_1's of the known DQ Her stars (including those with longer rotation periods) are significantly less than those of the known AM Her stars, unless all members of the two subclasses have $M_1 \gtrsim 1\ M_\odot$.

This differs from the conclusion reached by King (1985), and Hameury, King, and Lasota (1986) who suggest that the magnetic moments of the AM Her and the DQ Her stars are the same. Their view is motivated by the fact that most (8 of 10) of the DQ Her stars lie longward of the period gap, while most (9 of 12) of the AM Her stars lie shortward of the gap. They note that many more DQ Her stars would be expected shortward of the gap, and conclude that the known AM Her stars have come from systems like the known DQ Her stars.

However, the period distributions of the DQ Her and AM Her stars may be distorted by observational selection effects. For example, the number of AM Her stars shortward of the gap may be enhanced by the high interest in them and by the ease with which their classification can be confirmed by optical polarimetry. As a second example, DQ Her stars shortward of the period gap have rotation periods similar to their orbital periods; such periods are more difficult to detect and confirm. There may also be theoretical reasons for the disparate observed distributions of the two subclasses. For example, longward of the period gap, only low mass ($M_1 \lesssim 0.4 - 0.6\ M_\odot$) degenerate dwarfs are more likely to synchronize, even if the secondary has a strong ($B_2 \sim 10^2 - 10^3$ G) magnetic field (Lamb and Melia 1986). Since stable mass transfer occurs in such systems only for $P_b \lesssim 3 - 3\ 1/2$ hours (see Figures 7 – 10), very few AM Her stars may exist longward of the period gap. As another example, the μ_1's of magnetic CV's may increase with time if nova explosions progressively expose an internal fossil field (Chanmugam and Gabriel 1972, Lamb 1974), leading to more AM Her stars at short orbital periods.

ii) Magnetic CV's and isolated magnetic degenerate dwarfs

About 2 % of isolated degenerate dwarfs have magnetic fields $B_1 \gtrsim 10^6$ G (Angel, Borra, and Landstreet 1981; Schmidt and Liebert 1986). The number of such stars per logarithmic interval in magnetic field strength is consistent with a uniform distribution from $B_1 \approx 10^6 - 5 \times 10^9$ G, with ≈ 0.5 % per decade (Angel, Borra, and Landstreet 1981; Schmidt and Liebert 1986). There is *no* evidence that the distribution is bimodal, as has been claimed (King 1985).

The difference between the frequency of occurrence of strongly magnetic degenerate dwarfs in CV's and in the field is striking. This may indicate that something about the formation and evolution of degenerate dwarfs in close binaries is favorable to the creation of strong surface magnetic fields. Important differences in the distribution of magnetic field values in the two groups may exist, but unknown observational selection effects and low statistics prevent a meaningful comparison at present.

c) Existence of Accretion Disks

Figures 7 and 8 compare the magnetic moments μ_1 of the magnetic CV's with the criteria for the existence of an accretion disk in the system. The DQ Her stars are shown as solid bars and the AM Her stars as shaded bars. Figure 8, which is independent of the evolutionary model and variations in $\dot{M}_2$ among the individual stars, shows that the three AM Her stars longward of the period gap are expected to have no accretion disks if $M_1 \lesssim 0.9\ M_\odot$, while those shortward of the gap are expected to have no disks for any mass. These results are in good agreement with observations, which indicate that these systems have no accretion disks.

Figure 8 does not allow us to decide whether or not the known DQ Her stars have disks because of the large uncertainties in the actual values of μ_1 for these stars. However, there is strong observational evidence that accretion disks exist in these systems, which includes the following (Warner 1983, 1985 and references therein; Penning 1985):

- A power-law spectrum in the optical and UV light from the system like that expected for emission from a disk.
- Doubled emission lines which reflect the circular velocity of material in the outer disk where the lines originate, and which do not cross back and forth in wavelength

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with period P_b, as would be expected for accreting matter streaming toward the degenerate dwarf primary (as is seen, *e.g.* in the AM Her stars and in SS433).

• Broad emission line wings, with velocities frequently extending up to 2000 km s^{-1}, reflecting the circular velocities in the inner portion of the disk.

• Broad emission lines phased with pulse phase, due to light from reprocessing of pulsed X-ray emission by the inner edge and surface of the disk.

• Dwarf nova outbursts in EX Hya and SW UMa, the two DQ Her stars with orbital periods below the period gap, implying (within the framework of the disk instability model) disks in these systems.

The existence of disks in these systems indicates that $P << P_{eq}$ in many of them, and that the observed values of $|\dot{M}_2|$ and $\dot{P}$ do not reflect their long-term evolutionary values, as other evidence also indicates. It also implies that the μ_1's of the DQ Her stars are significantly less than the upper bounds derived from their spin behavior. This increases the likelihood that the DQ Her stars have smaller μ_1's than do the AM Her stars (compare Figures 13 and 14).

Recently, Tuohy *et al.*(1986) discovered a cataclysmic variable, H0542-41, which appears to show X-ray emission at the sideband period $P_{side} = (P^{-1} - P_b^{-1})^{-1} \approx 2050$ seconds as well as at $P \approx 1920$ and $P_b \approx 6.5$ hours. Lamb and Mason (1986) have suggested that this system may be a DQ Her star without an accretion disk, and that the modulation of the X-ray emission at P_{side} may be due to magnetospheric gating of the accretion stream. If this picture is verified, the system may teach us a great deal about the appearance of DQ Her stars without accretion disks.

d) Synchronization

Figures 9 and 10 compare the magnetic moments μ_1 of the magnetic CV's with the criteria for synchronization. Again, the DQ Her stars are shown as solid bars and the AM Her stars as shaded bars. Two of the three AM Her stars longward of the gap are not expected to be synchronized and the third (AM Her) is expected to be synchronized only if the degenerate dwarf mass $M \lesssim 0.4\ M_\odot$, assuming $\mu_2 = 0$. However, all three systems may be synchronized if $\mu_2 \sim 10^{33} - 10^{34}$ ($B_2 \sim 100 - 1000$ G) (Lamb and Melia 1986). The AM Her stars with orbital periods shortward of the gap are expected to be synchronized if they have moderate masses $M_1 \lesssim 0.7\ M_\odot$. Observations indicate that

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several of these systems are synchronously rotating, or nearly so; thus the criterion for synchronization is consistent with the behavior of the AM Her stars. The requirement $\mu_2 = 0$ for synchronous rotation longward of the period gap indicates that the secondary possess a significant magnetic field, as other evidence also indicates (see, e.g, Chanmugam and Dulk 1982).

Figures 9 and 10 show that, even taking into account $\mu_2 \neq 0$ and the theoretical uncertainty the synchronizatin criteria, it appears that the known DQ Her stars will not evolve into AM Her stars. This differs from the conclusion of Chanmugam and Ray (1984) and King, Frank, and Ritter (1986).

V. CONCLUSIONS

Our conclusions are as follows. We have identified four regimes: (1) When $\mu_1 \lesssim 10^{31}$ G cm^3, the magnetic field of the degenerate dwarf is unable to funnel the accretion flow; these systems are not DQ Her stars and may show little or no evidence of a magnetic field. (2) Systems with 10^{31} G cm$^3 \lesssim \mu_1 10^{33}$ G cm^3 are always DQ Her stars. (3) Systems with 10^{33} G cm$^3 \lesssim \mu_1 \lesssim 10^{35}$ G cm^3 are DQ Her stars which evolve into AM Her stars, assuming μ_1 is constant throughout their evolution (an assumption which may not be valid). These three regimes agree qualitatively with those of King, Frank, and Ritter (1985). (4) Systems with 10^{35} G cm$^3 \lesssim \mu_1$ are always AM Her stars.

We have confirmed that AM Her stars do not have accretion disks, while DQ Her stars may or may not have them, depending on the value of μ_1, as pointed out by King (1985) and Hameury, King, and Lasota (1986). The ranges of μ_1 for the known DQ Her and AM Her stars allowed by observation are very large, due to uncertainties in the theory of disk and stream accretion, and to the dependence of the ranges on the mass M_1 of the degenerate dwarf. Nevertheless, it is likely that the μ_1's of the known DQ Her stars (including those with longer rotation periods) are significantly less than those of the known AM Her stars, unless all known DQ Her and AM Her stars have $M_1 \gtrsim 1\ M_\odot$. This differs from the conclusion reached by King (1985) and King, Frank, and Ritter (1985). The criteria for synchronization and for the presence or absence of a disk in DQ Her systems are also uncertain, due to uncertainties in the theory of the MHD synchronization torque and the dependence of the criteria on M_1. Nevertheless, it appears that the known DQ Her stars will not evolve into AM Her stars, assuming that μ_1 is constant throughout their evolution. This differs from the conclusion reached by Chamugam and Ray (1984), King (1985), and King, Frank, and Ritter (1985).

The fact that the strong magnetic field of the degenerate dwarf dramatically alters the optical, UV, and X-ray appearance of magnetic CV's has come to be appreciated. That it can also alter the evolution of the binary itself is only now becoming recognized. Spin-up and spin-down of the magnetic degenerate dwarf temporarily speeds up and slows down the binary evolution of DQ Her stars (Ritter 1985; Lamb and Melia 1986). Co-operative magnetic braking may speed up the binary evolution of AM Her stars when the degenerate dwarf has a particularly strong ($B_1 \gtrsim 5 \times 10^7$ G) magnetic field (Schmidt, Stockman, and Grandi 1986). Here we have pointed out that synchronization of the magnetic degenerate dwarf injects angular momentum into the binary, driving the system apart and producing a *synchronization-induced period gap* (Lamb and Melia 1986). This process yields an entirely new way of producing ultra-short period binaries. The secondaries in these binaries are hydrogen-rich rather than hydrogen-depleted or pure helium.

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