

Fundamentals of linear algebra, by A. H. Lightstone. ix + 340 pages. Appleton-Century-Crofts, New York, 1969. U.S. \$8.95.

Intended as a one-semester contemporary course in linear algebra, the book covers the usual material—determinants, matrices, an elementary theory of groups, rings, polynomials, linear spaces, linear operators, characteristic equations, lines, planes and quadratic surfaces. The deviation from other books is only in style and not in content. Most of the material of this book may be found in “Linear Algebra, An Introductory Approach” by C. W. Curtis. But the present book lacks the elegance and neatness of Curtis’ book. The author is in a hurry to introduce as many new ideas and results. The result is a packed course, with not sufficient time for digestion of the ideas. For example, it is difficult to find much virtue in the discussion of cos and arcs functions or the generalization of the notion of cross product in the context. The student is unlikely to grasp this generalization in a first course. The notations tend to be complicated. For example, on p. 73, $aB \cdot r$ could have been taken as the definition of “matrix with multipliers”. A lot of emphasis is laid on the virtue of an ordered basis. But everything can be done equally well with a fixed basis.

There is a nice way of computing the inverse of a matrix on pp. 84–85. Each chapter is followed by a large number of exercises. Each chapter is well motivated. There are one or two technical errors and some typographical errors.

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The origins of the infinitesimal calculus, by Margaret E. Baron. viii + 304 pages. Oxford Univ. Press, New York; Pergamon Press, New York, 1969. \$13.

This book offers material not easily available elsewhere in English for the mathematician interested in knowing the results reached concerning areas and volumes from 1635 (Cavalieri) to 1687 (Newton). Concerning Cavalieri and his predecessors, it is less reliable. Even the latter half of the book may be exasperating to historians by reason of its free use of modern notation, giving rise to many sentences like this (p. 181): “More important, however, is the geometrical transformation through which, by means of the relation $t/x = dy/dx$, Roberval transforms the integral $\int_0^a x dy$ into $\int_0^a t dx$.”

The statement that Roberval, a quarter-century before Leibnitz, made use of any such relation as $t/x = dy/dx$ is hard to reconcile with the author’s claim in her preface that “historical development is central and the methods which emerge are treated strictly within their historical context”. It is even harder to square with her remark (p. 153) that Roberval’s “style is obscure, verbose and difficult for, although he abandons any attempt to adhere to the rigorous geometric methods of his

predecessors, he makes no move to order his ideas through the use of algebraic symbols". The reader is given no clue to the evolution of algebraic notation; Vieta and Harriot are not mentioned.

The first half of the book attempts to sketch developments from classical antiquity to the publication of Cavalieri's *Geometry by Indivisibles of the Continuum*. This part suffers not from notation, but from neglect of some conceptual shifts. The use of modern notation for Greek geometrical achievements is unobjectionable, since the dominant role of Eudoxian theory of proportion is preserved, and summation rather than integration is symbolized. But the medieval theory of proportion was not Eudoxian, and the author appears not to know this. The chief Latin translators of and commentators on Euclid's definition of proportionality in the fifth book of the *Elements* misunderstood the concept, and medieval theory of proportion, developed on an arithmetical basis, gave rise to a totally different concept of continuity from ours. Neglect of this oddity has produced some peculiar views about medieval influences on later mathematics, views that are nowhere more misleading than in the history of the infinitesimal calculus. Bradwardine's celebrated function-concept, interesting from the standpoint of physics, by no means introduced the mathematical idea of a *continuous* function, and even Oresme's masterful extension did not carry it beyond fractional powers and algebraic numbers.

Assuming a connection between medieval and 17th-century investigations, the author declares (p. 123) that "the term indivisible was mediaeval in origin [and] had been familiar since Bradwardine". Whether Bradwardine discussed indivisibles in opposition to Aristotle, I do not know (no citation is offered), but the pseudo-Aristotelian treatise *On Indivisible Lines*, unknown to the Middle Ages, was resurrected in the 16th century and its ideas were rejected by Galileo and Cavalieri. Likewise the author asserts (p. 117) that Galileo "finds support for . . . mathematical indivisibles from . . . the possibility of the existence of a vacuum", citing his *Two New Sciences*. But in that work he clearly says that both empty space and interpenetrability of bodies are objectionable assumptions, and that "Both of these objections . . . are avoided if we accept the . . . view of indivisible constituents" (TNS, p. 48). To her statement in the same place that "Galileo makes no distinction between physical atoms and line and surface elements", it may be replied that it was for this distinction that he coined the terms *parti quante* and *parti non quante*, or countable (finite) and uncountable (infinitesimal) parts. This distinction was also clear to his pupil, Cavalieri, who, when charged with borrowing Kepler's method, replied: "Kepler somehow compounds greater bodies out of very tiny ones, treating these by considering them as adherent . . ., while I say that planes are related as are the aggregates of all their parallel lines, and bodies are related as are the aggregates of all their parallel planes. . . . No one can fail to see how different these are." In regarding Cavalieri's "aggregates" as sums, or totals by addition (p. 125), rather than as entities capable only of comparison by ratios in pairs of like

kind (as Galileo, like Cavalieri, had viewed them), the author loses the essential distinction between the method of the German mathematician and that of his Italian contemporaries.

The second half of the book, dealing with 17th-century work after Cavalieri, is less controversial with regard to conceptual problems and contains much of interest relating to lesser-known men of the era.

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Geometric transformations, II, by I. M. Yaglom. 189 pages. Translated from the Russian by Allen Shields. New Mathematical Library No. 21, Random House, New York, 1969. Paper U.S. \$1.95.

The first volume treated isometries of the plane. The present volume treats similarity transformations: central similarity; spiral similarity; dilative reflection. The description of "theory" is very brief (only a few pages) but adequate. Most of the book is devoted to problems and their solutions, the latter together in the last half of the book. The reviewer has used both volumes in an undergraduate geometry course and found that the students were challenged by the many interesting problems. This book is recommended for use at the senior high school level or undergraduate university level.

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