THEOREM 1.3. *Given f, g in C, $\lambda < \omega_1$ a limit ordinal and n a natural, we have*

1. If $\mathrm{CB}(f) = \mathrm{CB}(g) = \lambda$ then $f \equiv g$,
2. If $\mathrm{CB}(f) = \lambda + n$ and $\mathrm{CB}(g) = \lambda + 2n + 1$, then $f \leq g$.

In particular, if $(\mathsf{C}_\alpha, \leq)$ is a well-quasi-order (wqo) for all $\alpha \in \omega_1$ then $(\mathsf{C}_\infty, \leq)$ is a wqo. The question of whether C_∞ is a wqo or not remains open.

Applying Theorem 1.3 to the subclass of identity functions, we obtain an alternative proof that topological embeddability on P0D spaces is a wqo. This result is also obtained as a corollary of Laver's celebrated result on labelled trees.

Raphaël Carroy.
E-mail: raphael.carroy@univie.ac.at
URL: http://www.logique.jussieu.fr/∼carroy/papers/These.pdf

EVANDRO LUÍS GOMES, ***Sobre a história da paraconsistência e a obra de da Costa: a instauração da Lógica Paraconsistente [On the history of paraconsistency and da Costa's work: the establishment of paraconsistent logic]. (December, 2013). 535p + appendixes***. Philosophy Ph.D., Institute of Philosophy and Human Sciences and Centre for Logic, Epistemology and the History of Sciences, The University of Campinas, Campinas, São Paulo, Brazil, 2013. Supervised by Itala M. Loffredo D'Ottaviano. MSC: 01A85, 01–02, 03A05, 03B53. Keywords: logic, history and philosophy of logic, nonclassical logics, paraconsistent logic and history of philosophy.

Abstract

The establishment of paraconsistent logic, the quest for and the description of its historical background, by means of an analysis of their philosophical foundations are here presented by the way of contemporary historiography of logic. Nowadays the mature development of paraconsistent logic helps to write the history of its instauration and, in a quite archeological way, to rebuild the prehistory of this approach as well as its theoretical schemata. We especially study the effects of contradiction on rational contexts and the logical and theoretical tools for its suppression, handling or absorption. Broad and strict paraconsistency are also confirmed by the level of refusal of the *ex falso*. Such a logical law affirms that every formula follows from a contradiction and is related to the formal trivialization of the theories whenever their underlying theories are, e.g., classical or intuitionist. In paraconsistent theories, however, the *ex falso* does not hold in general. Such theories can be inconsistent without being trivial after all. We consider texts, contexts, and historical landmarks from analytical, descriptive and historical point of view in order to understand its stages of development. In Part I, we tell the history of the antecedents of paraconsistent approach. In Chapter 1, *Paraconsistent logical elements in ancient authors*, we identify and gather meaningful texts to the prehistory of paraconsistency in Western thought. From those elements, we outline an interpretation that matches paraconsistent elements with the theoretical achievements of Heraclitus of Ephesus, Aristotle, and by the Stoics. In Chapter 2, *Paraconsistent logical elements in medieval authors*, we study the growing process concerning of handling with contradiction in the rational thought, most of the latter being inspired by the treatment of the issue in the former tradition. There are paraconsistent grounds, for instance, in the work of Peter Abelard, Peter of Spain, and William of Ockham. Such authors, by their own efforts, could elevate such elements into sophisticated arguments found in their discussion whether the *ex falso* is or not admissible. In Part II, we introduce the history of the strictly paraconsistent positions properly said together with its prelude. In Chapter 3, *The dawn of contemporary paraconsistency*, we study statements concerning inconsistency in some studies of Leibniz, Hume, Kant, and Hegel, as well as we analyze other authors' contribution to that debate such as Jan Łukasiewicz and Nicolaj Vasiliev. Those thinkers were forerunners in the defense of a theoretical possibility of nonclassical logics, in the sense that some of their ideas has lead to paraconsistent approaches thereafter. In Chapter 4, *The stage of paraconsistency*, we focus on rebuilding the historical and theoretical context that occurs before the

introduction of the first paraconsistent logics. In the 1930s, 1940s, and 1950s, in a somehow latent way, some scholars have debated formal and logical theses of paraconsistent flavour without effectively starting up such a theoretical field. Stanisław Jaśkowski is one of the first to contribute to the paraconsistent field in a very intentional way, with full awareness of the meaning of his contribution to it. In fact, his discussive logic D_2 is a mature propositional paraconsistent logic. In 1950s, in an independent and more radical way, the initial investigations of Newton da Costa will lead him to conceive and introduce his paraconsistent logics $C_n, 1 \leq n \leq \omega$, definitely published in 1963. Such a full-fledged system of paraconsistent logics, whose later investigation has opened up the field of paraconsistency to the worldwide community of scholars, is an important event to the recent history of logic and constitutes the first meaningful contribution of a Brazilian thinker to logic and to the Western philosophy.

Abstract prepared by Evandro L. Gomes and Itala M. Loffredo D'Ottaviano
E-mail: elgomes@uem.br, evgomes@gmail.com
URL: http://repositorio.unicamp.br/jspui/handle/REPOSIP/281158

Matthew Harrison-Trainor, ***The Complexity of Countable Structures***, University of California, Berkeley, CA, USA, 2017. Supervised by Antonio Montalbán. MSC: 03D45. Keywords: Computability theory, computable structure theory.

Abstract

We prove various results about the complexity of countable structures, both computable and arbitrary. We will describe some of the more important results.

§1. Scott ranks. We begin by investigating descriptions of countable structures in the infinitary logic $\mathcal{L}_{\omega_1\omega}$. Given a countable structure $\mathcal{A}$, we can find a sentence φ, a Scott sentence for $\mathcal{A}$, which describes $\mathcal{A}$ up to isomorphism in the sense that $\mathcal{A}$ is the unique countable model of φ. We can assign a complexity, the Scott rank of $\mathcal{A}$, to $\mathcal{A}$; this is the quantifier complexity of the simplest Scott sentence.

Given an $\mathcal{L}_{\omega_1\omega}$ sentence φ, which we think of as a theory defining a class of structures, what might the set of Scott ranks of the models of φ be? We call such a set of Scott ranks the Scott spectrum of φ. Under projective determinacy, we get a complete descriptive-set-theoretic characterization of which sets of ordinals are the Scott spectra of a sentence.

Theorem 1.1 (Harrison-Trainor; ZFC + PD). *The sets of ordinals which are the Scott spectra of $\mathcal{L}_{\omega_1\omega}$-sentences are exactly the sets of the following forms, for some $\mathbf{\Sigma}^1_1$ class of linear orders $\mathcal{C}$:*

1. *The well-founded parts of orderings in $\mathcal{C}$,*
2. *The orderings in $\mathcal{C}$ with the non-well-founded part collapsed to a single element, or*
3. *The union of (1) and (2).*

Using the same ideas, we also solve three open questions. We answer a question of Montalbán by showing, for each $\alpha < \omega_1$, that there is a Π^{in}_2 theory with no models of Scott rank less than α. We also answer a question of Knight and Calvert by showing that there are computable models of high Scott rank which are not computably approximable by models of low Scott rank. Finally, we answer a question of Sacks and Marker by showing that δ^1_2 is the least ordinal α such that if the models of a computable theory T have Scott rank bounded below ω_1, then their Scott ranks are bounded below α.

We also look at Scott sentences for finitely generated groups. Every finitely generated structure automatically has a Σ^{in}_3 Scott sentence, which is relatively simple. It turns out that many groups have a simpler d-Σ^{in}_2 Scott sentence—a conjunction of a Σ^{in}_2 and a Π^{in}_2 sentence—and hence have Scott rank at most 2. This led Knight to conjecture that every finitely generated group has a d-Σ^{in}_2 Scott sentence. To resolve this conjecture, we first give a general characterization of the finitely generated structure with d-Σ^{in}_2 Scott sentences, and