

THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

MAX BLACK

ALONZO CHURCH

LEON HENKIN

*Managing Editor:* ROBERT E. LUCE

*Consulting Editors:*

W. ACKERMANN

CARL G. HEMPEL

EVERETT J. NELSON

C. A. BAYLIS

LÁSZLÓ KALMÁR

RÓZSA PÉTER

PAUL BERNAYS

JOHN G. KEMENY

W. V. QUINE

G. D. W. BERRY

S. C. KLEENE

BARKLEY ROSSER

EVERT BETH

J. C. C. MCKINSEY

THORALF SKOLEM

ROBERT FEYS

ANDRZEJ MOSTOWSKI

A. R. TURQUETTE

F. B. FITCH

H. E. VAUGHAN

VOLUME 14

1949

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC.

WITH THE AID OF SUBVENTIONS FROM

HARVARD UNIVERSITY

PRINCETON UNIVERSITY

INSTITUTE FOR ADVANCED STUDY

SMITH COLLEGE

UNIVERSITY OF PENNSYLVANIA







THE  
JOURNAL  
OF  
SYMBOLIC LOGIC

EDITED BY

MAX BLACK                      ALONZO CHURCH                      LEON HENKIN

*Managing Editor:* ROBERT E. LUCE

*Consulting Editors:*

W. ACKERMANN	CARL G. HEMPEL	EVERETT J. NELSON
C. A. BAYLIS	LÁSZLÓ KALMÁR	RÓZSA PÉTER
PAUL BERNAYS	JOHN G. KEMENY	W. V. QUINE
G. D. W. BERRY	S. C. KLEENE	BARKLEY ROSSER
EVERT BETH	J. C. C. MCKINSEY	THORALF SKOLEM
ROBERT FEYS	ANDRZEJ MOSTOWSKI	A. R. TURQUETTE
F. B. FITCH		H. E. VAUGHAN

VOLUME 14

1949

PUBLISHED QUARTERLY BY THE ASSOCIATION FOR SYMBOLIC LOGIC, INC.  
WITH THE AID OF SUBVENTIONS FROM  
HARVARD UNIVERSITY                      PRINCETON UNIVERSITY  
INSTITUTE FOR ADVANCED STUDY                      SMITH COLLEGE  
UNIVERSITY OF PENNSYLVANIA

The four numbers of Volume 14 were issued at the following dates:

Number 1, pages 1–80, May 16, 1949.

Number 2, pages 81–144, June 23, 1949.

Number 3, pages 145–208, October 6, 1949.

Number 4, pages 209–283, January, 1950.

All numbers of this volume are copyrighted by the Association for Symbolic Logic, Inc. Reproduction of copyrighted numbers of the Journal by photostat, photo-print, microfilm, or like process is forbidden, except by written permission of the Managing Editor.



## TABLE OF CONTENTS

Foundations of mathematics for the working mathematician. By N. BOURBAKI.....	1
The Heine-Borel theorem in extended basic logic. By FREDERIC B. FITCH.....	9
Constructible falsity. By DAVID NELSON.....	16
A note on nominalism and recursive functions. By R. M. MARTIN.....	27
The logical simplicity of predicates. By NELSON GOODMAN.....	32
Fragments of the propositional calculus. By LEON HENKIN.....	42
Reviews.....	49
Eleventh meeting of the Association for Symbolic Logic.....	73
Abstracts of papers.....	73
Elections.....	80
Institutional contributing subscribers to the JOURNAL.....	80
On natural numbers, integers, and rationals. By FREDERIC B. FITCH.....	81
On ternary logic. By PAUL DIENES.....	85
On an implication function in many-valued systems of logic. By Z. P. DIENES.....	95
Definability and decision problems in arithmetic. By JULIA ROBINSON.....	98
The word problem for semigroups with two generators. By MARSHALL HALL, Jr.....	115
Reviews.....	119
Nicht konstruktiv beweisbare Sätze der Analysis. By ERNST SPECKER.....	145
The completeness of the first-order functional calculus. By LEON HENKIN.....	159
Computational logic. By NATHAN P. LEVIN.....	167
A syntactical characterization of S5. By GUSTAV BERGMANN.....	173
Note on an idea of Fitch. By JOHN R. MYHILL.....	175
$m$ -valued sub-system of $(m + n)$ -valued propositional calculus. By TZU-HUA HOO.....	177
Reviews.....	182
A further consistent extension of basic logic. By FREDERIC B. FITCH.....	209
A note on the deductive completeness of $m$ -valued propositional calculi. By J. B. ROSSER and A. R. TURQUETTE.....	219
A note on nominalistic syntax. By R. M. MARTIN.....	226
An improvement in the theory of simplicity. By NELSON GOODMAN.....	228
Results concerning the decision problem of Lewis's calculi S3 and S6. By SÖREN HALLDÉN.....	230
Reviews.....	237
Index of reviews.....	268



## ERRATA

### VOLUME 3

Page 134, line 32. For the second occurrence of « (IV) », read « (V) ».

### VOLUME 6

Page 19, line 31. For « to to », read « to ».

Page 93. In each of the last five lines on this page, replace the last occurrence of = by  $\neq$ .

Page 188, second column, line 19 from the bottom and line 6 from the bottom. For 31, read 30.

### VOLUME 7

Page 103, line 7 from the bottom. For « Kuratowski », read « Kuratowskiego ».

Pages 111, 112. See the author's footnote on page 9 of volume 14.

### VOLUME 9

Page 104, line 25. For « Tarksi's », read « Tarski's ».

### VOLUME 10

Page 12, line 7. For « moduc », read « modus ».

### VOLUME 11

Page 115. In the second line of XXVI, for the last  $\neg$ , read  $\supset$ .

Page 116, line 23. For  $(\alpha_1)(\alpha_2)B \dots (\alpha_m)$ , read  $(\alpha_1)(\alpha_2) \dots (\alpha_m)B$ .

Page 118, footnote 7, line 3. For  $A_n$ , read  $A_i$ .

### VOLUME 13

Page 133. In †741, change  $\cup$  to  $\cap$ , and insert a bar over  $\cup$ .

Page 168, lines 16–17. For « Adjukiewicz », read « Ajdukiewicz ».

Page 176, line 7. For « dan », read « dans ».

Page 225, line 23. For « mathematiche », read « matematiche ».

Page 225, line 34. For « does », read « goes ».

Page 233, line 6 from the bottom. Delete the first comma in the line.

### VOLUME 14

Page 56. Supply the page number, « 56 », in the upper left corner.

Page 63, line 21. For « SMIELEW », read « SZMIELEW ».

Page 71, line 12 from the bottom. For « sciences », read « science ».

Page 70, line 36. For « esprimibilità », read « esprimibilità ».

Page 114. In [8], for « Congrès », read « Congrès ».

Page 116, line 5 from the bottom. For  $PTQ_1$  read  $PRQ_1$ .

Page 118, line 2. For  $B_1U_2^*$ , read  $B_1^*U_2^*$ .

Page 118, line 4. For  $p + 1 \geq i$ , read  $i \geq p + 1$ .

Page 118, line 3 from the bottom. For  $T baab$ , read  $R baab$ .

Page 139, line 13. For « basés », read « basées ».

Page 140, line 16. For « paraissantes », read « paraissant ».

Page 143, line 2 from the bottom; page 144, lines 2, 4, 7. For XIV (62), XIV (63), XIV (64),

Page 163, line 5. For  $\Gamma_i$ , read  $\Gamma_1$ . [read respectively XIV 62, XIV 63, XIV 64.

Page 163, line 7. For  $\Gamma_i$ , read  $\Gamma_i$ .

Page 165, line 24. For  $(\Lambda, E_1, E_2)$ , read  $(\Lambda, E'_1, E'_2)$ ; and in the latter part of the line, for

Page 165, line 27. For «  $\Lambda, E_1, E_2$  », read «  $\Lambda, E'_1, E'_2$  ». [ $E_1$ , read  $E'_1$ ].

Page 177. In 2.41, for  $i^n$ , read  $i^m$ .

Page 189, line 18. For « relation numbers », read « relation-numbers ».

Page 193, line 21. For « Structure », read « Structures ».

Page 197, line 23 from the bottom. After « of », insert « the ».

Page 205, line 6 from the bottom. For « AND'S », read « AND's ».

Page 208, line 20. For « methodes », read « méthodes »; and for « de », read « du ».

Page 208, line 23. For « deductive », read « déductive ».