

Correspondence

DEAR EDITOR,

Most scientific calculators have a button for selecting the angle units to be radians, degrees or *grads* when using trigonometrical functions. Who uses *grads*?

Yours sincerely,

A. ROBERT PARGETER

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DEAR EDITOR,

In a recent note (77.15) I stated a result that the Fermat point of a tetrahedron has an equianqular property. I would like to make clear that in the proof I *assumed* the existence of such a point, and that for some tetrahedra this may not be a valid assumption to make. Nevertheless, for tetrahedra with a degree of symmetry, say with one equilateral triangular face and with three other identical isosceles triangular faces, the result is certainly true.

Yours sincerely,

PAUL GLAISTER

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Editor's note

I received Paul's letter before note 79.21.

DEAR EDITOR,

Looking through some back numbers of the 'Gazette' for something else, I came across a note (77.5) by R. H. Macmillan in the March 1993 issue, entitled 'Area of a triangle'. Since as far as I can see this did not occasion any response, I am emboldened to stick my neck out — I do so with some trepidation, given that I am very much an amateur amidst the professionals — and offer the following comments.

I was taught what is effectively this formula when I was at school some 50 years ago, but it was expressed in a different form. Specifically, the area of a triangle with coordinates (x_1, y_1) , (x_2, y_2) and (x_3, y_3) is given by:

$$0.5 \times \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}.$$

From this it follows that if one regards one of the points — say (x_1, y_1) — as a variable (x, y) then the necessary and sufficient condition for (x, y) to lie on the line joining (x_2, y_2) and (x_3, y_3) (i.e. the equation of the line through them) is that the area of the triangle is zero, i.e.

$$\begin{vmatrix} 1 & 1 & 1 \\ x & x_2 & x_3 \\ y & y_2 & y_3 \end{vmatrix} = 0.$$

This relation is, of course, normally derived by solving the pair of simultaneous equations requiring for example $y = ax + b$ to go through (x_2, y_2) and (x_3, y_3) .

Yours sincerely,

ALAN D. COX

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Editor's Note

Robert Pargeter mentioned in a letter that he remembers teaching the formula to sixth-formers a few years ago. Overseas candidates for 'Additional Maths' continue to use essentially the same method in papers I have recently marked.

DEAR EDITOR,

While not being a school teacher, I get the impression that there has been a steady decline in the teaching of geometry in schools over the past decade, despite moves towards graphic communication in, for example, user interfaces for computers. With the lack of geometric education, I feel there could be a new literacy problem arising. It is therefore laudable that there have been a number of articles in recent editions of the *Mathematical Gazette* related to art and geometry. While the article "The Portrait of Fra Luca Pacioli" by Nick MacKinnon was an interesting piece of research, it contains errors which illustrate the misunderstanding that can arise when mathematicians do not understand the geometry of perspective.

Pacioli's ellipse

On page 139 of the Pacioli article there are two sentences (just above the diagram) which are in conflict with elementary geometry: "In fact a circle drawn in linear perspective does not give an ellipse. The difference here is immaterial."

Linear perspective is concerned with projection and section. An artist takes lines from the eye which are the reverse of the path of light rays followed from an object. These rays intersect the picture plane to get the image. Normally, the picture plane is a plane perpendicular to the main line of sight, but need not necessarily be so. If you are creating the image of a circle, the circle together with the eye-point form a cone. The picture plane intersects this cone in a conic as has been known from Greek times – indeed the modern term conic is a shortening of "conic section". In normal cases the conic is an ellipse, though it could equally well be a hyperbola or parabola, or even a circle if the circle lies in a plane which is parallel to the picture plane.

The article then goes on to the method used to produce ellipses for the diagrams. "I stretched the diagram until the circle was an ellipse ..." However, in linear perspective this does not happen. If a circle is stretched, the centre of the circle is transformed into the centre of the ellipse. In perspective, because of