Canad. Math. Bull. Vol. **20** (1), 1977

A NOTE ON THE CATEGORY OF THE TELESCOPE

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Let X be an infinite connected CW-complex which is the union of an increasing sequence of subcomplexes X_r. Let cat X denote the Lusternik–Schnirelmann category of X, normalized to take the value 0 on contractible spaces. Suppose that cat $X_r \leq k (r \geq 1)$. In his problem list [1], T. Ganea proved that cat $X \leq 2k+1$ and asked (Problem 5) whether this is the best possible upper bound. The purpose of this note is to prove that cat $X \leq 2k$.

As pointed out by Ganea, we may replace X by the telescope $Y = \cup\ (r \geq 1) X_r \times [r-1, r]$. The required inequality will be obtained by representing Y as a double mapping cylinder and applying the main result of [2].

Let $W = \bigvee\ (r \geq 1) X_r$, $A = \bigvee\ (r \geq 1) X_{2r-1}$, $B = \bigvee\ (r \geq 1) X_{2r}$ be wedges as indicated and let $f: W \to A$ map X_{2r} by inclusion into X_{2r+1} and map X_{2r-1} identically. Similarly let $g: W \to B$ map X_{2r-1} by inclusion into X_{2r} and map X_{2r} identically. Then certainly Y is homeomorphic to the double mapping cylinder $Z = Z(f, g)$. By [2; (2)], we have cat $Z \leq$ cat $X +$ max (cat A, cat B). But cat $W \leq k$, cat $A \leq k$ and cat $B \leq k$. Hence cat $Y =$ cat $Z \leq 2k$.

Acknowledgement. Grants to the Topology Research Group from the University of Cape Town and the South African Council for Scientific and Industrial Research are acknowledged.

References

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2. K. A. Hardie, *On the category of the double mapping cylinder*, Tôhoku Mathematical Journal, **25** (1973), 355–358.

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Received by the editors June 10, 1976.