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W. LEWIS, M.A.  
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PROCEEDINGS  
OF THE  
CAMBRIDGE PHILOSOPHICAL SOCIETY  
VOLUME 28

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BY WALTER LEWIS, M.A.

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