ON WAVES IN NON-ISOTHERMAL, COMPRESSIBLE, IONIZED AND VISCOUS ATMOSPHERES*

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Abstract. A review is given of the properties of waves in atmospheres, with particular emphasis on (Section 1) the variation of amplitude and phase with altitude for propagating waves (Figures 1 to 4) and the waveforms of standing modes (Figure 5). The cases dealt with concern waves under the combined influences of gravity and compressibility, and examine the effects of: (Section 2) temperature gradients in a non-isothermal atmospheric model; (Section 3) external magnetic field, either vertical or horizontal; (Section 4) dissipation by viscosity and electrical resistance. The results are relevant to (Section 5) the assessment of atmospheric wave growth and shock formation, and to the calculation of heating functions describing the deposition of wave energy.

1. Approximate and Exact Theories of Atmospheric Waves

The study of waves in atmospheres has been strongly influenced by the basic case (Rayleigh, 1890; Lamb, 1932 § 309) of acoustic-gravity waves in isothermal atmospheres, which exhibit for propagating modes an amplitude growing exponentially on twice the scale height and a phase increasing linearly with altitude. The aim of the present communication is to indicate the extent to which these properties of atmospheric waves are modified by the presence of: (Section 2) temperature gradients; (Section 3) external magnetic fields; (Section 4) viscous or resistive dissipation.

In all these cases the linear wave equations describing small amplitude waves have variable coefficients, due to atmospheric stratification, viz., variation of density, temperature, wave speed or damping with altitude. Taking these coefficients approximately constant leads to the W.K.B.J. approximation (Brekhovskikh, 1961; Moore and Spiegel, 1964; Lighthill, 1967; McLellan and Winterberg, 1968; Yeh and Liu, 1974; Campos, 1982); this approximation leads necessarily to sinusoidal waves, which can be described in terms of dispersion relation, phase speed, group velocity, etc. The W.K.B.J. approximation applies only to high-frequency waves over short distances, i.e., is *invalid* for: (i) wavelengths comparable to or larger than the scale height, i.e., the main part of the wave spectrum in the solar photo- and chromospheres (Bray and Loughhead, 1974; Athay, 1976); (ii) asymptoticaly for all frequencies at large distances compared with the wavelength, e.g., for waves generated in the photosphere and propagating in the high corona.

In order to describe the wave fields for all wavelengths (including those comparable to the scale height), and all distances (including asymptotically), the wave equations must be solved *exactly*, i.e., taking into account the dependence of the coefficients on

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altitude (Lamb, 1908; Hide, 1956; Yanowitch, 1967; Thorpe, 1968; Zhugzhda, 1971; Nye and Thomas, 1976; Thomas, 1978; Leroy, 1982). Although the frequency is conserved (if atmospheric properties do not depend on time), there is no wavenumber since the waveform is generally not sinusoidal in altitude, and dispersion relation, group velocity, etc... do not exist. The wave equation can be solved exactly for some forms of the variable coefficients, corresponding to specific atmospheric models, for which the wavefields are expressed at all altitudes in terms of special functions, e.g., Bessel or hypergeometric. Besides, it is possible to determine asymptotic properties for a wide range of atmospheric models (Campos, 1983a, b, c).

Thus we are concerned with wave fields which are given: (i) exactly at all altitudes and frequencies for specific (isothermal or non-isothermal) atmospheric models; (ii) asymptoticaly at high altitude, for all frequencies, for any atmosphere with bounded temperature and vanishing density. The asymptotic form of the exact wave field for specific atmospheric models (i) can be used to check the general asymptotic laws for waves (ii).

2. Acoustic-Gravity Waves in a Non-Isothermal Atmosphere

Waves under the combined influences of compressibility and gravity have been studied extensively in connection with the Earth's atmosphere (Yih, 1965; Beer, 1974; Gossard and Hooke, 1975; Lighthill, 1978), besides solar applications (e.g., Biermann, 1948; Mein, 1978, 1980). We consider three-dimensional acoustic-gravity waves in a model atmosphere with the temperature profile

$$T(z) = T_\infty + (T_0 - T_\infty)e^{-\theta z}, \tag{1}$$

where we may choose at will: (i) the initial T_0 and asymptotic T_∞ temperature, and thus the degree of heating or cooling with altitude; (ii) the steepness parameter θ, allowing larger or smaller values of the maximum temperature gradient $\theta(T_\infty - T_0)$ or conductive heat flux. Some of the temperature profiles in the family (1) are illustrated in Figure 1, where we plot the ratio of temperatures $T(z)/T_0$ (or its logarithm) against altitude z (made dimensionless dividing by the asymptotic scale height $L \equiv RT_\infty/g$, where R is the gas constant and g the acceleration of gravity): (top) the steepness parameter is kept fixed at $\theta L = 1$, and the ratio of asymptotic to initial temperature is given four values $T_\infty/T_0 = 0.5, 1, 10, 100$ corresponding to cooling with altitude, the isothermal case, and moderate or intense heating with alititude; (bottom) the ratio of temperatures is kept fixed at $T_\infty/T_0 = 10$, and the steepness parameter is given four values $\theta L = 0.5, 1, 2, 4$ corresponding to an increasingly steeper approach to the asymptotic temperature, and larger maximum temperature gradient.

It can be shown (Campos, 1983a) that the vertical velocity W at altitude z, for a wave of frequency ω and horizontal wavevector $\mathbf{k}$, satisfies

$$L^2(1 - \beta e^{-\theta z})W'' - LW' - \{k^2L^2(1 - \beta e^{-\theta z}) + (\omega/2\omega_2)^2 + (k\omega_1/\omega)^2\}W = 0, \tag{2}$$

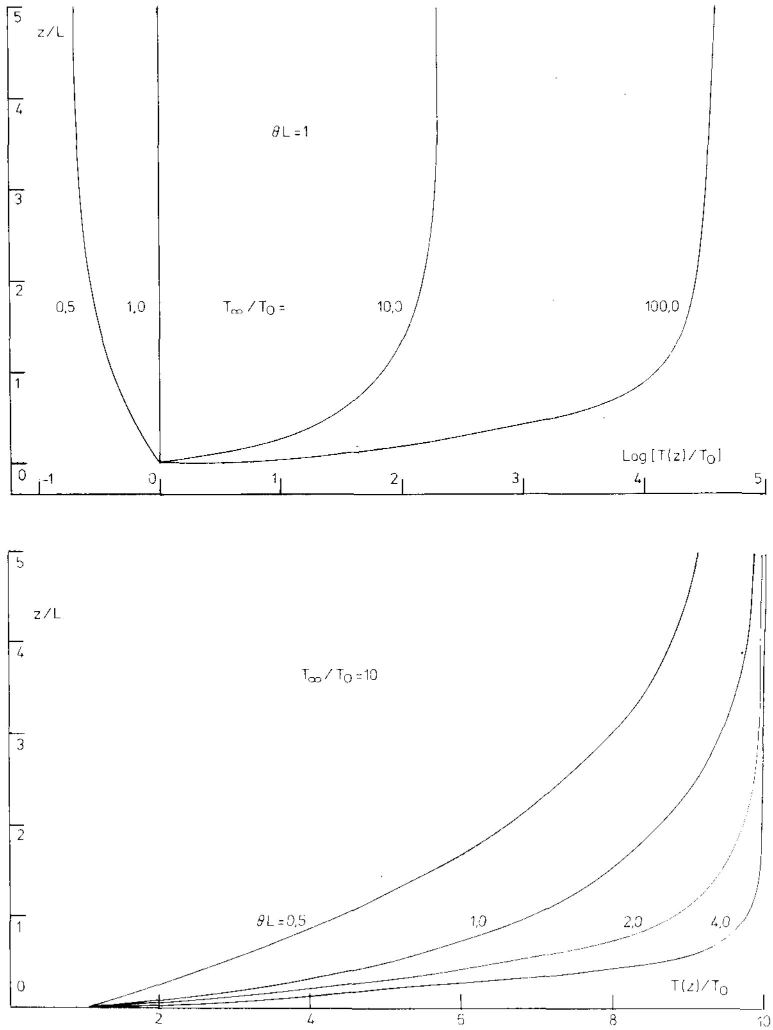

Fig. 1. Temperature profiles for a range of atmospheric models (1) with four ratios of asymptotic to intitial temperature (top) and four values of steepness parameter (bottom).

where $L = RT_\infty/g$ is the asymptotic scale height, $\beta = 1 - T_0/T_\infty$ a measure of the non-isothermality of the temperature profile, and ω_1, ω_2 denote the cut-off frequencies:

$$\omega_1 \equiv (g/c_\infty)\sqrt{(\gamma - 1)}, \qquad \omega_2 \equiv c_\infty/2L, \tag{3a, b}$$

where $c_\infty \equiv \sqrt{(\gamma R T_\infty)}$ is the asymptotic sound speed and γ the ratio of specific heats. Propagation is only possible below ω_1 (gravity mode) or above ω_2 (acoustic mode), since then the vertical wavenumber

$$K \equiv \{(\omega^2/\omega_2^2 - 1)/4L^2 + k^2(\omega_1^2/\omega^2 - 1)\}^{1/2} \tag{4}$$

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is real, and the wave velocity field is given by

$$W(z;\mathbf{k},\omega)=W(0;\mathbf{k},\omega)e^{z/2L}e^{iKz}\{E(z;\mathbf{k},\omega)/E(0;\mathbf{k},\omega)\}\,, \tag{5a}$$

where: (i) the first three factors are the initial wave velocity, the exponential amplitude growth and linear phase increase as for a Lamb wave in an isothermal atmosphere; (ii) the factor in curly brackets, which involves the hypergeometric function

$$E(z;\mathbf{k},\omega)\equiv F(-1/(2\theta L)-k/\theta-iK/\theta,\,-1/(2\theta L)+k/\theta-iK/\theta;\; 1-2iK/\theta;\,\beta e^{-\theta z})\,, \tag{5b}$$

reduces to unity in the isothermal case $\beta=0$, and otherwise concentrates all the effects of the non-uniform temperature profile.

It is clear from (5b) that the Lamb's wave (first three factors in (5a)) will be most modified in the region $e^{-\theta z}\sim 1$ of larger temperature gradient. At high altitude, as $z\to\infty$ and $e^{-\theta z}\to 0$, the hypergeometric function in (5b) tends to unity $E(\infty;\mathbf{k},\omega)=1$, so that Lamb's wave is regained (5a), with an extra constant factor $\{E(0;\mathbf{k},\omega)\}^{-1}$, which corresponds to a constant amplitude factor $|E(0;\mathbf{k},\omega)|^{-1}$ and a phase shift $-\arg\{E(0;\mathbf{k},\omega)\}$. This is illustrated for vertical waves (with horizontal wavenumber $k=0$) in Figure 2, where the logarithm of the ratio of the velocity spectrum at altitude z to the initial velocity $V\equiv\log\{W(z;0,\omega)/W(0;0,\omega)\}$ is plotted against dimensionless altitude z/L, for four values of the compactness parameter $\varepsilon\equiv kL=0.5,1,2,5$, corresponding to wavelengths $\lambda\equiv 2\pi L/\varepsilon$ larger than or comparable to the scale height: (left) the real part of V, which is the logarithm of the ratio of amplitudes, shows that waves grow exponentially in the asymptotic regime (straight lines for $z/L\to\infty$), but growth is faster than exponential in the region of larger temperature gradients ($z/L<2$); (right) the imaginary part of V, which is the difference in phase (or argument) between the wave spectrum $W(z;0,\omega)$ at altitude z and the initial value $W(0;0,\omega)$, increases linearly with altitude in the asymptotic regime (like Kz for $z/L\to\infty$), but the phase increases faster in the region of larger temperature gradient. The faster than exponential amplitude growth and greater than linear phase increase with altitude are more noticeable when propagating through positive temperature gradients, for higher frequency waves (smaller kL); negative temperature gradients can be shown (Campos, 1983a) to have the reverse effect.

In general, for any atmosphere (isothermal or not) with bounded asymptotic temperature, acoustic-gravity waves grow exponentially in amplitude and increase linearly in phase in the asymptotic regime; the growth is faster in positive temperature gradients and smaller in negative ones, the effect being more noticeable for higher frequency waves. Thus the amplitude and phase as function of altitude are similar in the asymptotic regime for waves in isothermal or non-isothermal atmospheres with bounded temperature; the effect of temperature gradients at lower altitudes adds up to a constant amplitude factor and phase shift which will be larger for high frequency waves.

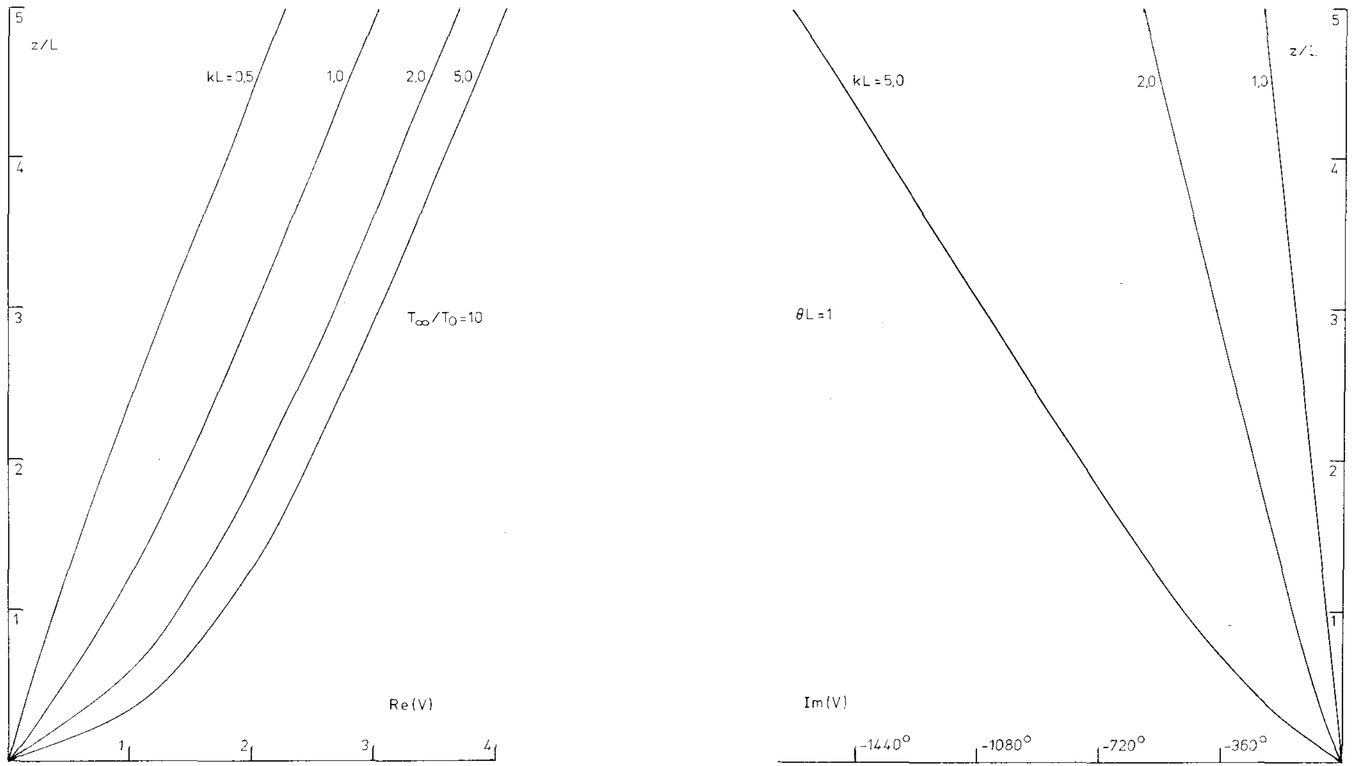

Fig. 2. Acoustic-gravity waves propagating (5a, b) vertically in the non-isothermal atmosphere of Figure 1. The logarithm of the ratio of amplitudes (left) and the phase shift (right) for the velocity perturbation at altitude z relative to initial value at altitude $z = 0$ are plotted against (dimensionless) altitude (divided by the scale height). Four values of the compactness parameter KL are considered.

3. Alfvén-Gravity and Magnetosonic-Gravity Waves

The effectsof an external magnetic field will be exhamined in the case of vertical waves, for which the wave fields depend only on altitude z and time t, but not on horizontal coordinate $\mathbf{x}$ (this is equivalent to vanishing horizontal wavevector $\mathbf{k} = 0$). We consider in the first instance a constant, vertical external magnetic field H_z, in which case (Campos, 1983b) the only propagating components of the velocity $\mathbf{v}$ and magnetic field $\mathbf{h}$ perturbations are horizontal and parallel, viz., along the x-axis, and satisfy

$$\partial^2 v_x/\partial t^2 - C_1^2\,\partial^2 v_x/\partial z^2 = 0 = \partial^2 h_x/\partial t^2 - \partial\{C_1^2\,\partial h_x/\partial z\}/\partial z\,, \tag{6a, b}$$

where C_1 denotes the Alfvén speed, which depends strongly on altitude, e.g., for an isothermal atmosphere under a constant magnetic field H it increases exponentially on twice the scale height for density:

$$C_1(z) \equiv \mu H/\sqrt{4\,\pi\rho(z)} = \{\mu H/\sqrt{4\,\pi\rho_0}\}\,\exp(z/2L)\,. \tag{7}$$

Thus, whereas in an homogeneous medium the Alfvén speed is constant, and the velocity and magnetic field perturbations satisfy the same equation (6a), in an atmosphere the Alfvén speed depends on altitude and the velocity and magnetic fields obey different laws, viz., (6b) has relative to (6a) an extra term $-2\,C_1\,\mathrm{d}C_1/\mathrm{d}z\,\partial h_x/\partial z$.

In any atmosphere for which the density vanishes at high altitude, viz. $\rho \to 0$ as $z \to \infty$, the Alfvén speed diverges $C_1 \to \infty$, and thus from (6a) $\partial^2 v_x/\partial z^2 \to 0$ and (6b) $\partial h_x/\partial z \to 0$, so that the velocity perturbation grows linearly and the magnetic field perturbation is asymptoticaly constant:

$$\begin{aligned} v_x(z;\,\omega) &\sim \{a(\omega)z + b(\omega)\}\,v_x(0;\,\omega)\,, \\ h_x(z;\,\omega) &\sim i(H_z/\omega)\,a(\omega)\,v_x(0;\,\omega)\,, \end{aligned} \tag{8a, b}$$

for propagating Alfvén-gravity waves of frequency ω (we have used the induction equation $\partial h_x/\partial t + H_z\,\partial v_x/\partial z = 0$ to derive (8b) from (8a)). This result can be checked for an isothermal atmosphere (7), in which case (6a, b) can be solved exactly to yield the velocity and magnetic field perturbations at all altitudes (Campos, 1983b):

$$v_x(z;\,\omega) = v_x(0;\,\omega)\,\{H_0^{(2)}((2\omega L/c_1)e^{-z/2L})/H_0^{(2)}(2\omega L/c_1)\}\,, \tag{9a}$$

$$h_x(z;\,\omega) = i(H_z/c_1)\,v_x(0;\,\omega)e^{-z/2L}\,\{H_1^{(2)}((2\omega L/c_1)e^{-z/2L})/H_0^{(2)}(2\omega L/c_1)\}\,, \tag{9b}$$

where c_1 is the Alfvén speed at altitude $z = 0$, and $v_x(0;\,\omega)$ the initial velocity perturbation for a wave of frequency ω. It can be checked that the asymptotic forms of (9a, b) for large altitude z are (8a, b) with:

$$H_0^{(2)}(2\omega L/c_1) \times \{a(\omega), b(\omega)\} = 2i/\pi L,\ 1 - i2\phi/\pi - i(2/\pi)\log(2\pi L/c_1)\,, \tag{10a, b}$$

where ϕ is Euler's constant, and $H_n^{(2)}$ the Hankel function of second kind order n.

The boundary conditions used to determine the constants of integration in the solution of (6a) are: (i) the initial velocity perturbation $v_x(0;\,\omega)$ for a wave of frequency ω at altitude $z = 0$; (ii) the velocity perturbation at low altitude and high-frequency corresponds to an upward propagating wave, i.e., scales on $\exp\{i\omega(z/c_1 - t)\}$. The wave fields (9a, b) are thus determined from the condition, which appears relevant to the solar case, that we know the velocity perturbation of upward propagating waves of all frequencies at a given level $z = 0$, e.g., in the photosphere. The wave fields (9a, b) are plotted in dimensionless form:

$$V \equiv v_x(z;\,\omega)/v_x(0;\,\omega)\,, \qquad H \equiv c_1 h_x(z;\omega)/H_z v_x(0;\omega)\,, \tag{11a, b}$$

against dimensionless altitude z/L in Figures 3 and 4, the former for amplitudes (or modulus of (11a, b)) and the latter for phase shifts (arguments of (11a, b)). Four values of the compactness parameter $\varepsilon \equiv \omega L/c_1$ are considered, corresponding to wavelengths λ larger than or comparable to the scale height, and density changes by a factor of $\Delta = \exp(\lambda/L) = \exp(2\pi/\varepsilon)$ within a wavelength that are large to moderate. It is clear from Figures 3 and 4 that asymptoticaly as $z \to \infty$: (i) the velocity perturbation grows linearly, faster for higher frequencies; (ii) the magnetic field perturbation is constant asymptoticaly, and smaller for higher frequencies; (iii) the phase difference between the wave fields at altitude z and 0 is finite asymptoticaly as $z \to \infty$, and generally larger for high frequency waves.

Besides propagating waves, which can transport energy from one atmospheric region to another, we also consider standing modes, which appear as atmospheric oscillations. The boundary conditions for a wave trapped between the atmospheric layers $z = 0$ and $z = a$ are $v_x(0;\,\omega) = 0 = v_x(a;\,\omega)$; as $a \to \infty$ we find that the wave reflected from infinity cannot give a zero but only a finite amplitude, i.e., a 'node at infinity' corresponds to a finite, non-zero velocity perturbation. The solution of (6a) which vanishes at $z = 0$ and is finite at infinity corresponds to standing modes with frequencies and wavelengths:

$$\omega_n = c_1 j_n/2L\,, \qquad \lambda_n = 4\pi L/j_n\,, \tag{12a, b}$$

where j_n are the roots of the Bessel function $J_0(j_n) = 0$. The wave fields for the nth mode are given by the velocity and magnetic field perturbations:

$$v_x(z,\,t) = (\pi c_1/2L)\,\mathrm{Im}\,\{v_0(\omega_n)e^{-i\omega_n t}\}\,\{J_0(j_n e^{-z/2L})/J_1(j_n)\}\,, \tag{13a}$$

$$h_x(z,\,t) = (\pi H_z/2L)\,\mathrm{Re}\,\{v_0(\omega_n)e^{-i\omega_n t}\}\,\{J_1(j_n e^{-z/2L})/J_1(j_n)\}\,, \tag{13b}$$

where $v_0(\omega)$ is the spectrum at altitude $z = 0$ and frequency ω. The first four standing modes of vertical Alfvén-gravity waves in an isothermal atmosphere are illustrated in Figure 5, showing that asymptoticaly: (i) the velocity perturbation is finite but non-zero, increasing with the order of the mode; (ii) the magnetic field perturbation decays exponentially to zero.

Having considered two magneto-acoustic-gravity wave modes, namely, an acoustic-gravity wave (Section 2) and an Alfvén-gravity wave (Section 3), we now turn to a mode coupling compressibility and magnetism (besides gravity). This is a vertical magneto-sonic-gravity wave, corresponding to a constant, horizontal external magnetic field H_x, which propagates an horizontal, parallel magnetic field perturbation h_x and a vertical

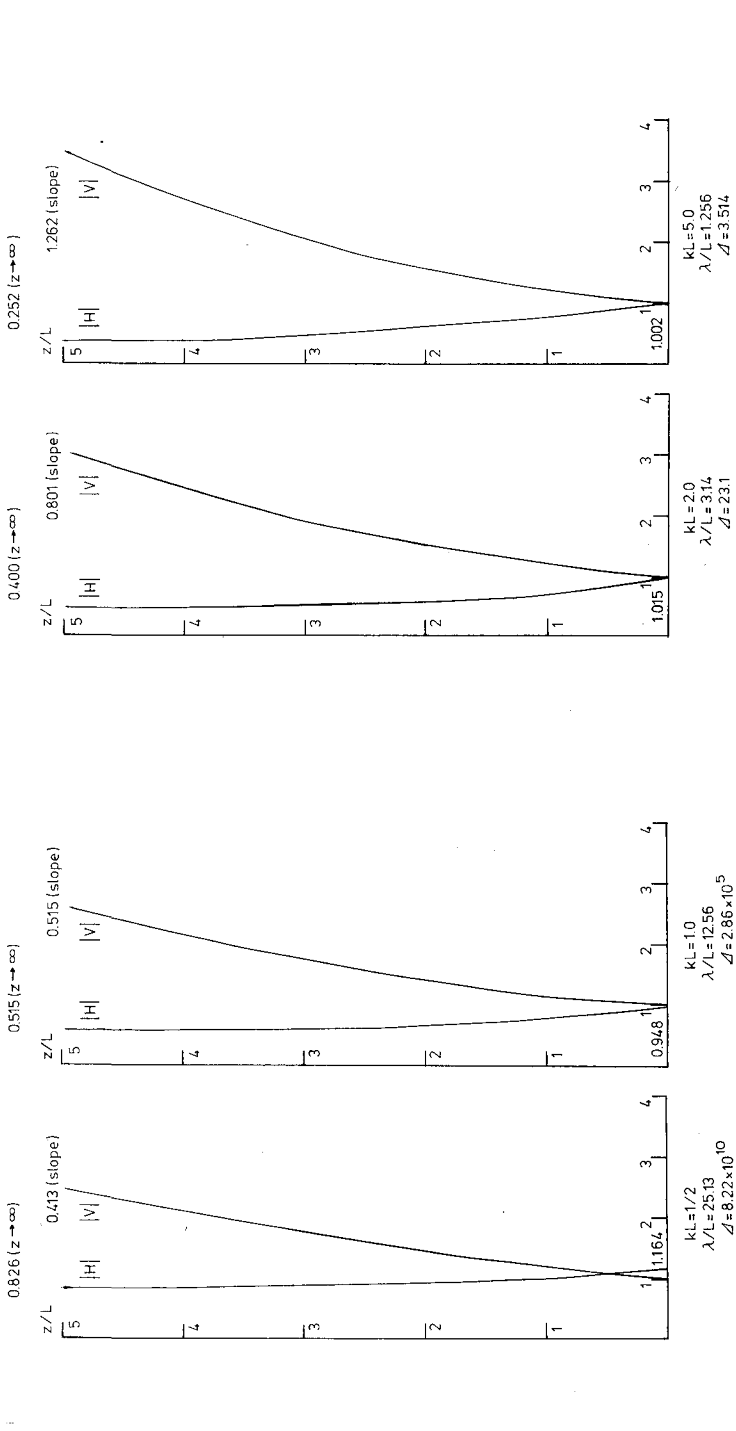

Fig. 3. Alfvén-gravity waves propagating vertically (9a, b) in an isothermal atmosphere: dimensionless (11a, b) amplitude of velocity (V) and magnetic field (H) perturbations versus dimensionless altitude, for four values of compactness parameter kL (or wavelength λ or density change Δ within a wavelength).

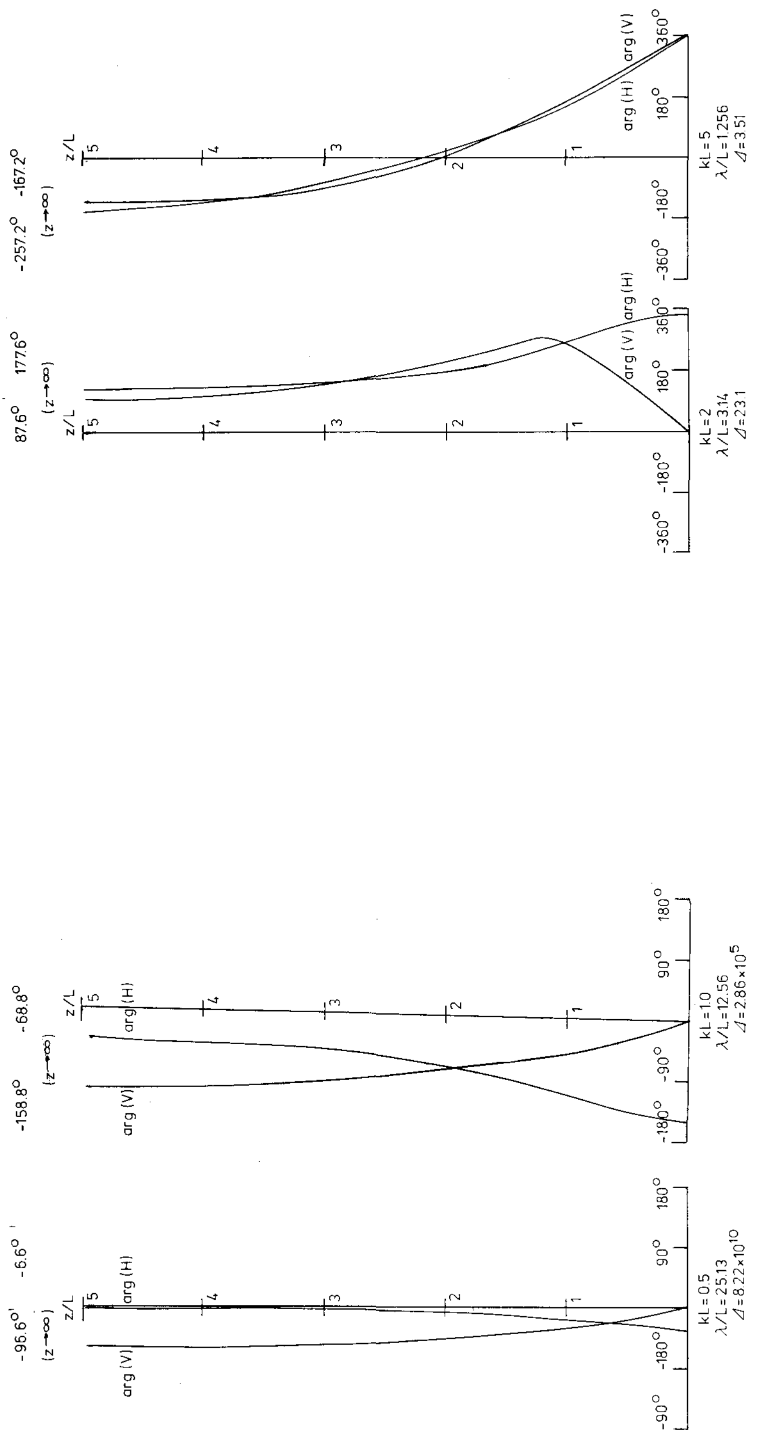

Fig. 4. As for Figure 3: difference in phases.

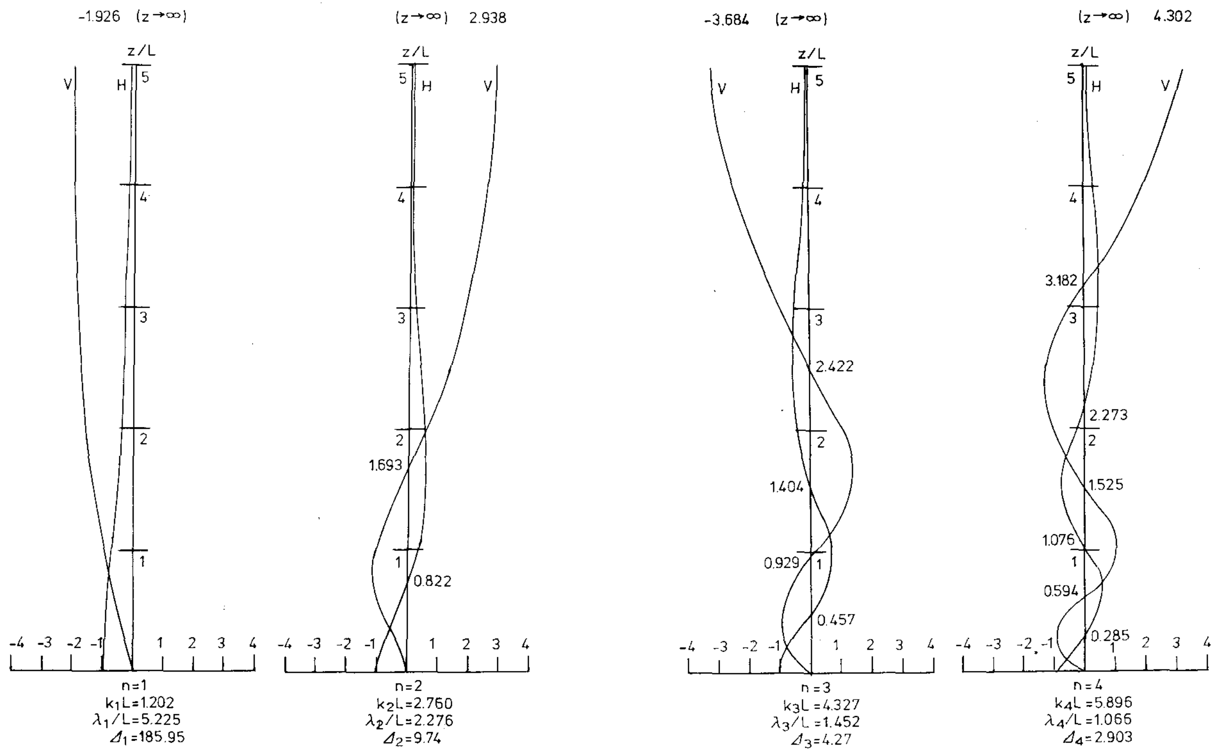

Fig. 5. Velocity (V) and magnetic field (H) waveforms for the first four standing modes (13a, b) of Alfvén-gravity waves in an isothermal atmosphere.

velocity perturbation v_z. The wave equations

$$\partial^2 v_z/\partial t^2 - (C_0^2 + C_1^2)\partial^2 v_z/\partial z^2 + \gamma g\,\partial v_z/\partial z = 0\,, \tag{14a}$$

$$\partial^2 h_x/\partial t^2 - \partial\{(C_0^2 + C_1^2)\,\partial h_x/\partial z\}/\partial z + \gamma g\,\partial h_x/\partial z = 0\,, \tag{14b}$$

where the sound $C_0(z)$ and Alfvén $C_1(z)$ speeds generally depend on altitude, can be used to derive asymptotic laws, and can be integrated exactly in terms of hypergeometric functions in the isothermal case. It can be shown (Campos, 1983b) that there exist two altitude ranges, separated by a transition layer, defined by the equality of sound and Alfvén speeds $C_0(z_*) = C_1(z_*)$: (i) below the transition layer the gas pressure predominates over the magnetic pressure, and the magnetosonic-gravity waves ressemble acoustic-gravity waves, which apart from modifications due to the magnetic field, exhibit exponential amplitude growth, whether standing or propagating; (ii) above the transition layer the magnetic pressure predominates, and the magnetosonic-gravity waves ressembles, apart from modifications due to compressibility, an Alfvén-gravity wave, exhibiting: (a) asymptoticaly finite velocity and decaying magnetic field for standing modes; (b) linearly diverging velocity and asymptoticaly constant magnetic field for propagating waves.

4. Dissipation by Viscosity or Electrical Resistance

As a first instance of dissipation of atmospheric waves we consider the effect of viscosity on a vertical acoustic-gravity wave. The velocity perturbation is vertical and satisfies the equation (Campos, 1983c)

$$\partial^2 v_z/\partial t^2 - C_0^2\,\partial^2 v_z/\partial z^2 + \gamma g\,\partial v_z/\partial z = \nu\,\partial^3 v_z/\partial z^2\,\partial t\,, \tag{15}$$

where the sound speed C_0 and kinematic viscosity ν depend generally on altitude. In an isothermal atmosphere the former is constant, and if the static viscosity is also constant, the kinematic viscosity varies inversely with density, i.e., grows exponentially with altitude, and (15) can be solved in terms of hypergeometric functions. The appearance of hypergeometric solutions once more (they also apply to magnetosonic-gravity waves) is associated with the fact that these are the simplest special functions with three singularities: (i) at $z = 0$, corresponding to the initial wave field; (ii) at $z = \infty$, corresponding to the asymptotic regime; (iii) at a transition or reflecting layer $z = z_*$ which separates two altitude ranges. In the case of viscous acoustic-gravity waves the reflecting layer is determined by the condition $\omega C_0(z) = \nu(z)$, so that it depends on frequency, and below it waves grow exponentially whereas above they tend to a finite asymptotic value.

For any atmosphere, isothermal or not, with asymptoticaly vanishing density $\rho \to 0$ for $z \to \infty$, the kinematic viscosity diverges $\nu \to \infty$ (if the static viscosity does not vanish), and it would appear from (15) that $\partial^2 v_z/\partial z^2 \to 0$, so that the velocity perturbation would grow linearly. However, this would correspond to $\partial v_z/\partial z$ asymptoticaly constant, so that the rate of dissipation of energy by viscosity, which scales on $(\partial v_z/\partial z)^2$ would be infinite when integrated over an atmospheric column from $z = 0$ to $z = \infty$, which is physically absurd since energy can be supplied to the waves only at finite rate.

The condition of finite dissipation rate (Yanowitch, 1967; Lyons and Yanowitch, 1974)

$$\int_0^\infty |\partial v_z(z;\,\omega)/\partial z|^2\,\mathrm{d}z < \infty\,, \tag{16}$$

implies that the velocity perturbation is asymptoticaly bounded for standing modes and propagating waves.

Having mentioned viscous dissipation of an hydrodynamic wave, we now turn to Ohmic resistive dissipation of Alfvén-gravity waves. In this case (16) is replaced by a condition of finite rate of dissipation by Joule effect, i.e., it holds with $\partial v_z(z;\,\omega)/\partial z$ replaced by the electric current $j(z;\,\omega)$ which scales on $\partial h_x(z;\,\omega)/\partial z$, where h_x is the magnetic field perturbation. The velocity v_x and the magnetic field h_x perturbations satisfy (Campos, 1983c):

$$\partial^2 v_x/\partial t^2 - C_1^2\,\partial^2 v_x/\partial z^2 - C_1^2\,\partial\{\chi\,\partial(C_1^{-2}\,\partial v_x/\partial t)/\partial z\}/\partial z = 0\,, \tag{17a}$$

$$\partial^2 h_x/\partial t^2 - \partial(C_1^2\,\partial h_x/\partial z)/\partial z - \chi\,\partial^3 h_x/\partial t\,\partial z^2 = 0\,, \tag{17b}$$

which reduces to (6a, b) when the electrical diffusivity vanishes $\chi = 0$. The electrical diffusivity χ is approximately independent of density, and thus can be taken as a constant in an isothermal atmosphere, in which case the Alfvén speed is given by (7). The exact solution of (17a, b) appears again in terms of hypergeometric functions, so that there is a transition layer, specified by $\omega C_1(z) = \chi(z)$, separating: (i) a low altitude region where diffusion predominates, and the velocity and magnetic fields have a wavenumber k given by $k^2 = \omega/2\chi$; (ii) a high-altitude region where propagation predominates and the amplitude and phase laws are similar to those for non-dissipative hydromagnetic waves (Section 3).

It may be concluded that exponential amplitude growth is the exception rather than the rule for atmospheric waves, since it applies only to acoustic-gravity waves (standing or propagating), and is modified: (i) in the presence of an external magnetic field, with or without resistance, to linear growth for propagating waves and asymptoticaly finite,

TABLE I

Asymptotic laws[2] for the amplitude[1] of atmosphere waves

Type	Standing mode	Propagating wave
Acoustic-gravity	Exponential growth	Exponential growth
Alfvén-gravity	Finite, non-zero[3]	Linear growth[4]
Magnetosonic-gravity	Finite, non-zero[3]	Linear growth[4]
Viscous acoustic-gravity	Finite, non-zero	Finite, non-zero
Resistive Alfvén-gravity	Finite, non-zero[3]	Linear growth[4]

[1] The phase shift for propagating waves is linear in the acoustic-gravity case and asymptoticaly finite and non-zero in all other cases.
[2] These asymptotic laws apply to the velocity perturbation.
[3] The magnetic field perturbation decays exponentially.
[4] The magnetic field perturbation is asymptoticaly finite, non-zero.

non-zero amplitude for standing modes; (ii) in the presence of dissipation by viscosity to finite asymptotic amplitude for standing modes or propagating waves. The magnetic field perturbation (for magnetic and coupled modes) is asymptoticaly finite for propagating waves and decaying for standing modes. These results are summarized in the Table I, and apply to waves in isothermal and non-isothermal atmospheres with asymptoticaly bounded temperature and vanishing density. A consequence for all atmospheric magnetic or coupled modes is: (i) the kinetic energy $E = \rho v^2/2$ (per unit volume) tends asymptoticaly to zero, since the density decays (exponentially) faster than the square of velocity may grow (algebraically); (ii) the magnetic energy $G = \mu h^2/8\pi$ (per unit volume) decays to zero for some standing modes, but for propagating waves is asymptoticaly bounded. Thus a magnetic or coupled (magneto-acoustic) wave propagating upward in an atmosphere violates the equipartition of energy which holds in an homogeneous medium, since asymptoticaly all energy is magnetic.

5. Shock Fomation and Atmospheric Heating

As an indication of the relevance of these results to solar physics we recall that it is generally accepted that waves are generated in the solar photosphere; these consist (Campos, 1977) of three magneto-acoustic-gravity wave modes, and it is often argued that they grow into shocks during propagation upward into the chromosphere. If waves grow exponentially, then, even if the initial amplitude is small, after a few scale heights, the non-linear effects which lead to shock formation come into play. It should be borne in mind, however, that in the solar atmosphere both magnetic fields and dissipation mechanisms are present, and thus waves may grow only linearly or have bounded amplitude. In these cases it is necessary to calculate, for the particular wave mode and in the physical conditions considered, whether waves grow enough for shocks to form within the height range of interest or: (i) shock formation is delayed by the magnetic field until higher altitude; (ii) the dissipation mechanisms limit amplitude to a level at which non-linear effects are small and shocks do not form.

We have concerned ourselves with the properties of waves in non-isothermal, compressible, viscous and ionized atmospheres, and leave the detailed application to solar phenomena (Osterbrock, 1961; Uchida, 1968; Meyer, 1968; Hollweg, 1972; Mein and Mein, 1980) to subsequent works. We do however mention another consequence of these results, concerning the calculation of atmospheric heating functions, specifying the rate of deposition of wave energy with altitude. An accurate law of variation of wave amplitude and phase with altitude is necessary for the computation of the heating function, which in turn is critical for the establishment of solar atmospheric models. In this respect the W.K.B.J. approximation is of limited use, since it assumes sinusoidal waveforms and linear phases, and does not apply to wavelengths comparable to or larger that the scale height nor does it hold after several scale heights. Instead an exact theory should be used, applying to all frequencies and altitudes, yielding amplitude and phase laws from which the heating functions can be calculated as input to solar atmospheric models.

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