Where does the book fit? It's fine for a U.S. graduate program but harder to place in the U.K. or Australia. Much of it would be accessible to a good third or fourth year undergraduate and I would love to see all mathematicians learning measure theory in this way. In practice, however, the case for *any* measure theory (as opposed to integration theory) can be hard to argue particularly when statisticians often settle for discrete random variables at undergraduate level. A graduate student in analysis or mathematical statistics would find "Probability and Measure" suitable for self-study and highly rewarding. I recommend it strongly. (The text itself is horribly disfigured by broken formulae on page after page but appears remarkably free from misprints and errors.)

GAVIN BROWN

SAH, C.-H., *Hilbert's third problem: scissors congruence* (Pitman, 1979), pp. 240, £9·95.

If two planar polygons have the same area then one of them can be cut into triangular pieces which can be rearranged to cover the other polygon exactly. This fact, which is the two dimensional version of Hilbert's third problem, is usually attributed to F. Bolyai (1832) and to P. Gerwein (1833). It is pointed out in this book that the problem was solved about twenty years earlier by William Wallace (who was later a Professor at Edinburgh). The above problem was posed by Wallace as Question 269 in volume 3 of the new series of Leybourn's Mathematical Repository in 1814 and the printed solution is by Lowry. The solution given is simple and elegant. Gauss considered the three dimensional version of the problem and pointed out that to prove that two prisms with the same base and equal altitudes have the same volume it seems essential to use an infinite process of some kind. Hilbert's third problem asked for a rigorous justification of Gauss's assertion. An attempt at such a proof had already been made by R. Bricard in 1896 but Hilbert's publicity of the problem gave rise to the first correct proof—that by M. Dehn appeared within a few months. The third problem was thus the first of Hilbert's problems to be solved. Although several improvements and clarifications of Dehn's proof have appeared, this prompt solution seems to have led mathematicians to regard the third problem as rather an uninteresting one. Indeed, the problem seems not to have been discussed at all at the American Mathematical Society's 1974 Symposium on Hilbert's problems.

The book under review seems to be the third book entirely devoted to this problem. The previous ones were both written by V. G. Boltianskii (the first was published in Russian in 1956 and in English in 1963, the second and larger book in Russian in 1972 and the English translation was published by J. Wiley in 1978). Someone who wants a quick, clear and elementary account of the problem and its solution should read one of Boltianskii's books; indeed the books are probably accessible to a bright sixteen year old. Someone who wants to find a research problem in geometry might profitably read Sah's book.

The solution of the problem can be explained as follows. If P is a polyhedron in $\mathbb{R}^3$ whose edges have lengths $l_1, l_2, \ldots, l_n$ and whose corresponding dihedral angles are $\theta_1, \theta_2, \ldots, \theta_n$, define its Dehn invariant $d(P)$ to be $\sum l_i \otimes \theta_i$ whose values are in $\mathbb{R} \otimes (\mathbb{R}/\pi\mathbb{Z})$. Two polyhedra P_1, P_2 are equidecomposable if each can be cut into pieces $P_{i1}, \ldots, P_{ir}$ with P_{1j} congruent to P_{2j}. It is not hard to check that if P_1, P_2 are equidecomposable then their Dehn invariants are equal. To give an example for Hilbert's problem it is enough therefore to find two polyhedra P_1, P_2 with equal volume but $d(P_1) \neq d(P_2)$. Two such are the cube and the regular tetrahedron of unit volume. They have Dehn invariants $12 \otimes (\pi/2) = 0$ and $12.\sqrt[3]{3} \otimes \cos^{-1}(1/3) \neq 0$ respectively. In 1965, J.-P. Sydler proved that two polyhedra in $\mathbb{R}^3$ are equidecomposable if and only if they have the same volume and the same Dehn invariant. This would seem to be the final word on Hilbert's third problem in its original form.

Sah's book discusses many variants of the original problem and ties them in with other parts of mathematics. He starts by giving an axiomatic and abstract framework in which to discuss the problem and its variants. These variants are many: the real numbers are replaced by as general a field as possible, the n-dimensional version is treated, the affine, euclidean, hyperbolic and spherical cases and even versions for more general manifolds are handled. As well as the usual congruence in space, equivalence of polyhedra under various restricted groups of transformations (such as translations) are considered. All these possibilities give a large number of problems and many of them have not been solved yet. I found some of these variants more interesting than I had expected at a first glance. The treatment of these problems given in this account is rather algebraic and the original geometry is sometimes hard to find. The main interest of Sah's account is that he constantly tries to make connections with other topics in mathematics. Some of these connections may turn out to be fruitful. Among the subjects that arise are group representations, Hopf algebras and the cohomology of groups. As an example of how a simple geometric problem can be generalised and abstracted this book could hardly be beaten.

ELMER REES

STRĂTILĂ, S. and ZSIDÓ, L., *Lectures on von Neumann algebras* (Abacus Press, 1979), pp. 478, £29·95.

Although the study of operator algebras commenced in the 1920's, it was not until 1957 that the first comprehensive monograph on the subject, J. Dixmier's "Les algèbres de'opérateurs dans l'espace hilbertien", appeared, to be followed in 1964 by the same author's "Les C^*-algèbres et leurs représentations" (both Gauthier-Villars, Paris). In 1971 a unified, if idiosyncratic, treatment was given by Sakai in his "C^*-algebras and W^*-algebras" (Springer-Verlag, Berlin). Since these works appeared there has been considerable and spectacular progress in understanding the structure of von Neumann algebras, largely as a result of the formulation in the late 1960's of the Tomita-Takesaki theory of modular Hilbert algebras. Until recently there was no readily accessible exposition of the theory of von Neumann algebras which included the Tomita-Takesaki theory. The situation has now changed dramatically with the publication, in the last year or so, of works by Bratteli and Robinson, Pedersen, Takesaki, and the volume under review.

In their book Strătilă and Zsidó, unlike some of the other authors just mentioned, concentrate on von Neumann algebras; their aim is to give a clear and self-contained exposition of the theory up to and including the Tomita-Takesaki results. They have, in many ways, been successful in this. The first eight chapters are devoted to a careful presentation of the classical, i.e. pre-Tomita-Takesaki, theory, the approach being in the Murray–von Neumann tradition: underlying Hilbert spaces are always clearly in view, and comparison of projections is to the fore. This approach is, in my opinion, easier for a newcomer to the subject than the elegant, but less transparent, approach of Dixmier using traces, or the non-spatial formulation of Sakai. The authors write clearly and include concise alternative proofs of certain important results (notably Yeadon's proof of the existence of a trace in a finite von Neumann algebra).

Chapter 9 prepares the ground for the Tomita-Takesaki theory with a systematic study of unbounded linear operators on Hilbert space. The chapter contains much useful material in a very accessible form. As well as presenting the standard results, such as polar decomposition and Stone's theorem, the authors discuss a certain operator equation and its solution in the form of an integral. It is interesting to see this result, the key to the fundamental results of the Tomita-Takesaki theory, set in this wider context.

The lengthy final chapter, entitled "the theory of standard von Neumann algebras", is devoted to an exposition of the Tomita-Takesaki theory. The basic objects in the theory, such as Hilbert algebras and the modular operator, are defined and their properties established. It is shown that a normal, faithful, semifinite weight on a von Neumann algebra satisfies the KMS condition with respect to a suitable modular automorphism group. Finally, Conne's remarkable unitary cocycle result is proved, and some of its applications given. At the end of each chapter there are exercises and comments amplifying earlier points. The book ends with a short appendix on fixed point theorems, followed by a very up to date bibliography of some 120 pages.

It is a pity that such a substantial portion of the book is given over to the bibliography, as many of the references do not relate directly to matters in the text, and moreover some important topics have been omitted. There is, for example, no treatment of direct integral decomposition. I am also disturbed at the lack of concrete examples of von Neumann algebras of types II and III. Surely any comprehensive treatment of the subject should include a proof, at least as an exercise, that the objects under consideration actually exist (to be fair to the authors, they do give several references). A minor criticism I would make is of the English, which, though always clear, is sometimes not idiomatic. Also, some of the terminology is unconventional, for example "of