Appendix B

Cross sections and traces

B.1 Cross sections

The matrix element $\mathcal{M}$ appearing in cross sections and decay rates is Lorentz-invariant and dimensionless. The expression for the cross section is

$$d\sigma = \frac{1}{|\vec{v}_1 - \vec{v}_2|} \frac{1}{2E_{p_1}} \frac{1}{2E_{p_2}} |\mathcal{M}|^2 (2\pi)^4 \delta^4\left(p_1 + p_2 - \sum_{i=1}^{n} k_i\right) \frac{d^3k_1}{2E_1(2\pi)^3} \cdots \frac{d^3k_n}{2E_n(2\pi)^3} s. \tag{B.1}$$

The flux factor is frequently computed in the laboratory frame or the center-of-mass frame. In general,

$$\frac{1}{|\vec{v}_1 - \vec{v}_2|} = \frac{mM}{\left[(p_1 \cdot p_2)^2 - m^2M^2\right]^{1/2}}. \tag{B.2}$$

The factor s is

$$s = \prod_i \frac{1}{k_i!} \tag{B.3}$$

if there are k_i identical particles of species i in the final state.

The decay width for a particle moving with energy E is

$$d\Gamma = \frac{1}{2E} |\mathcal{M}|^2 (2\pi)^4 \delta^4\left(p - \sum_{i=1}^{n} k_i\right) \frac{d^3k_1}{2E_1(2\pi)^3} \cdots \frac{d^3k_n}{2E_n(2\pi)^3} s. \tag{B.4}$$

B.2 Contraction identities and traces

$$\not{a}\not{b} = 2a \cdot b - \not{b}\not{a}, \tag{B.5}$$

$$\gamma^\lambda \gamma_\lambda = 4, \tag{B.6}$$

$$\gamma^\lambda \gamma^\mu \gamma_\lambda = -2\gamma^\mu, \tag{B.7}$$

$$\gamma^\lambda \gamma^\mu \gamma^\nu \gamma_\lambda = 4g^{\mu\nu}, \tag{B.8}$$

$$\gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\rho \gamma_\lambda = -2\gamma^\rho \gamma^\nu \gamma^\mu, \tag{B.9}$$

$$\gamma^\lambda \gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma \gamma_\lambda = 2(\gamma^\sigma \gamma^\mu \gamma^\nu \gamma^\rho + \gamma^\rho \gamma^\nu \gamma^\mu \gamma^\sigma), \tag{B.10}$$

$$\gamma^\lambda \sigma^{\mu\nu} \gamma_\lambda = 0, \tag{B.11}$$

$$\gamma^\lambda \sigma^{\mu\nu} \gamma^\rho \gamma_\lambda = 2\gamma^\rho \sigma^{\mu\nu}. \tag{B.12}$$

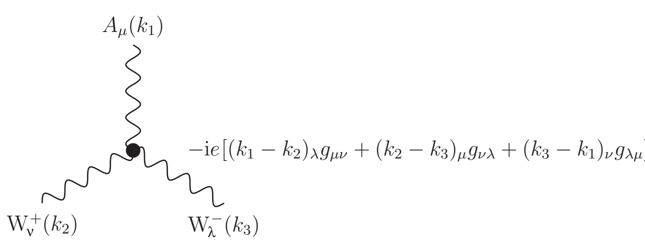
The trace of an odd product of γ^μ-matrices vanishes:

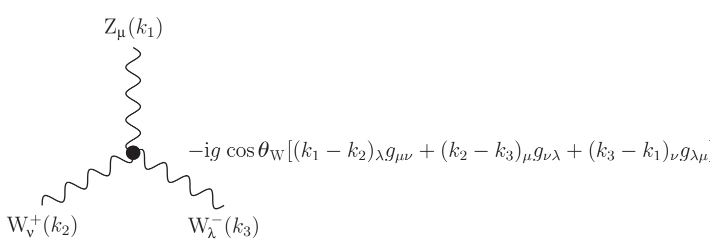
$$\mathrm{Tr}(\gamma^5) = 0 \tag{B.13}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu) = 4g^{\mu\nu} \tag{B.14}$$
$$\mathrm{Tr}(\sigma^{\mu\nu}) = 0 \tag{B.15}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^5) = 0 \tag{B.16}$$
$$\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho}) \tag{B.17}$$
$$\mathrm{Tr}(\gamma^5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = -4\mathrm{i}\varepsilon^{\mu\nu\rho\sigma} = 4\mathrm{i}\varepsilon_{\mu\nu\rho\sigma} \tag{B.18}$$
$$\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n}) = \mathrm{Tr}(\not{a}_{2n}\ldots\not{a}_2\not{a}_1) \tag{B.19}$$
$$\begin{aligned}\mathrm{Tr}(\not{a}_1\not{a}_2\ldots\not{a}_{2n}) &= a_1\cdot a_2\,\mathrm{Tr}(\not{a}_3\ldots\not{a}_{2n}) - a_1\cdot a_3\,\mathrm{Tr}(\not{a}_2\not{a}_4\ldots\not{a}_{2n}) + \cdots \\ &\quad + a_1\cdot a_{2n}\,\mathrm{Tr}(\not{a}_1\ldots\not{a}_{2n-1}) \\ &= 4\sum \varepsilon(a_{i_1}\cdot a_{j_1})\ldots(a_{i_n}\cdot a_{j_n}).\end{aligned} \tag{B.20}$$

ε is the signature of the permuatation $i_1 j_1 \ldots i_n j_n$ and the sum runs over the $(2n)!/(2^n n!)$ different pairings satisfying $1 = i_1 < i_2 < \cdots i_n, i_k < j_k$.

B.3 Some Feynman rules

In the text we gave Feynman rules for several vertices. We present here additional rules for vertices of gauge bosons:





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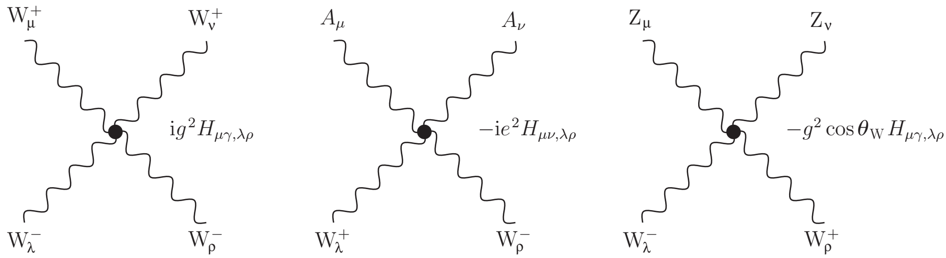

with $H_{\mu\nu,\lambda\rho} = 2g_{\mu\nu}g_{\lambda\rho} - g_{\mu\lambda}g_{\nu\rho} - g_{\mu\rho}g_{\nu\lambda}$. In graphs all momenta are taken to be entering *into* the vertices.

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