ORIGIN OF THE SUN'S DIFFERENTIAL ROTATION

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ABSTRACT. The present status of the Sun's differential rotation theories is reviewed. Attention is mainly focused on mechanisms for differential rotation based on the anisotropic viscosity concept and their modern develop ments within the framework of the mean-field hydrodynamics. The models with latitude-dependent heat transport and non-axisymmetric numerical simulations are briefly discussed.

1. INTRODUCTION

Differential rotation of the Sun is a phenomenon known for more than a century (Carrington, 1863). There exists the so-called equatorial acceleration with a decrease in angular velocity of global rotation with increasing latitude. Extensive observational information on details of this latitudinal dependence and its variation with phase of a solar cycle has been accumulated using both direct Doppler measurements and observations of motions of various tracers. Detailed discussions of these data may be found in recent reviews by Howard (1984) and Schröter (1985).

The purpose of this article is to review the theory of solar differential rotation. Theoretical efforts in this area also have a long history and are stimulated by its own interest and by the well-known relation to solar magnetism. Unfortunately, the rapid increase of observational information and theoretical work did not serve to choose between the known mechanisms for differential rotation the most effective one. Relatively independent models of solar rotation centered around different effects do co-exist nowadays.

Nevertheless, there is almost complete agreement between different approaches in appreciation of the fact that a fundamental cause of the Sun's differential rotation is the interaction between solar convection and rotation.

J. O. Stenflo (ed.), Solar Photosphere: Structure, Convection, and Magnetic Fields, 297–307.

The Rossby number for giant solar convection is smaller than unity (Tyler, 1973; Durney and Latour, 1978), i.e., convection is strongly influenced by Coriolis forces. The back reaction disturbs rotation and makes it differential.

In what follows we will consider mainly the anisotropic viscosity models of differential rotation and their modern develop ments made within the framework of the mean-field hydrodynamics. The models with latitude-dependent heat transport and non-axisymmetric simulations are discussed more briefly. These approaches concentrate the main theoretical efforts and dominant number of papers. Nevertheless, strict volume limitations place some interesting mechanisms for differential rotation beyond the scope of this treatment. Wholly ignored are variations of rotation over a solar cycle and torsional oscillations where theory is only incipient.

2. AXISYMMETRIC THEORIES

The axisymmetric theories consider the steady mean flow which possesses axial symmetry about the rotation axis. Velocity **V** of this flow is a superposition of meridional circulation $V_m = [V_r(r,\theta), V_\theta(r,\theta), 0]$ and rotation $V_R = [0, 0, \Omega(r,\theta) r \sin\theta]$. (Here and below the usual spherical coordinates are used.) The full velocity **v** is the superposition of global flow **V** and random convective velocity **u** with zero mean value: $\mathbf{v} = \mathbf{V} + \mathbf{u}$, $\langle \mathbf{v} \rangle = \mathbf{V}$, $\langle \mathbf{u} \rangle = 0$. The zonal component of the averaged equation of motion

$$\mathrm{div}\left(\rho r \sin\theta \langle u_\varphi \mathbf{u} \rangle + \rho r^2 \sin^2\theta\, \Omega\, \mathbf{V}_m\right) = 0 \quad (1)$$

is the basic equation of theories of differential rotation. Eq.(1) is contributed by Reynolds stress tensor $R_{ij} = -\rho \langle u_i u_j \rangle$. We shall distinguish in the correlation $T_{ij} = \langle u_i u_j \rangle$ the dissipative (D_{ij}) and non-dissipative (Λ_{ij}) parts

$$T_{ij} = D_{ij} + \Lambda_{ij} \quad (2)$$

The tensor D_{ij} is linear in spatial derivatives of angular velocity and represents the contribution of eddy viscosities. We accept the simple expression for the tensor D_{ij}:

$$D_{ij} = -\nu_T \left(\partial V_i / \partial r_j + \partial V_j / \partial r_i - \tfrac{2}{3} \delta_{ij}\, \mathrm{div}\, \mathbf{V}\right) \quad (3)$$

The simplifications connected with (3) are not essential for the qualitative discussion to follow.

The term Λ_{ij} in (2) represents nondissipative contributions which do not depend on spatial derivatives of angular velocity. Eq. (1) now reads

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$$div(\rho \nu_T \sin^2\theta\, r^2 \nabla\Omega) = \nabla_j (\rho r \sin\theta\, \Lambda_{\varphi j}) + \\ + div(\rho r^2 \sin^2\theta\, \Omega\, \underset{\sim}{V}_m) . \qquad (4)$$

The left-hand side of Eq. (4) describes viscouos damping of rotational inhomogeneity. If (4) includes only this term, the solution would be a rigid-body rotation, Ω = const, for any reasonable boundary conditions.

However, the difference from zero of the right-hand side of (4) prevents such a rotation from being solution of this equation. In other words, the right-hand side of (4) displays the sources of differential rotation. Two different terms represent two sources of a different nature.

First, convective motions can transport angular momentum and create inhomogeneous rotation. The quantity $\rho \Lambda_{\varphi j}$ in (4) is the nondissipative part of the j-th component of convective flux $\rho\langle u u_\varphi \rangle$ of angular momentum. Second, meridional circulation may also serve as a transporter of angular momentum; the quantity $\rho V_\varphi V_m = \rho r \sin\theta\, \Omega V_m$ being the corresponding flux.

The steady meridional flow V_m satisfies the continuity equation, $div\, \rho V_m = 0$, and can be characterized by a single stream function. The scalar equation for meridional circulation can be obtained by taking the zonal component of curl of the averaged equation of motion (Kippenhahn, 1963)

$$D(V_m) = r \sin\theta\, \partial\Omega^2/\partial z + \frac{1}{\rho^2} (\nabla\rho \times \nabla P)_\varphi \qquad (5)$$

where $\partial/\partial z = \cos\theta\, \partial/\partial r - (\sin\theta/r)\partial/\partial\theta$ is spatial derivative along the axis of rotation, $D(V_m)$ signifies the contribution of effective viscosities.

The right-hand side of (5) represents sources of meridional circulation. There are two different sources again. The first term is the nonpotential part of centrifugal force. The second one is brought about by nonpotential pressure force: $-\mathrm{curl}(\nabla P/\rho) = (\nabla\rho \times \nabla P)/\rho^2$.

The full system of equations comprises also equations of state and energy transport. However, they are not needed in the qualitative discussion to follow.

2.1 Models with anisotropic viscosity

Lebedinski (1941) was probably the first to note that anisotropy in the velocity distribution of solar convection should lead to differential rotation. The preferred direction of the anisotropy is singled out by gravity. Hence, it is natural to assume that

$$\langle u_\theta^2 \rangle = \langle u_\varphi^2 \rangle = s \langle u_r^2 \rangle \;, \; s \neq 1 .$$

Influence of the Coriolis force on the anisotropic convection gives rise to convective fluxes of angular momentum.

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Reynolds stresses for slowly rotating turbulent fluids with radial anisotropy were derived by Wasiutinski (1946) and Bierman (1951):

$$R_{r\varphi} = \rho \nu_T \sin\theta \, [r \, \partial\Omega/\partial r + 2\Omega \, (1-s)], \qquad (6)$$
$$R_{\theta\varphi} = \rho \nu_T \sin\theta \, \partial\Omega/\partial\theta$$

where $\nu_T = \tau \langle u_r^2 \rangle$ is the eddy viscosity (τ is a turnover time of convective eddy). Eqs.(6) have been repeatedly used to model solar differential rotation (Kippenhahn, 1963; Sakurai, 1966; Cocke, 1967; Köhler, 1970). Let us consider the basic effects involved in these models. Eqs (6) do not include meridional Λ-effect, i.e., $\Lambda_{\theta\varphi}$=0. Hence, the neglect of meridional circulation in Eq.(4) together with usually imposed boundary conditions of vanishing stress, $R_{r\varphi}$= 0, at the upper and lower boundaries of convection zone would lead to purely radial inhomogeneity of rotation. Hence, the equatorial acceleration can be obtained only with meridional circulation included. Almost all anisotropic viscosity models assume the adiabatic stratification of convection zone (a very important assumption). The term $\nabla\rho \times \nabla P$ in Eq.(5) vanishes in this case. The only cause of meridional flow which remains is the nonpotential part of centrifugal force. It is certainly different from zero for radially-inhomogeneous rotation caused by radial Λ-effect. Therefore, meridional circulation is unavoidable to occur. This circulation redistributes angular momentum and creates the latitudinal inhomogeneity of rotation.

The anisotropic viscosity models were capable of reproducing the observed surface distribution of angular ve-

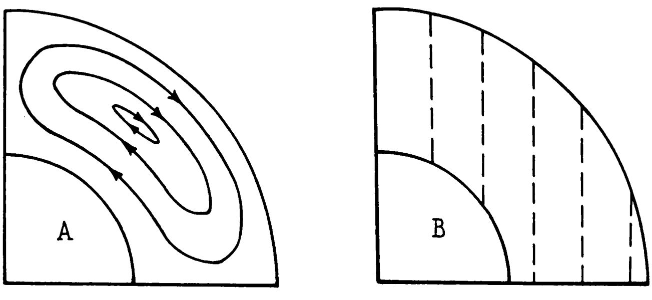

Fig.1. Meridional circulation (A) and isorotational surfaces (B) typical of the anisotropic viscosity models.

locity with the value of the anisotropy parameter s ≃ 1.2. The meridional circulation had a structure shown in Fig.1A. The typical isorotational surfaces are shown in Fig.1B. The angular velocity is nearly constant on cylinder surfaces co-axial with respect to the axis of rotation. Such a distribution is brought about by large values of Taylor number for the Sun, $T = 4\Omega^2 R^4/\nu_T^2 \simeq 3 \cdot 10^7$.

In other words, turbulent viscosity is relatively small and viscous drag cannot stabilize meridional circulation on the level where it does not dominate in the process of angular momentum transport. Under these conditions the circulation is stabilized by exhausting their source. The nonpotential part of centrifugal force provides this source in anisotropic viscosity models. The force is proportional to the spatial derivative of angular velocity along the axis of rotation. Hence, the derivative should be small, and we arive at the distribution of Fig.1B.

This result is the consequence of barytropic stratification. The nonpotential part of centrifugal force can be compensated for by viscous force only when $\nabla\rho \times \nabla P = 0$, and the distribution of Fig.1B applies for the solar case of large Taylor numbersindependently of the particular model accepted. Hence, the isorotational surfaces of the type shown in Fig.1B indicate that curl $\langle \nabla P/\rho \rangle = 0$ for approximations adopted.

Let us consider whether the assumption $\nabla\rho \times \nabla P = 0$ applies for the Sun. The relative value of deviation from barytropy in a rotating convection zone cannot exceed the relative deviation from adiabaticity of stratification which is extremely small ($\lesssim 10^{-}$) except for the thin ($\sim$1000 km) surface layer (Baker and Tamesvary, 1966; Spruit, 1974; Gough and Weiss, 1976). However, the centrifugal force for the Sun is about five orders of magnitude smaller than the force of pressure. Hence, the deviation from barytropy can be neglected if only $|\nabla\rho \times \nabla P| \ll 10^{-5} |\nabla\rho||\nabla P|$. It is rather questinable whether this neglect can be justified for the Sun.

It is natural to anticipate that allowance for deviations from barytropy should suppress meridional circulation. Recent numerical simulations by Schmidt (1982) support this point of view. Schmidt treated explicitly the pressure force in his anisotropic viscosity model. The meridional circulation was very slow and latitudinal inhomogeneity of rotation was very weak for Prandtl number unity and reasonable boundary conditions of constant heat flux at the bottom and black-body radiation at the top of convection zone.

The anisotropic viscosity models are capable of reproducing the observed equatorial acceleration but seem to disagree with other observational data. The global equatorward meridional circulation on the surface of the Sun is not observed (Duvall, 1979; LaBonte and Howard, 1982; Howard, 1984; Tuominen et al., 1983). The distribution of Fig.1B seem to disagree with helioseismology data (Deubner et al., 1979; Duvall et al., 1984,1986; Brown, 1985; Brown end Morrow, 1987). On using Eqs (6) we find for the solar surface

$$\langle u_\theta u_\varphi \rangle \cos\theta = - s \nu_T \sin\theta \cos\theta \, \partial\Omega / \partial\theta \leqslant 0 \, .$$

This contradicts the positive values of the covariance $<u_\theta u_\varphi>\cos\theta$ infered from sunspot statistics (Ward, 1965; Gilman and Howard, 1984).

Nevertheless, these models provide important insights into the problem at hand, and comparison of their results with observations show some promising directions of theoretical progress.

2.2. Mean-field hydrodynamics and differential rotation theory

A natural way of refining the semi-qualitative anisotropic viscosity models seem to be the derivation of equations for global flows from hydrodynamic equations. Such an approach requires these equations to be averaged over an ensemble of realizations of random convective motions and is termed the mean-field hydrodynamics.

It is necessary to know the properties of convection to perform the averagings required. However, the problem of highly nonlinear rotating solar convection is still impossible to solve. Some properties of convection are again assigned from qualitative consideretions. The interaction between convection and rotation is of primary importance for the differential rotation problem. For this reason attemts have usually been made to describe the rotationally-induced properties of convection in a most consistent way. Other properties, not directly related to rotation, are assumed given. Actually, two different turbulences (convections) are considered. One, to be referred to as "original turbulence", would take place in the presence of real sources of turbulence but with no rotation present. Properties of original turbulence do not explicitly depend on rotation and are assumed to be given. The other one, i.e., a real turbulence perturbed by rotation, will be named "background turbulence". The properties of background turbulence are derived from given properties of original turbulence.

The usually used approximations lead to a linear relation:

$$u_i = B_{ij}(\Omega)\, u_j^\circ$$

where **u** and **u°** are velocities for background and original turbulences, respectively; rotational influence is involved through the tensor B_{ij} (Rüdiger, 1977,1989).

The main advantage of the approach discussed here as compared those of preceding Section is the possibility of taking account of nonlinearities in the parameter $\omega = 2\tau\Omega$. (Note that $\omega > 1$ holds for giant solar convection.) Various nonlinear derivations (Iroshnikov, 1966; Rüdiger, 1977, 1982; Vandakurov, 1982; Kichatinov, 1986, 1987) being different in details, lead to the same structure of the nondissipative part, $\Lambda_{j\varphi}$, of velocity covariances:

$$\Lambda_{r\varphi} = \Omega \nu_T [V_0(\Omega) + V_1(\Omega)\cos^2\theta]\sin\theta ,$$
$$\Lambda_{\theta\varphi} = \Omega \nu_T H(\Omega)\sin^2\theta\cos\theta \qquad (7)$$

where V_0, V_1, and H are dimensionless functions.

Comparison of (7) with (6) shows that allowance for nonlinearities in lead to the appearance of meridional Λ-effect ($\Lambda_{\theta\varphi} \neq 0$). This opens the possibility of establishing agreement of the theories with the Ward profile.

In the rapid rotation limit ($\omega \gg 1$), the Λ-effect induced by convection anisotropy is proportional to ω^{-2} (Rüdiger, 1983; Kichatinov, 1986). Hence, the anisotropy of original convection is efficient in generating differential rotation only when ω is of order unity or smaller. It was found recently that not only anisotropy of convection but also inhomogeneity of convection zone can lead to differential rotation (Kichatinov, 1987, 1988). Note that Λ-effect induced by (density) inhomogeneity tends to a constant value in the rapid rotation limit. Therefore, in the case of rapid rotation ($\omega \gg 1$) the inhomogeneity is more effective in generating differential rotation as compared with convection anisotropy. However, the relation of the roles played by stratification and convection anisotropy in generating differential rotation of the Sun is still uncertain.

The developements of differential rotation theory made within the framework of mean-field hydrodynamics seem to be quite promising. Solutions of the equation for angular velocity using the components (7) of the Λ-tensor were found and requirements imposed by observational data upon the functions V_0, V_1 and H of Eqs (7) were determined (Rüdiger, 1989). However, it is still uncertain whether these requirements can be met with an original turbulence having realistic properties.

2.3. Models with latitude-dependent heat transport

The perturbation of convection by Coriolis forces depends on latitude. For this reason, convective heat flux must also be latitude-dependent. This fact was used by Weiss (1965) and Durney and Roxburg (1971) to explain the differential rotation of the Sun. Inhomogeneous heat flux produces latitudinal temperature inhomogeneity. The meridional circulation arises under these conditions and drives differential rotation.

The dependence of heat flux $\mathbf{F}_c$ on latitude was involved through the latitude dependence of the heat transport coefficient K_c:

$$\mathbf{F}_c = - K_c C_p(\nabla T - \nabla T_{ad}), \qquad (8)$$

$$K = K(r)\left[1 + \varepsilon f(r) P_2(\cos\theta)\right] \qquad (9)$$

where $\nabla T - \nabla T_{ad}$ is the superadiabatic temperature gradient, P_2 is the Legendre polynomial and ε is an adjustable parameter.

Early models required equator-to-pole temperature differences of several tens of degrees for the observed equatorial acceleration to be reproduced. However, the attempts to measure the temperature difference between equator and poles (Altrock and Canfield, 1972; Noyes et al., 1973; Falciani et al., 1974) revealed a high degree of homogeneity of global temperature distribution with no differential temperature confidently detected. The upper bound of about 5 K for the pole-equator temperature difference was established.

The latest models (Belvedere and Paterno, 1977, Belvedere et al., 1980) removed this contradiction. However, unreasonably small Prandtl numbers were required to keep the equator-to-pole temperature differences within observational constraints. These models were shown to be highly dependent on the choice of boundary conditions and on whether centrifugal forces are included or not (Moss and Vilhu, 1983).

A combined model was considered by Pidatella et al. (1986) which included both anisotropic viscosities and latitude-dependent heat transport (see also Schmidt, 1982). This permits the comparison of the two mechanisms for differential rotation. Anisotropic viscosities were found to be more effective in generating differential rotation.

The important achievement of the models with latitude-dependent heat transport is the allowance for thermodynamic properties of solar convection, which are mainly ignored by the anisotropic viscosity models. However, the relations (8) and (9) are not satisfactorily substantiated. More rigorous approaches (Durney and Spruit; 1979; Rüdiger, 1982) show that in contrast to (9) the heat transport coefficient is a tensor for rotating convection. Moreover, the heat conductivities are proportional to the intensity of convection and therefore must be dependent on superadiabatic temperature gradient. In other words, the convective heat transport is an essentially nonlinear process. It is rather questionable whether this process can be adequately treated by traditional linear approaches.

3. NONAXISYMMETRIC MODELS

The nonaxisymmetric approaches try to simulate (numerically) the global flows on the Sun as well as the smaller-scale three-dimensional convective motions using the fundamental hydrodynamic equations. In practice, however, some

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approximations of these equations are used because of computers limitations.

Extensive nonaxisymmetric simulations in Boussinesq approximation were carried out (see, e.g., review by Gilman, 1980). We shall not consider the results of the Boussinesq models but shall proceed with discussion of probably more realistic recent simulations allowing for density stratification of convection zone (Gilman and Miller, 1986; Glatzmaier, 1984, 1985 a,b). These simulations start from particular variant (Gilman and Glatzmaier, 1981) of anelastic approximation (Ogura and Phillips, 1962; Gough, 1969) of gas dynamics equations; Glatzmaier (1985a,b) take also magnetic fields into account. Only giant-scale convection

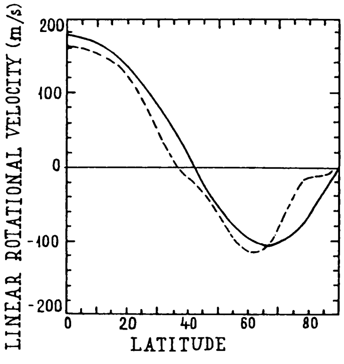

Fig.2. A comparison of observed (Howard and Harvey, 1970) differential rotation of solar photosphere (solid) with the results of nonaxisymmetric model by Gilman and Miller (1986)(broken). Linear velocities of rotation on the background of solid-body rotation (Ω = 2.6 rad/s) are shown.

and global flows have been simulated explicitly. The convective motions of smaller scales were parametrized by effective viscosities and conductivities.

The models by Glatzmaier (1984, 1985 a,b) and Gilman and Miller (1986) though different in numerical methods adopted yield essentially the same results. Quite satisfactory agreement with the observed rotation of photosphere has been found (Fig.2), except for high (λ > 70°) latitudes. The amplitudes of meridional circulation and equator-to-pole temperature differences were within observational constraints. Covariances of zonal and meridional velocities of simulated convection agreed with the Ward profile.

However, the models discussed did not yield the increase of angular velocity with depth at high latitudes as suggested by helioseismology. The isorotational surfaces were nearly cylindrical (Fig.2B). As has been noticed in Section 2.1, such isorotational surfaces suggest that $\langle \nabla \rho \times \nabla P \rangle = 0$ under approximations adopted.

The nonaxisymmetric models discussed here are probably the best-developed ones at this moment and agree quite well with observations. Nevertheless, we may confidently state that something important is missing in them because

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the toroidal magnetic field in Glatzmaier's (1985a) model migrated poleward in contrast to the observed equatorward drift of sunspot activity. It is very temting to suggest that this results from a nearly-cylindrical form of isorotational surfaces.

4. CONCLUDING REMARKS

It seems to follow from the above discussion that the question "whether we know basic mechanisms driving differential rotation of the Sun?" should be answered "yes", but the question "whether a reliable model for the solar rotation exists which reproduces all relevant observational data?" should be answered "no". This is because we are aware of principal physical mechanisms but not of the details of them.

The differential rotation can be produced by meridional circulation and Reynolds stresses. The meridional circulation can be excited by pressure forces and centrifugal forces. The desired nondissipative part of Reynolds stresses can be produced by two causes again: by anisotropy of convective motions and by inhomogeneity of convection zone. Any model of solar rotation faces the choice between one or the other or both of these effects at each step of its develop ment. It may be stated with relatively high confidence that solar differential rotation is the result of the above-mentioned effects, but it is not clear which of them play a dominant role.

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