Death of Massive Stars: Supernovae and Gamma-Ray Bursts
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A shallow water analogue of asymmetric core-collapse, and neutron star kick/spin

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Abstract. Massive stars end their life with the gravitational collapse of their core and the formation of a neutron star. Their explosion as a supernova depends on the revival of a spherical accretion shock, located in the inner 200km and stalled during a few hundred milliseconds. Numerical simulations suggest that the large scale asymmetry of the neutrino-driven explosion is induced by a hydrodynamical instability named SASI. Its non radial character is able to influence the kick and the spin of the resulting neutron star. The SWASI experiment is a simple shallow water analog of SASI, where the role of acoustic waves and shocks is played by surface waves and hydraulic jumps. Distances in the experiment are scaled down by a factor one million, and time is slower by a factor one hundred. This experiment is designed to illustrate the asymmetric nature of core-collapse supernova.

Keywords. supernovae, neutron stars, instabilities, numerical simulations, experiments

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1. Introduction

The stellar core-collapse and explosion signing the death of a massive star and the birth of a neutron star is a complex problem involving nuclear densities, neutrino interactions and transport, general relativistic corrections and multi-dimensional magnetohydrodynamics. The favored explosion scenario for moderately rotating cores relies on the delayed neutrino driven mechanism proposed by Bethe & Wilson (1985). A spherical accretion shock stalls during a few hundred milliseconds immediately after the formation of a neutron star, while neutrinos diffuse out of the neutrinosphere and eventually revive the shock. Transverse motions are generated by hydrodynamical instabilities developing after the shock bounce: entropy gradients induced by neutrino heating generate convection on the scale of the gain region (e.g. Janka & Müller 1996), while the Standing Accretion Shock Instability (SASI) is responsible for large scale oscillations (Blondin *et al.* 2003).

Some potential consequences of SASI. Large scale transverse motions contribute to the explosion by lengthening the exposure time of large coherent regions of the post shock flow to the neutrino flux, leading to an asymmetric explosion (Marek & Janka 2009). The $l = 1$ character of SASI allows for a significant pulsar kick, as demonstrated by numerical simulations in 2D and 3D (Scheck *et al.* 2004, 2006, Nordhaus *et al.* 2010, Wongwathanarat *et al.* 2010). Whether the spiral mode of SASI could significantly influence the spin of the neutron star is more debated (Blondin & Mezzacappa 2007, Yamasaki & Foglizzo 2008, Iwakami *et al.* 2009, Fernandez 2010, Rantsiou *et al.* 2011).

The hydrodynamics of core-collapse. Interpreting the outcome of the most realistic simulations benefits from understanding much simpler models where neutrino transport and interactions are replaced by ad-hoc cooling functions, and the infalling gas is approximated as time-independent. Blondin & Mezzacappa (2007) simply considered an ideal adiabatic gas for their first 3D simulation of SASI. They discovered a dominant spiral mode and its potential consequences on the spin the neutron star. The experimental approach described below and in more details in Foglizzo *et al.* (2012, hereafter FMGD) is built upon a similar degree of simplification, in the 2D equatorial plane of the stellar core.

2. SWASI, a Shallow Water Analogue of a Shock Instability

Inviscid shallow water equations. In a gravitational field g, surface gravity waves with a wavelength longer than the depth H of the fluid propagate with a velocity $c = (gH)^{\frac{1}{2}}$. The classical hydraulic jump seen in a kitchen sink marks the sudden transition between a thin layer of fast fluid ($v > c$) and a thicker layer of slower fluid ($v < c$). The variations of depth in the liquid are analogous to the variations of density in a compressible gas. The Froude number $\mathrm{Fr} \equiv v/c$ is equivalent to the Mach number comparing the gas velocity to the velocity of sound waves. A hydraulic jump in shallow water is analogous to a shock in a gas. Averaged over the depth of the fluid, the inviscid shallow water equations describing the flow of water along a surface $z = H_{\rm grav}(r)$ are identical to the equations describing an isentropic gas with an adiabatic index $\gamma = 2$ in a potential $\Phi(r) = gH_{\rm grav}(r)$:

$$\frac{\partial H}{\partial t} + \nabla \cdot (Hv) = 0, \tag{2.1}$$

$$\frac{\partial v}{\partial t} + (\nabla \times v) \times v + \nabla \left(\frac{v^2}{2} + c^2 + \Phi \right) = 0. \tag{2.2}$$

Choosing a surface shape $H_{\rm grav}(r) \equiv H_{\rm grav}^{\rm jp} \times r_{\rm jp}/r$ such that $\Phi(r)$ mimics the gravitational potential $GM_{\rm NS}/r$ of the central neutron star, the free fall timescale $t_{\rm ff}^{\rm jp}$ in the 2D shallow water model is related to the free fall time scale $t_{\rm ff}^{\rm sh}$ in its astrophysical analogue as follows:

$$\frac{t_{\rm ff}^{\rm sh}}{t_{\rm ff}^{\rm jp}} \equiv \left(\frac{r_{\rm sh}}{r_{\rm jp}} \right) \left(\frac{r_{\rm sh} g H_{\rm grav}^{\rm jp}}{GM_{\rm NS}} \right)^{\frac{1}{2}}. \tag{2.3}$$

Remembering that the mechanism responsible for SASI in a gas is based on the interaction of acoustic waves and advected perturbations (Foglizzo *et al.* 2007, Scheck *et al.* 2008, Fernandez & Thompson 2009, Foglizzo 2009, Guilet & Foglizzo 2012), we expect a similar instability in shallow water based on the interaction of surface gravity waves and vorticity perturbations.

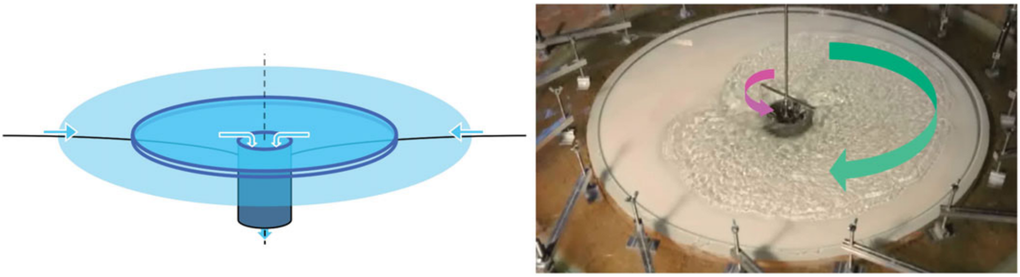

Figure 1. In the SWASI experiment, water flows radially inward from an annular reservoir down a potential well, and spills over the edge of an inner cylinder. On the right picture, the dynamical evolution of the hydraulic jump is dominated by a spiral mode. A light horizontal bar is used to visualize the budget of angular momentum.

Inner boundary. Like in the adiabatic simulations of Blondin & Mezzacappa (2007), the flow is extracted at the inner boundary rather than settling down onto the neutron star surface after intense neutrino cooling. In the experiment, the height of the upper edge of the inner cylinder defines a pressure threshold over which the fluid is evacuated by spilling over.

Viscous drag in the experiment. The viscous drag has been measured in the experiment in the stationary flow and modeled as a drag force $\bar{\nu}v/H^2$ in Eq. 2.2, with an effective viscosity coefficient $\bar{\nu} = 0.03\text{cm}^2/\text{s}$. The viscous drag is negligible after the hydraulic jump. The injection radius is 32cm, the radius of the inner cylinder is 4cm, the radius of the hydraulic jump is typically 20cm, and the flow rate is typically 1L/s.

Perturbative analysis. The perturbative analysis of the 2D model of the experiment revealed a dominant $m = 1$ instability in most of the parameter space (FMGD). For a given radius of the hydraulic jump, the variation of the injection slit and injection velocity in the stationary flow correspond to variations of the pre-jump Froude number Fr_{jp} and flow velocity v_{jp} , which we measure in units of the local inviscid free fall velocity v_{ff}. The flow is linearly unstable over a domain approximately described by $v_{\text{jp}}/v_{\text{ff}} > 0.5$ and $\text{Fr}_{\text{jp}} > 3$ (Fig. 4 in FMGD), which is barely affected by the viscous drag.

Experimental and numerical results in the linear regime. The experiment shows a robust $m = 1$ instability which grows from random noise. The oscillations of the hydraulic jump can either be sloshing of rotating, as expected from the linear stability analysis. Oscillation periods in the range 2-4 seconds have been measured in the experiment for a range of initial jump radii from 15 cm to 25 cm. Comparing these measurements to the prediction of the perturbative analysis and to the linear phase of 2D simulations showed very good agreement (Fig. 3 of FMGD), thus validating the 2D model of the experiment.

Experimental and numerical results in the non linear regime. When the instability is vigorous enough to reach non linear amplitudes, a systematic transition to a spiral mode has been observed, both in the experiment and in the 2D simulations. This non linear symmetry breaking had also been observed in the 3D numerical simulations of Blondin & Mezzacappa (2007).

Angular momentum budget. A freely rotating horizontal bar has been used in the experiment to visualize the angular momentum in the inner regions of the flow (Fig. 1). As the radius of the rotating hydraulic jump increases, the rotation of this bar is opposite to the rotation of the hydraulic jump. Both the experiment and the 2D simulations illustrate the partition of angular momentum between a rotating wave and the vorticity advected towards the accretor. The SWASI experiment illustrates the possible impact of SASI on the pulsar spin (Blondin & Mezzacappa 2007, Fernandez 2010).

3. Perspectives

Theory of core-collapse. The experimental limitations being different from the numerical ones, the SWASI experiment is a complementary tool to address some hydrodynamical questions in the theory of core-collapse. The non linear evolution of the 2D inviscid shallow water model presented here is remarkably similar to the 3D evolution of SASI in an adiabatic gas, despite the different value of the adiabatic index and the absence of entropy perturbations. A further comparison between these dynamical systems can help us characterize the role of the inner boundary, test the saturation mechanism of SASI (e.g. Guilet *et al.* 2010), and identify the physical mechanism responsible for the non linear symmetry breaking. This can help us understand the degree of asymmetry of the explosion, the strength of the pulsar kick and the possible effect of SASI on the pulsar spin. A new experiment is under construction at CEA-Saclay with improved accuracy.

Allowing for a global rotation, it should be able to describe the 2D evolution of SASI during the collapse of a rotating stellar core.

Public outreach. The SWASI experiment is simple and robust. It demonstrates some dynamical aspects of supernova theory using human timescales and sizes with a low construction cost. This experiment could contribute to scientific outreach towards students, researchers and the general public. A simplified version, currently designed at CEA-Saclay, will be proposed to universities and science museums.

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Discussion

SUWA: Do you have any plan to extend the experiment in 3D ?

FOGLIZZO: Unfortunately not. The shallow water analogy is fundamentally restricted to a 2D approximation of the astrophysical flow.

COUCH: I would have thought viscous forces would be important in the experiment, in contrast to the core-collapse context.

FOGLIZZO: The viscous drag appears to be significant only in the shallowest regions of the flow, ahead of the hydraulic jump. The unstable eigenmodes are barely affected by viscosity, but their non linear saturation could be.