*Active OB stars: structure, evolution, mass loss, and critical limits*
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# Rapid rotation and mixing in active OB stars – Physical processes

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**Abstract.** In the standard description of stellar interiors, O and B stars possess a thoroughly mixed convective core surrounded by a stable radiative envelope in which no mixing occurs. But as is well known, this model disagrees strongly with the spectroscopic diagnostic of these stars, which reveals the presence at their surface of chemical elements that have been synthesized in the core. Hence the radiation zone must be the seat of some mild mixing mechanisms. The most likely to operate there are linked with the rotation: these are the shear instabilites triggered by the differential rotation, and the meridional circulation caused by the changes in the rotation profile accompanying the non-homologous evolution of the star. In addition to these hydrodynamical processes, magnetic stresses may play an important role in active stars, which host a magnetic field. These physical processes will be critically examined, together with some others that have been suggested.

**Keywords.** stars: early-type, stars: interiors, stars: magnetic fields, stars: rotation

## 1. The need for non-standard stellar models

Until recently, stellar models ignored the possibility that some mixing could occur in the radiation zones of stars. But the situation is evolving because there is increasing evidence for such mixing, of which we shall give here only two examples.

It is well known that some A-type stars display anomalies in their surface composition, when they are compared to other, called 'normal' stars. These peculiarities were ascribed successfully to radiative acceleration and gravitational settling, by Michaud (1970) and his collaborators. But it turns out that these microscopic processes are so efficient that they would produce surface anomalies that are much more pronounced than those that are observed. For instance, helium would disappear from the surface of A-type stars in about one million years, as was pointed out by Vauclair *et al.* (1974). Since this is not observed, Vauclair *et al.* (1978) suggested that some mild turbulence - other than thermal convection - must operate near the surface to smooth the composition gradients. This was again emphasized and illustrated by Richer & Michaud (2000).

Another proof of such mixing is the overabundance, observed at the surface of massive stars, of chemical elements that are synthesized in the nuclear core. The overabundance of $^4$He and $^{14}$N, can only be explained if the radiative envelope has undergone some mixing (cf. Meynet & Maeder 2000). Interestingly, these overabundances seem to be correlated with the rotation velocity of the stars (Herrero *et al.* 1992); this property was again stressed by Maeder *et al.* (2009), who re-discussed the data gathered by Hunter *et al.* (2008).

There are thus strong indications that radiation zones undergo some kind of mixing, and several teams are now taking that mixing into account in their stellar evolution code. There are two ways of dealing with the problem. One is to model the mixing by an *ad hoc* diffusion, which is allowed to depend on one or more parameters, and to adjust these parameters to fit the observations. Our preference goes instead to the physical approach, which strives to implement as well as possible the physical processes that are likely to cause the mixing. We believe that they have been identified: these are the large scale circulation required by the transport of angular momentum, and the turbulence generated by the shear of differential rotation.

## 2. Rotational mixing

### 2.1. *Meridional circulation*

In its original treatment (Eddington 1925; Vogt 1925), the meridional circulation was ascribed to the fact that the radiative flux is no longer divergence-free in a rotating star, due to the centrifugal force. The characteristic time of the circulation was derived by Sweet (1950), and has since been named the Eddington-Sweet time: $t_{\rm ES} = t_{\rm KH}(GM/\Omega^2 R^3)$, with $t_{\rm KH} = GM^2/RL$ being the Kelvin-Helmholtz time. $R$, $M$, $L$ designate respectively the radius, mass and luminosity, $\Omega$ the angular velocity and $G$ the gravitational constant. Sweet's result suggested that rapidly rotating stars should be well mixed by this circulation and thus would remain homogeneous; therefore they would not evolve to the giant branch, contrary to what is observed.

However, these early studies overlooked the fact that the circulation carries angular momentum and thus modifies the rotation profile: starting from arbitrary initial conditions, the star undergoes a transient phase which lasts indeed about an Eddington-Sweet time, after which it settles into a quasi-stationary regime where the circulation is governed solely by the torques applied to the star. For instance, when the star loses angular momentum through a strong wind, the circulation adjusts precisely such as to transport that momentum to the surface (Zahn 1992). The resulting rotation is then non-uniform, and a baroclinic state sets in, with the density varying with latitude along isobars. When the angular velocity depends on the radial coordinate only, as we shall assume for reasons that will be explained later on, the density perturbation is given by $\delta\rho(r,\theta) = \tilde{\rho}(r) P_2(\cos\theta)$, and the baroclinic equation takes the simple form

$$\frac{\tilde{\rho}}{\bar{\rho}} = \frac{1}{3}\frac{r^2}{\bar{g}}\frac{d\Omega^2}{dr}, \tag{2.1}$$

with $\bar{\rho}$ and $\bar{g}$ being the mean density and gravity on the level surface, and $P_2(\cos\theta)$ the Lengendre polynomial of degree 2. On the other hand, when the star does not exchange angular momentum, the circulation would die altogether, as predicted by Busse (1982), if it had not to compensate the effects of structural adjustments (contraction, expansion) as the star evolves, and also the weak turbulent transport down the gradient of angular velocity that will be discussed next.

### 2.2. *Shear turbulence caused by differential rotation*

Since the rotation regime that results from the applied torques is not uniform, the shear of that differential rotation is prone to various instabilities, which generate turbulence and therefore mixing. Here we shall consider only those instabilities that apparently play a major role, namely the shear instabilities, also called Kelvin-Helmholtz instabilities.

#### 2.2.1. *Turbulence produced by the vertical shear*

Let us first examine the instability produced by the vertical shear, $\Omega(r)$. This instability is very likely to occur, because the Reynolds number characterizing such flows in stellar interiors is extremely high, due to the large sizes involved. However the stable entropy stratification acts to hinder the shear instability: in the absence of thermal dissipation, it occurs only if locally

$$\frac{N_T^2}{(\mathrm{d}V_\mathrm{h}/\mathrm{d}z)^2} \leqslant Ri_\mathrm{c}, \tag{2.2}$$

where $V_\mathrm{h}$ is the horizontal velocity, $z$ the vertical coordinate, and $N_T$ the buoyancy frequency defined by $N_T^2 = (g\delta/H_P)(\nabla_\mathrm{ad} - \nabla)$, with the classical notations and $\delta = -(\partial \ln \rho/\partial \ln T)_P$. This condition is known as the *Richardson criterion*; $Ri_\mathrm{c}$, the critical Richardson number, is of the order of unity and it depends somewhat on the rotation profile.

In a stellar radiation zone, this criterion is modified because the perturbations are no longer adiabatic, due to thermal diffusion. When the radiative diffusivity $K$ exceeds the turbulent diffusivity $D_\mathrm{v} = w\ell$ ($\ell$ and $w$ represent the size and the vertical rms. velocity of the largest eddies), the instability criterion takes the form (Dudis 1974; Zahn 1974)

$$\frac{N^2}{(\mathrm{d}V_\mathrm{h}/\mathrm{d}z)^2}\left(\frac{w\ell}{K}\right) \leqslant Ri_\mathrm{c}. \tag{2.3}$$

From the largest eddies that fulfill this condition, one can derive the turbulent diffusivity $D_\mathrm{v}$ acting in the vertical direction in the radiation zone of a star. However this instability criterion (2.3) holds only in regions of uniform composition, where the stability is enforced solely by the temperature gradient; when the molecular weight $\mu$ increases with depth, it seems at first sight that one should replace this criterion by the original one, expression (2.2), where now the buoyancy frequency is controlled by the gradient of molecular weight:

$$N^2 \approx N_\mu^2 = \frac{g\varphi}{H_P}\frac{\mathrm{d}\ln\mu}{\mathrm{d}\ln P},$$

with $\varphi = (\partial \ln \rho/\partial \ln \mu)_{P,T}$. However, as Meynet and Maeder (1997) pointed out, this condition is so severe that it would prevent any mixing in massive main-sequence stars, contrary to what is observed. We shall see below how that stabilizing action of $\mu$-gradients can be overcome.

### 2.2.2. *Turbulence produced by the horizontal shear*

Likewise, the horizontal shear $\Omega(\theta)$ will also generate turbulence, and this turbulence will probably be highly anisotropic, due to the vertical stratification, with much stronger transport in the horizontal than in the vertical direction, i.e. $D_\mathrm{h} \gg D_\mathrm{v}$. This shear instability tends to suppress its cause, namely the differential rotation in latitude, and it will thus lead to a 'shellular' rotation state, where the angular velocity depends on the radial coordinate only: $\Omega \sim \Omega(r)$.

Such anisotropic turbulence interferes with the meridional circulation, turning the advective transport into a vertical diffusion (Chaboyer & Zahn 1992). To lowest order, the vertical velocity of the circulation is given by $u_r(r,\theta) = U(r)P_2(\cos\theta)$, where $P_2$ is the Legendre polynomial of degree 2; then the resulting diffusivity is

$$D_\mathrm{eff} = \frac{1}{30}\frac{(rU)^2}{D_\mathrm{h}}, \tag{2.4}$$

when $D_\mathrm{h} \geqslant rU$. Unfortunately, a reliable prescription for that horizontal diffusivity $D_\mathrm{h}$ is still lacking, in spite of recent attempts to improve it (see Maeder 2003; Mathis *et al.* 2004).

Another property of such anisotropic turbulence is that, by smoothing out chemical inhomogeneities on level surfaces, it reduces the stabilizing effect of the vertical $\mu$-gradient. The Richardson criterion for the vertical shear instability then involves the horizontal diffusivity $D_h$, and the vertical component of the turbulent viscosity can be derived as before (Talon & Zahn 1997):

$$D_{\rm v} = Ri_{\rm c} \left[ \frac{N_T^2}{K + D_{\rm h}} + \frac{N_\mu^2}{D_{\rm h}} \right]^{-1} \sin^2\theta \left( \frac{{\rm d}\Omega}{{\rm d}\ln r} \right)^2 . \tag{2.5}$$

## 2.3. *Rotational mixing of type I*

The two transport processes that have just been discussed (meridional circulation and shear-induced turbulence) are both linked with the differential rotation. Therefore, when modeling the evolution of a star including these mixing processes, it is necessary to calculate also the evolution of its rotation profile $\Omega(r)$ (since $\Omega$ is a function of $r$ only, due to the anisotropic turbulence mentioned above). Then all perturbations separate in $r$ and colatitude $\theta$, as illustrated already above for the vertical component of the meridional velocity: $u_r(r,\theta) = U(r)P_2(\cos\theta)$. For a detailed account of how this mixing may be implemented in stellar evolution codes, we refer to Zahn (1992), Maeder & Zahn (1998) and Mathis & Zahn (2004).

We first examine the simplest case, that we call 'rotational mixing of type I', where the angular momentum is transported by the same processes that are responsible for the mixing, namely meridional circulation and turbulent diffusion. The angular velocity then obeys the following advection/diffusion equation, obtained by averaging over latitude:

$$\frac{\partial}{\partial t}\left[\rho r^2\,\Omega\right] = \frac{1}{5r^2}\frac{\partial}{\partial r}\left[\rho U r^2\,\Omega\right] + \frac{1}{r^2}\frac{\partial}{\partial r}\left[\rho \nu_{\rm v} r^4 \frac{\partial\Omega}{\partial r}\right] + \text{applied torques}, \tag{2.6}$$

with the turbulent viscosity $\nu_{\rm v} \approx D_{\rm v}$ given by (2.5). In spite of the fact that this equation is one-dimensional only, it captures the advective character of the angular momentum transport by the meridional circulation: depending on the sense of the circulation, angular momentum may be transported up the gradient of $\Omega$, which is never the case when the effect of meridional circulation is modeled just as a diffusive process, as it is done most often.

At first sight, and neglecting evolutionary effects, the circulation is governed mainly by the applied torques. When the star loses little angular momentum, or none, it settles into a regime of differential rotation where a weak inward advection of angular momentum compensates the turbulent diffusion directed outwards. Integrating (2.6) over $r$, we have then

$$U(r) \approx -5\nu_{\rm v}\frac{d\ln\Omega}{dr}, \tag{2.7}$$

which shows that the strength of the meridional circulation scales with the vertical component of the turbulent diffusivity. In the limit $\nu_{\rm v} \to 0$, the circulation would vanish altogether, as was first pointed out by Busse (1982).

On the other hand, when the star loses angular momentum, the circulation adjusts itself such as to transport precisely that amount towards the surface (Zahn 1992). The balance reads then

$$\frac{3}{8\pi}\frac{\partial J(r,t)}{\partial t} = \frac{1}{5}\rho r^4 \Omega U + \rho\nu_{\rm v} r^4 \frac{\partial\Omega}{\partial r}, \tag{2.8}$$

where $J$ is the flux of angular momentum. Near the surface, angular momentum is transported mainly by the meridional circulation, since $\partial\Omega/\partial r \to 0$, but this holds also in the bulk of the radiation zone.

However, as the star evolves, the inner regions contract and the envelope expands; this results in readjustments of the angular velocity, associated with transport of angular momentum, which are accounted for by the l.h.s. of (2.6).

Massive main-sequence stars belong to the first category examined above, that with relatively modest angular momentum loss, and their models have been seriously improved by the implementation of rotational mixing (Maeder & Meynet 2000). The theoretical isochrones agree with the observed ones, and such rotational mixing accounts well for the observed enhancement of He and N at the surface of early-type stars (Talon *et al.* 1997; Meynet & Maeder 2000 and subsequent papers). Finally, combined with a suitable description of the mass loss, this type of mixing also predicts the observed proportion of blue and red giants in open clusters.

Note that this good agreement between observations and models of massive stars is obtained without having to invoke other processes than meridional circulation and shear turbulence to transport angular momentum and thus shape the rotation profile. This is not the case for solar-type stars, where gravity waves (or rather gravito-inertial waves) probably participate in what may be called 'rotational mixing of type II'. As it has been shown by Talon & Charbonnel (2003 - see also Charbonnel & Talon 2005), these waves, which are emitted at the base of the convection zone of such stars, are able to extract angular momentum from their radiative interior, and to enforce there the nearly uniform rotation detected through helioseismology. To what extend this wave transport also occurs in massive stars remains for the moment an open question, for lack of observational constraints on their rotation profile; the situation should improve soon thanks to asteroseismology.

## 3. Magnetism of massive stars: most likely of fossil origin

When we were describing the effect of rotation on the structure and evolution of massive stars, we made implicitly the assumption that magnetism plays a negligible role in them. This approach is no longer valid for the active stars to which this symposium is dedicated, because these are the seat of strong magnetic fields.

What we know about stellar magnetism will be addressed in several contributions during this symposium; we shall just recall here the salient facts. Magnetic fields are observed at the surface of all solar-like low-mass stars, namely stars that possess a thick outer convection zone. These fields are variable on the scale of months or years, and they are highly structured; most probably they are generated through dynamo action driven by the convective motions.

In contrast, among the more massive stars, that do not possess such convection zones, only a small fraction, less than 5% according to Power *et al.* (2008), are hosting magnetic fields: these are the so-called Ap and Bp stars. These fields display much simpler topologies, and they seem unchanged over long timescales (at least when compared to human life span); that is why they are believed to be of fossil origin (Wade *et al.* 2009), resulting from the contraction, during star formation, of the primeval, Galactic field. Moreover, all magnetic stars manifest anomalies of their surface composition, and most of them are the seat of slow uniform rotation.

Since they are rooted in the radiative interior, fossil fields evolve on a considerably longer timescale than dynamo fields; the Ohmic decay time for a dipolar field is given by $R^2/2\pi\eta$, $R$ being the radius and $\eta$ the magnetic diffusivity. This time amounts to 10 Gyr for the Sun (Cowling 1957), and it scales with mass $M$ and radius roughly as $M^{3/2}/R^{1/2}$. Therefore these fields are extremely long lived, even if they are of high spherical degree (the decay rate varies as $\ell(\ell+1)$, $\ell$ being the order of the multipole). Unless the fields are destroyed by some instability, a possibility that we shall examine next.

### 3.1. *MHD instabilities in stellar radiation zones*

Magnetic fields are liable to various instabilities, which could perhaps lead to mixing and may therefore have an impact on stellar evolution. These instabilities have been discussed by Spruit (1999) in a comprehensive review; he concluded that the most likely to play a role in stellar radiation zones are those studied by Tayler and his collaborators in the 1970's. These affect axisymmetric poloidal and toroidal fields and they are linked to the classical pinch and kink instabilities.

Describing the fully non-linear regime of the instability represents a difficult task. Spruit (2002) suggested that the instability should saturate when the turbulent magnetic diffusivity $\eta_t$, that supposedly is accompanying it, yields a zero growth rate. We had an opportunity to verify his prediction by examining the results of actual non-linear simulations performed with the 3-dimensional ASH code (for Anelastic Spherical Harmonics). The purpose of this calculation was to study the interaction of a fossil magnetic field with the solar tachocline (Brun & Zahn 2006), in order to check whether such a field could prevent this transition layer from expanding into the deep interior, as it had been suggested by Gough and McIntyre (1998).

We started the simulations with a deeply buried poloidal field of dipole type and a uniformly rotating radiation zone, on the top of which we imposed the differential rotation of the convection zone. As time proceeds, the poloidal field diffuses upward and the differential rotation expands downward; their interaction produces a toroidal field which is antisymmetric with respect to the equatorial plane. Once the poloidal field reaches the convection zone, it imprints the differential rotation of that region on the radiative interior below, according to Ferraro's law (Ferraro 1937). Since this is not observed, we concluded that such a fossil poloidal field, as it was postulated by Gough and McIntyre, does not exist in the Sun.

The benefit of carrying out these simulations with a 3-dimensional code was to capture the non-axisymmetric instabilities that affect the large scale magnetic field. We observed indeed the instabilities that had been described by Tayler and his collaborators. The first instability to appear was that of the initial poloidal field, with an azimuthal number $m \approx 40$. It was followed by that of the toroidal field, once that field had been generated through shearing the poloidal field by the differential rotation. The second instability, of azimuthal order $m = 1$, is clearly that studied by Pitts and Tayler (1985), although here it occurred in the presence of both the toroidal field and of the poloidal field, a configuration which is deemed to be more stable than that of just a toroidal field.

This instability saturates when its energy reaches that of the fossil field, but it is not clear whether this is just a coincidence. An unexpected result was to find that the Ohmic decline of the poloidal field is not accelerated by the instability. The 3-dimensional perturbations associated with this instability behave as Alfvén waves, rather than as turbulence, and they do not produce the turbulent diffusion invoked by Spruit. We conclude therefore that they cannot achieve either any mixing of the stellar material.

But other instabilities may arise in magnetic stars, and this was illustrated recently by a series of numerical simulations performed by Braithwaite (Braithwaite & Spruit 2004; Braithwaite & Nordlund 2006; Braithwaite 2009). He showed that an arbitrary, small-scale initial field relaxes into a twisted torus configuration which combines a poloidal and a toroidal field. The quest for such stable configurations is pursued actively, both through numerical simulation and through theory (cf. Duez & Mathis 2010); we shall hear more of it during this symposium.

### 3.2. *Other causes for the magnetic field*

The properties of the magnetic field in Ap-Bp stars strongly suggest that the field is of fossil origin: their simple topology, their constancy in time, the fact that the surface rotates uniformly. But other possible causes are worth considering.

#### 3.2.1. *A dynamo in the radiation zone?*

When Spruit examined the MHD instabilities in stellar radiation zones, he made an interesting conjecture, namely that such instability could regenerate the toroidal field which initially triggers it, and would thus operate a dynamo, much as that driven by the convective motions in a convection zone. However the dynamo loop cannot work as he describes it (cf. Spruit 2002): it requires some type of $\alpha$-effect to generate a mean electromotive force, as is well-known in dynamo theory (Parker 1955). As in the case of the Sun, the only way to check whether this $\alpha$-effect is actually present is through numerical simulations.

But so far these have yielded conflicting results: Braithwaite (2006) claims to observe dynamo action, with field reversals, contrary to us, who don't see any regeneration of either the poloidal or the toroidal field (Zahn *et al.* 2007). The main difference between our simulations lies in the way the equations are solved. Our code is of pseudo-spectral type, which allows us to reach, with enhanced diffusivities, a magnetic Reynolds number of $10^5$ for a resolution of 128x256x192. That should be more than sufficient to detect a dynamo, if it exists at all. Braithwaite uses instead a 6th order finite difference scheme, with a resolution of 64x64x33; his numerical diffusivity is tuned to ensure stability and it is not easy to infer from it the effective Reynolds number. Moreover, it appears that his calculations have not been run long enough beyond the transient phase to establish without any doubt that a dynamo is at work. Clearly further simulations are needed to settle this issue.

#### 3.2.2. *A dynamo operated through thermal convection?*

Recent 3-D simulations have taught us that convective regions very easily generate magnetic fields. A good example is the simulation of core convection in a rotating A-type star of two solar masses which has been performed by Brun *et al.* (2005). They introduce a seed field in a core with fully developed convection, and observe its growth to an amplitude where its energy is comparable to that of the flows. The result does not seem to depend much on the parameters used. So there is little doubt that massive stars can produce a dynamo field in their convective core.

The question which remains is whether such a field would be visible at the surface. If the field has reversals as observed in the Sun, say of period $P$, it would penetrate only to a distance $d = (\eta P)^{1/2}$, which amounts to 1 km for $P = 10$ years. Only the DC component would reach the surface, and this after a global diffusion time exceeding a few Gyr. Admittedly, flux tubes could rise from the core through magnetic buoyancy, but this was not observed in the simulations by Brun *et al.*.

Another possibility is that a local dynamo could operate in the subsurface convection zones due to the ionization of He and Fe, as it was suggested by Cantiello *et al.* (2009). We will certainly hear more about that during this symposium.

Thus one cannot rule out that the magnetism of Ap-Bp stars is due to a dynamo generated by thermal convection, although it would then be difficult to explain why the observed fields are so constant in time - at least since they have been observed, which is at most a few decades.

## 3.3. *Impact of magnetic fields on stellar structure*

In order to have an impact on the overall structure of a star, the magnetic force (often called Lorentz force although it was Laplace who first gave its expression) ought to be able to compete with the two main forces that govern its hydrostatic equilibrium, namely gravity and the pressure force. This would occur if the Alfvén velocity $V_A = B/\sqrt{4\pi\rho}$ equaled the sound speed; in the Sun that would require the field to be of order $2\,10^8$ G, which is considerably stronger than any field observed on the surface of main sequence stars.

However, it is quite possible that such extremely strong fields are hiding in the deep interior of some stars. Presumably the only way to detect them would be through asteroseismology. The frequencies of the oscillation eigenmodes - either $p$ or $g$-modes, depending on the restoring force (pressure or buoyancy) - are split and displaced by a magnetic field, much like what occurs under the effect of rotation, and this may be used to estimate the magnetic field. In massive stars, where only few eigenmodes are observed, moreover of low spherical degree, the task is rather difficult, except perhaps for the slowly pulsating B stars, where high order $g$-modes could bear the signature of a 100 kG field located near the boundary of the convective core (Hasan *et al.* 2005). The effect of a dipole field on the p-modes in a rotating star was considered in full detail by Bigot *et al.* (2000), with application to the rapidly rotating Ap stars.

### 3.3.1. *Strong magnetic fields can suppress thermal convection*

It is well known that thermal convection can generate magnetic fields, but magnetic fields can also suppress the convective instability. This can be seen by examining the stability of a stratified fluid in presence of magnetic field. A perturbation described by its displacement $\vec{\xi} \propto \exp\,[st + i\vec{k}\cdot\vec{x}]$ has a growth-rate $s$ that obeys the following dispersion relation, where we neglect all kind of dissipation (thermal, Ohmic, viscous):

$$s^2 = \left(\frac{k_h}{k}\right)^2 N_t^2 - (\vec{k}\cdot\vec{V}_A)^2, \qquad N_t^2 = \left[\frac{g}{H_P}\,(\nabla - \nabla_{\rm ad})\right], \tag{3.1}$$

$N_t$ being the buoyancy frequency. Here $g$ is the local gravity, $H_P$ the pressure scale height, $\vec{k}$ the wavenumber, $k_h$ its horizontal component, and $\nabla = \partial \ln T/\partial \ln P$ designates as usual the logarithmic temperature gradient.

At first sight it would seem that, to suppress convection, it requires a field strength of

$$\frac{B^2}{4\pi\rho} = V_A^2 \gtrsim N_t^2 R^2 \tag{3.2}$$

according to (3.1), which would translate into around $10^7$ G ($10^3$ Tesla) in the radiation zone of a massive star, or $10^3$ G near its surface. However that threshold is lowered by two orders of magnitude when diffusion is taken in account; the stability condition then becomes (Chandrasekhar 1961)

$$\frac{B^2}{4\pi\rho} = V_A^2 \gtrsim \frac{\eta}{K} N_t^2 R^2, \tag{3.3}$$

with $\eta$ and $K$ being the Ohmic and thermal diffusivities (typically $\eta/K \approx 10^{-4}$ in stellar interiors). The instability is of double-diffusive type, with heat diffusing much faster than the magnetic field.

In the sunspots we see a striking proof of that inhibition of thermal convection by a field of kilogauss strength. That is also why we observe surface inhomogeneities in the Ap-Bp stars: due to the presence of magnetic field, there is no convection to smooth out such inhomogeneities, as in normal stars where convection exists in the thin superadiabatic layers due to the ionization of hydrogen and helium.

3.3.2. *Magnetic fields amplify the loss of angular momentum by stellar winds*

A well established fact is that solar-like stars spin down as they evolve. At the ZAMS, some of such stars have equatorial velocities that reach 100 km/s, while that of the Sun is only 2 km/s. The only way stars can achieve this is by losing mass, through a wind whose mechanism has been elucidated by Parker (1958). In the absence of magnetic field this is not a very efficient process, but the picture changes when one includes the magnetic field, as was pointed out by Schatzman (1962). The wind is then forced to rotate with the star up to a distance where the wind speed exceeds the Alfvén velocity: the lever arm is then no longer the radius $R$, but the so-called Alfvén radius, which in the present Sun is about $R_A \approx 15\,R$.

This mechanism works also for massive stars, where the magnetic field interacts likewise with their radiation-driven wind; it has been studied in detail in a series of papers by ud-Doula and Owocki (ud-Doula & Owocki 2002; Owocki & ud-Doula 2004; ud-Doula *et al.* 2006, 2008). They introduce a 'wind magnetic confinement parameter' $\eta* = B_{\rm eq}^2 R^2/\dot{M} v_\infty$, which measures the ratio between the magnetic field energy density (at the equator) and the kinetic energy density of the wind ($v_\infty$ is its terminal speed and $\dot{M}$ the mass loss rate). They show that for $\eta^* \gg 1$ the latitude dependent Alfvén radius $R_A(\theta)$ scales with the confinement parameter according to a rather simple law:

$$\left[\frac{R_A(\theta)}{R}\right]^{2q-2} - \left[\frac{R_A(\theta)}{R}\right]^{2q-3} = \eta^*[4 - 3\sin^2(\theta)], \qquad (3.4)$$

where $\theta$ is the colatitude, and $q$ the magnetic exponent. For a dipole field $q = 3$ and therefore $R_A \propto \eta^{*1/4}$.

3.3.3. *Magnetic fields inhibit the rotational mixing*

We recalled above that in the absence of magnetic field, the radiation zone of a rotating star undergoes mild mixing through internal motions - a combination of turbulence and large-scale flows - that are due to several causes. These are mainly the loss of angular momentum by a wind, eventually the coupling with an accretion disk, and the structural adjustments the star undergoes as it evolves (such as the moderate core contraction and envelope expansion on the main sequence).

Let us examine now what occurs in the presence of magnetic field. One effect of such a field is to enhance the angular momentum loss, and at first sight this should speed up the rotational mixing. But the field acts also in the deep interior: when it is axisymmetric with respect to the rotation axis, it tends to render the rotation uniform along the field lines of the meridional field (Ferraro 1937). To estimate the strength of the axisymmetric field above which that rotational mixing is inhibited, we refer to the horizontally averaged angular momentum transport equation; we neglect the turbulent transport, since there is no shear anymore, but include the torque exerted by the Lorentz force $\vec{j} \times \vec{B}/c$ :

$$\rho \frac{d}{dt}\left(r^2\overline{\Omega}\right) = \frac{1}{5r^2}\partial_r\left(\rho r^4 \overline{\Omega} U\right) + \overline{\vec{B}_p \cdot \vec{\nabla}(rB_\varphi/4\pi)}, \qquad (3.5)$$

with $\vec{B}_p$ and $B_\varphi$ being the meridional and azimuthal components of the magnetic field.

Introducing the characteristic time $t_{\rm AM}$ for the angular momentum evolution (due to a wind, or to structural adjustments), we see that the Lorentz torque balances the l.h.s.

when

$$B_p B_\varphi \gtrsim B_{\rm crit}^2 = 4\pi\rho \, \frac{R^2\Omega}{t_{\rm AM}}. \qquad (3.6)$$

If we apply this result to a 15 $M_\odot$ star, taking $\rho = 0.07$ g/cm$^3$, $R = 4.5\,10^{11}$ cm, $R\,\Omega = 3\,10^7$ cm/s and setting somewhat arbitrarily $t_{\rm AM} = 10^6$ yr, we find that the critical field strength for suppressing rotational mixing is of the order of $B_{\rm crit} \approx 600$ Gauss, a level which is probably exceeded in the interior of Ap-Bp stars. This crude estimate needs to be confirmed by more thorough calculations, for which the formalism has been derived by Mathis and Zahn (2005).

When the field is non-axisymmetric, as is often observed, uniform rotation tends to be enforced everywhere in the radiation zone. Little more is known about this oblique rotator model, as can be seen in Mestel (1999), and this is clearly a promising field for future research.

## 4. Conclusion and perspectives

Despite all the work that has been accomplished since the 1970's, notably by Leon Mestel, Roger Tayler and their collaborators, we still do not fully understand the origin of magnetism in massive stars and the role it plays in their evolution. But let me summarize the few results that are now firmly established.

Only a minority of massive stars host magnetic fields - less than 5% according to the latest survey. In those stars, the magnetic field is probably of fossil origin, and it is deeply rooted in the radiative interior. We don't know why magnetic stars are so few: presumably all stars had initially a fossil field, which they gathered from the interstellar medium during the process of formation, but it must have been later destroyed in most of them, presumably through MHD instabilities.

It has been known since the 1960's that magnetic fields enhance the loss of angular momentum through stellar winds (Schatzman 1962), which explains why most magnetic stars are slow rotators. One would thus be tempted to conclude that rotational mixing is enhanced by this angular momentum loss; but it turns out on the contrary that the internal motions - either large scale circulations or turbulence - are frozen by the interior field. The field itself can become unstable, but the instability saturates in a regime that is wave-like and does not lead to turbulent diffusion. Thus one suspects that magnetic stars will not show signs of internal mixing - we are eagerly waiting observational data that will tell us whether this prediction is correct.

The study of stellar magnetism is still in its infancy, but the situation is improving rapidly. The new generation of extremely powerful spectro-polarimeters is delivering much wanted observational constraints of high quality, while the steadily progressing computer resources allow numerical simulations with a increasing degree of realism. There will be much to discover and to explain, for the young astrophysicists entering this exciting field!

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**Discussion**

OWOCKI: In your estimate of the field needed to suppress differential rotation and mixing, you assumed a fixed angular momentum loss time. But that time should itself depend on field strength (and mass loss rate). If one includes this, it would give a different scaling and estimate for the required field

ZAHN: You are right: one should indeed include that dependence in deriving the scaling.

BOMANS: You stressed the importance of fossil magnetic fields. Observations imply low strength ordered magnetic fields in lower mass galaxies, which means low metallicity galaxies. Would you therefore expect to see low magnetic field OB stars in these galaxies?

ZAHN: From the crude scaling of the magnetic field as density to the power 2/3, one should expect a lower field in low metallicity galaxies. But will that scaling hold over so many orders of magnitude?

PULS: Did I understand correctly that your recent calculations show that the Spruit-Taylor dynamo does not work?.

ZAHN: Yes, as we explain in Zahn *et al.* (2007), we observe the Taylor instability, but there is no dynamo action, i.e. the mean axisymmetric field is not regenerated.

PULS: However, present evolutionary codes include this mechanism. What would happen if the fields were more correctly described?

ZAHN: This is precisely what we want to find out.