

**ASTEROSEISMOLOGY:  
THEORY AND METHODS**

# SOUNDING STELLAR INTERIORS

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## 1. Introduction

Multimode pulsations have been discovered in a large variety of main sequence stars and white dwarfs. There are abundant and accurate data on p- and g- mode frequencies in these objects. So far, however, use of these data for sounding interiors has been very limited. The main obstacle is the difficulty in identification of detected modes that is in assignment of spherical harmonic degree,  $l$ , azimuthal order,  $m$ , and radial order,  $n$ , to the modes detected in power spectra of variable stars. This has never been a problem in helioseismology. Even in the case of whole disc measurements a simple structure of power spectra enables an unambiguous mode identification. Such a situation is very rare in asteroseismology. Perhaps oscillating white dwarfs PG 1159-035 and GD 358 are the only examples. Methods of determining the  $l$  and  $m$  from observational data on the excited modes are available. However, their implementation to – typically low amplitude – multimode pulsators is difficult. In most cases mode identification cannot be separated from determination of stellar parameters.

The number of modes detected in individual stars, and in fact in all stars put together, is by orders of magnitude smaller than that in the Sun. Again the two white dwarfs each with over 100 excited modes are on top of the list of multimode pulsating stars. Not surprisingly, astrophysics made best use of asteroseismic data on these objects. I will return to these two stars in section 3.

Most of this review, in particular sections 4 and 5, is devoted to  $\delta$  Scuti and  $\beta$  Cephei stars. This reflects primarily my personal interest. The truth is that for no object of these two types credible seismic sounding is available. Here the maximum number of excited modes is 20, which is much less than the number of unstable modes with  $l \leq 2$  found in corresponding models. The possibility of identification with higher degree modes is usually

neglected. This simplifies mode identification but, unfortunately, is not fully justified.

The ultimate aim of asteroseismology is to construct stellar seismic models i.e. stellar models with adjusted parameters to fit observed frequencies within the measurement errors. We are not at this stage yet. In section 7 I outline the methodology which, I hope, will be soon applied.

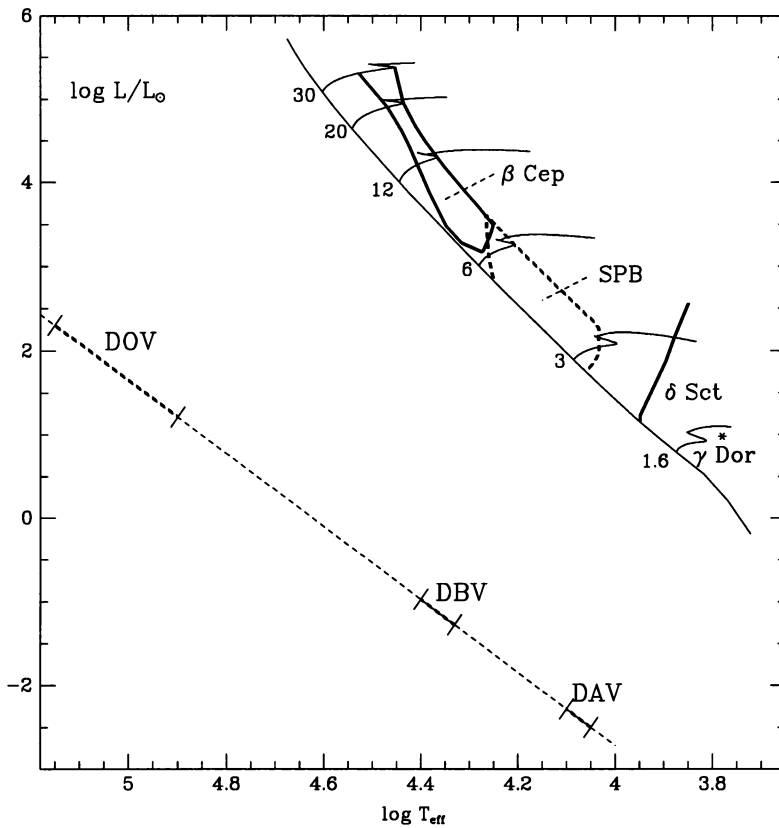
The true asset of stellar seismology are g-modes which are definitely detected in quite a variety of pulsating stars. These modes are indeed much better design to probe interiors than p-modes – still the only modes available for helioseismic sounding. In many stars, like in the Sun, the g-modes have large amplitudes in deep interior. Furthermore, their frequencies are very sensitive to the mean molecular weight gradient. The possibility of probing a chemically inhomogeneous interior is crucial for testing stellar evolution theory.

## 2. White Dwarf and Main Sequence Pulsators

Naturally, asteroseismologists are interested in stars with many excited modes and such stars are found almost exclusively among white dwarfs and stars in the main sequence band. The exceptions are some of  $\delta$  Scuti stars which are in the phase of hydrogen burning in a thick shell surrounding a helium-rich core. Basic data on types of pulsating stars of interest are summarized in Table 1 and in Fig. 2, showing positions of the objects in the H-R diagram.

Oscillating white dwarfs are spread over a wide range of effective temperatures. Objects denoted as DAV and DAB have, respectively, pure H and He atmospheres. The theory predicts excitation of p-modes in these stars but the presence of such modes has not been confirmed. All oscillating white dwarfs are g-mode pulsators. Abundant data on these stars have been collected in campaigns of the Whole World Telescope (Nather *et al.*, 1990).

A large diversity of pulsation is found among the upper main sequence stars. There are two types of high-order g-mode pulsators: Slowly Pulsating B stars (SPB) (Waelkens, 1991) and  $\gamma$  Doradus stars (Balona *et al.*, 1994). These long-period variable stars are only of potential interest for asteroseismic sounding. Years of observations are required to gather sufficient amount of data. Use of automatic telescopes seems the best option. Data on  $\beta$  Cep and  $\delta$  Sct stars collected for decades are far more abundant. Two networks – STEPHI (Belmonte *et al.*, 1993) and DSN (e.g. Breger *et al.*, 1994)– are devoted to observations of the latter type. Their operation led to the discovery of truly multimode objects. High-order p-mode pulsators in the upper main sequence are represented by roAp stars. The objects share



*Figure 1.* The position of the pulsating stars in the H-R diagram. Evolutionary tracks for Population I stars with indicated masses are shown. For  $\beta$  Cep and SPB stars theoretical instability domains are shown. For  $\delta$  Sct only the high temperature boundary is given. The asterisk marks the  $\eta$  Boo position.

their H-R position with  $\delta$  Sct stars but are chemically peculiar and have strong magnetic fields. Prospects for the roAp star seismology are discussed in these proceedings by J. Matthews.

In all these objects the most likely (and in many cases certain) cause of mode excitation is the opacity mechanism. In cooler stars we expect excitation of solar-like oscillations. Christensen-Dalsgaard and Frandsen (1983) provided a crude prediction of the amplitudes and period ranges. Unfortunately, none of the announced detections has been confirmed by independent teams of observers. The most convincing case is that of  $\eta$  Boo (Kjeldsen *et al.*, 1995).

In Table 1 I provide some data on white dwarf and main sequence pulsators (N\* denotes the number of objects). Figure 1 shows the objects

TABLE 1.

TYPE	$M/M_{\odot}$	$\log T_{eff}$	$N^*$	Periods	Modes
DAV	0.4 - 0.8	4.05 - 4.10	23	2 - 15 min	g
DBV	$\sim 0.6$	4.33 - 4.40	8	2 - 15 min	g, $n \gg 1$
DOV	0.6	4.9 - 5.15	5	5 - 16 min	g, $n \gg 1$
$\gamma$ Dor	$\sim 1.5$	3.84 - 3.88	10	1 - 2 d	g, $n \gg 1$
roAp	1.8 - 2	$\sim 3.9$	27	6 - 15 min	p, $n \gg 1$
$\delta$ Sct	1.5 - 2.5	3.84 - 3.93	350	0.02 - 0.3 d	p, g
SPB(53 Per)	3 - 7	4.10 - 4.25	10	0.5 - 4 d	g, $n \gg 1$
$\beta$ Cep	8 - 16	4.35 - 4.45	60	0.07 - 0.3 d	p, g
solar type	1.6	3.78	1	12 - 16 min	p, $n \gg 1$

on the H-R diagram.

### 3. PG 1159-035 and GD 358

These two stars represent the greatest success of asteroseismology. PG 1159-035 is the prototype object for the DOV type. Its power spectrum obtained with a WET campaign (Winget *et al.*, 1991) is the most revealing power spectrum ever obtained for a star. It is fully resolved into 125 frequencies of which 101 has been identified with high-order g-modes of  $l = 1$  and 2 degrees. The identification was possible without a reference to a star model because the frequencies obey approximately the two following asymptotic relations valid for high-order g-modes:

$$P_{l,n+1,0} - P_{l,n,0} \approx \frac{2\pi^2}{\sqrt{l(l+1)} \int_0^R N d \ln r}$$

and

$$\nu_{l,n,m} - \nu_{l,n,0} \approx m \frac{\Omega}{2\pi} \left(1 - \frac{1}{l(l+1)}\right),$$

where  $P$  is the period and  $N$  is the Brunt-Väisälä frequency.

The same relations were earlier used by Delache and Scherrer (1983) in their search for g-modes in solar oscillation spectra. However, in the g-mode frequency range the solar power spectrum is far less clean than that of PG 1159-035. The latter is more similar to that of solar p-modes obtained with the whole disc measurements. This analogy goes further. In both cases the main patterns of the power spectra are modeled with just two numbers. The approximate validity of asymptotics allows to separate mode identification from model fitting.

The preliminary analysis of the power spectrum yields very important information about star rotation: the value of period,  $P_{rot} = 1.35$  d, and the evidence that it is nearly uniform. Let us recall that we had no useful information about solar rotation from the early whole-disc data and even now there are discrepant inferences. The measured mean period separation enabled an estimate of the mass:  $M/M_{\odot} \approx 0.59$ .

If the data had satisfied Eq.(1) exactly then that would have been the end of seismic analysis and no seismic probing of the internal structure would have been possible. Fortunately, this is not the case. There is a departure from the asymptotic value, which plotted as function of  $P$  exhibits oscillatory behavior which is caused by a nearly discontinuous transition between regions of different chemical composition. Detailed models reproducing the observed behavior were constructed by Kawaler & Bradley (1994).

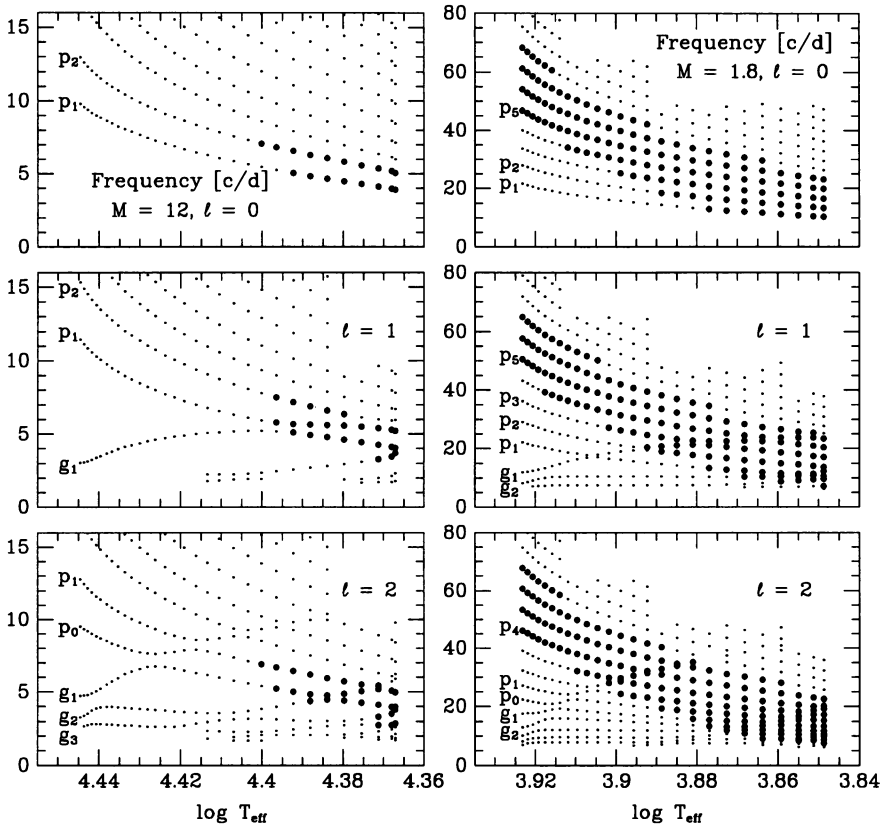
GD 358 is a DBV type object. Its power spectrum obtained with a WET campaign (Winget *et al.*, 1994) is the richest ever obtained for a distant star. It is more complicated than that of PG 1159-035 and a smaller fraction of 180 peaks has been connected to oscillation mode frequencies. A sequence of hypothetical  $l = 1$  triplets was identified. However, such an identification requires nonuniform rotation and the presence of a strong magnetic field. If confirmed, these findings would have interesting ramifications.

#### 4. Unstable Modes in Upper Main Sequence Stars

Figure 2 shows how the instability to low degree modes appears in  $\beta$  Cep star models ( $12 M_{\odot}$  sequence) and in  $\delta$  Sct star models ( $1.8 M_{\odot}$  sequence). In the former case the driving effect arises in the metal opacity bump at  $T \approx 3 \times 10^5$  K and in the latter in the HeII ionization zone at  $T \approx 5 \times 10^4$  K.

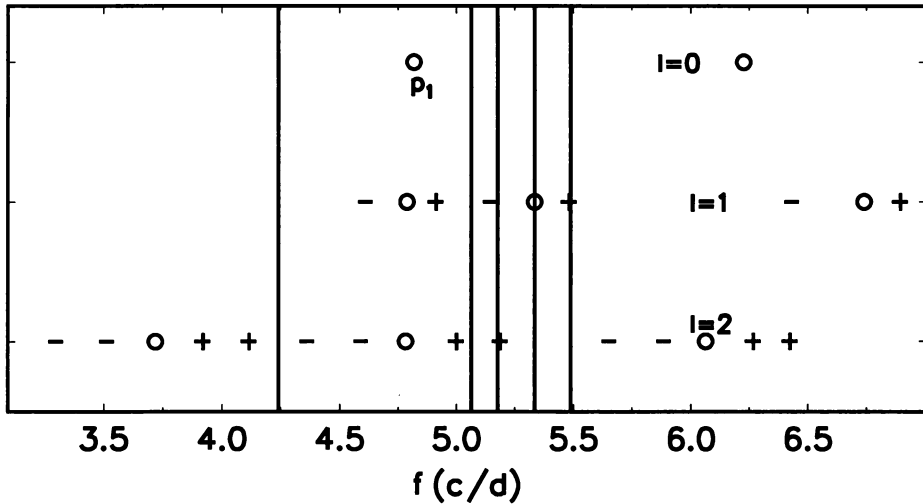
Typically, a large number of modes is simultaneously unstable. The instability is essentially independent of the azimuthal order. Thus, each dot actually represents  $2l + 1$  modes. Unstable modes with  $l > 2$  occur in the same frequency ranges as those for  $l \leq 2$ . Even if, invoking visibility arguments, we disregard high degree modes we still find in cooler  $\delta$  Sct stars over 80 unstable modes – more than four times as many as detected in the richest periodograms. For  $\beta$  Cep stars the number of unstable modes is significantly smaller than in  $\delta$  Sct stars. However, it is still much larger than the maximum number of detected modes which is six. In both types, the frequency range of the detected modes agrees with the theoretical prediction. What we do not understand is why we do not see most of the unstable modes. We do not know whether they are not excited or have amplitudes below the detection level.

Only at  $l = 0$  do we see an almost equidistant frequency separation



*Figure 2.* Frequencies (in c/d units,  $1 \text{ c/d} = 11.57 \mu\text{Hz}$ ) of low-order p- and g-modes of low degree for models of  $12 M_{\odot}$  and  $1.8 M_{\odot}$  stars in the main sequence evolutionary phase. Big dots denote unstable modes.

between consecutive modes, which is a property of p-modes. At  $l > 0$  the picture is complicated by the occurrence of unstable g-modes, whose frequencies increase during the initial phases of the evolution. Except close to ZAMS all the unstable nonradial modes have a mixed g- and p-mode character. Instability of g-modes is a good news for prospects of seismic probing but it complicates mode identification procedure. Another complication is caused by rotation. At the rates typical for these stars the calculated frequency splitting departs significantly from the simple equidistant pattern seen in the PG 1159-035 power spectra.



*Figure 3.* Oscillation frequencies in DD Lac (shown as vertical lines) compared with those of unstable modes in an approximate model ( $M = 12M_{\odot}$ ,  $\log T_{eff} = 4.378$ ,  $V_{rot} = 92$  km/s,  $\log g = 3.74$ ). Frequencies of calculated modes with  $m < 0$ ,  $m = 0$ , and  $m > 0$  are shown with -, o, and +, respectively.

## 5. Selected Power Spectra

Here I compare periodograms for one  $\beta$  Cep star and two  $\delta$  Sct stars with model predictions. Chosen models have parameters within the range allowed by the photometric and spectroscopic data for the objects but in no case effort was made to fit frequencies except of one mode in  $\beta$  Cep type star. Thus, these are not even approximate seismic models in the sense explained in section 7.

In this comparison I consider only modes with  $l \leq 2$ . Modes of higher degrees are also unstable but they are less likely to reach detectable amplitudes. For such modes the integration over the stellar disk includes comparable terms having opposite signs and this leads to a considerable reduction of observable amplitudes. As clearly seen in the whole disc solar data, a large increase in the amplitude reduction takes place between  $l = 2$  and 3 degrees. Assuming  $l \leq 2$ , identification is a useful working hypothesis but it lacks a solid justification.

### 5.1. DD LAC

This  $\beta$  Cep star is a subject of an ongoing analysis done in collaboration with Mike Jerzykiewicz. The figure and the discussion were presented in a poster at this meeting. The equidistant triplets suggest the  $l = 1$  interpre-



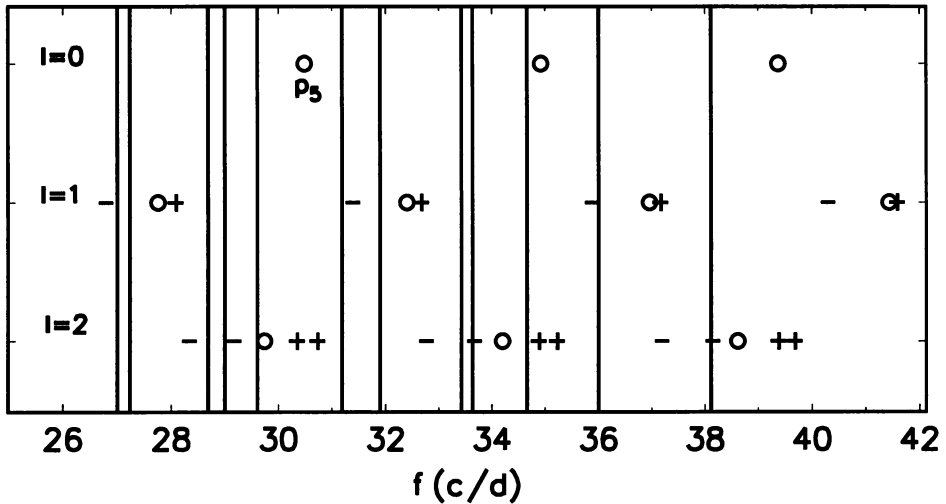


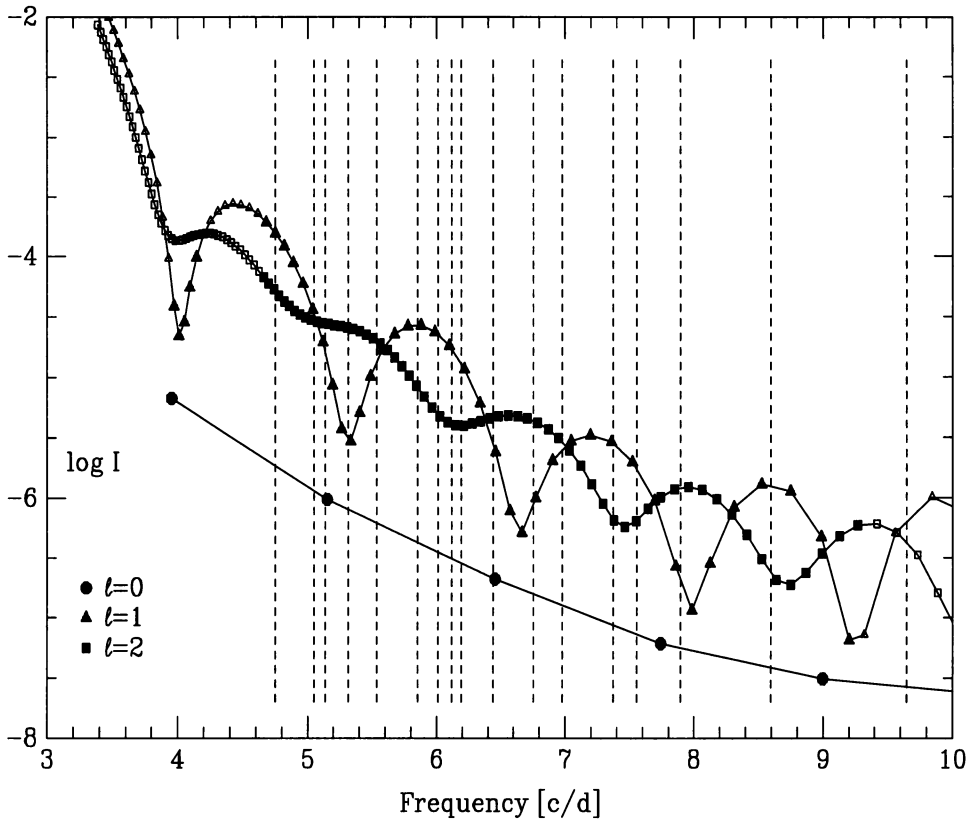
Figure 4. Oscillation frequencies in CD-24 7599 (shown as vertical lines) compared with those of unstable modes in an approximate model ( $M = 1.95M_{\odot}$ ,  $\log T_{eff} = 3.910$ ,  $V_{rot} = 70$  km/s,  $\log g = 4.06$ ). Frequencies of calculated modes with  $m < 0$ ,  $m = 0$ , and  $m > 0$  are shown with -, o, and +, respectively.

tation, which is also consistent with the photometric data. A sequence of models was constructed that fits the central peak of the triplet to  $l = 1$ ,  $m = 1$  modes. The model shown in Figure 3 is an example. A common problem for all these models is that the quadratic effect of rotation causes departure from constant separation far greater than in the data. One possible explanation suggested is the phase locking resulting from resonant interaction between the three modes (Buchler *et al.*, 1995).

In all models of DD Lac consistent with the data two close  $l = 1$  triplets occur in the frequency range of the observed peaks, because all these models are close to avoided crossing between  $g_1$  and  $p_1$  modes. The occurrence of this effect may be seen in Figure 2. Such modes are of special interest for asteroseismology because their frequencies are very sensitive to the extent of convective overshooting (Dziembowski & Pamyatnykh, 1991; Audard *et al.*, 1995).

## 5.2. CD-24 7599

This recently discovered  $\delta$  Scuti star in a joint campaign of WET and DSN networks (Handler *et al.*, 1996) was selected by us (a collaboration including Pamyatnykh, Handler, Goode and Pikall) to practice the methodology of constructing seismic stellar models. The object seems a good choice. It has many detected modes and is located near ZAMS. One may see in Fig. 2



*Figure 5.* Oscillation frequencies in 4 Canum Venaticorum (shown as vertical lines) compared with those in an approximate model ( $M = 2.3M_{\odot}$ ,  $\log T_{eff} = 3.84$ ,  $V_{rot} = 88$  km/s,  $\log g = 3.39$ ). The ordinate gives moment of inertia ( $I$ ) which is evaluated assuming the same radial displacement at the surface. Modes partially trapped in the envelope are characterized by low  $I$ . Only the  $m = 0$  modes are shown. Empty symbols denote stable modes.

how the complexity of the oscillation spectra increases with the progress of the evolution. The project is not finished. Some preliminary results are included in a paper which is now in press (Handler *et al.*, 1996). In Fig. 4 I use one of the approximate models used in that project.

The star is located near the high- $T_{eff}$  boundary of the  $\delta$  Sct domain in the H-R diagram. In such a case only higher order modes are unstable beginning with  $p_4$  or  $p_5$ , at  $l = 0$ . Stability calculations yield in this case useful constraints on stellar parameters, which come within the ranges allowed by the data.

The departure from a constant rotational splitting in the model spectrum is larger than in the case of DD Lac. However, a nearly constant

frequency separation between  $l = 0$  and  $l = 1$  modes is seen. A pronounced peak in the power spectrum has been identified with this separation. This identification yields yet another constraint on the star parameters (Handler *et al.*, 1996). Some of the measured frequencies nearly coincide with those of the model. However, a closer look reveals that, in fact, we are still far from a unique mode identification.

### 5.3. 4 CANUM VENATICORUM

4 CVn is an evolved  $\delta$  Sct star. The presence of at least 17 modes was revealed by Fourier analysis of the recent DSN campaign data (Breger, private communication). In contrast to the previous case this object is not a good choice for asteroseismic sounding, at least at the present stage of understanding stellar pulsations. I choose this star to illustrate what remains to be understood and is perhaps the most difficult problem of the stellar pulsation theory.

The parameters of the model used in Fig. 4 are within the range allowed by observational data. However, what matters here is only that this is a post-main sequence star. In deep interiors of such objects the Brunt-Väisälä frequency attains very large values. The consequence is the large density of oscillation spectra for nonradial modes seen in Fig. 4. The actual density is  $2l + 1$  higher because only modes with  $m = 0$  are shown. The total number of unstable modes with  $l \leq 2$  is about 450.

The oscillatory behavior of the inertia reflects the effect of partial mode trapping in the acoustic cavity. We do not know whether this effect has anything to do with mode selection. It is possible that the amplitudes are random quantities and that there is no rule of mode selection. Without such a rule we will never be able to make use of nonradial mode frequencies for seismic probing.

## 6. Mode Identification Using Additional Observables

The way to a seismic model of an oscillating star is much easier if we know the spherical harmonics for any of the excited modes. This information may be, in principle, inferred from observational data.

Stamford and Watson (1977) first noted that ratios of intensity amplitudes measured at various wavelengths and ratios involving the amplitude of radial velocity are sensitive to the  $l$ -value. Balona & Stobie (1979) showed that the color to intensity amplitude ratio *versus* corresponding phase difference diagrams yields a particularly revealing diagnostic. This method of  $l$  determination has been developed in a number of subsequent papers and applied to various objects. Recently, Cugier *et al.* (1994) published an extensive survey of diagnostic diagrams for  $\beta$  Cep stars based on accu-

rate calculations of stellar oscillations. In several types of the diagrams the loci of  $l = 0, 1,$  and  $2$  modes are well separated. Hence, it is possible to discriminate between the three identifications without knowledge of stellar parameters. For  $l = 0,$  having some information about the parameters, one may use the diagrams to discriminate between the  $n = 1$  and  $n = 2$  modes.

The amplitude ratios and the phase differences are independent of the mode azimuthal order,  $m,$  and of the inclination angle of the rotation axis,  $i.$  Both quantities, which are very useful in constructing seismic models, may be inferred from line profile changes. A systematic method of determination of  $m, l,$  and  $i$  from line profile data was developed by Balona (1986) and by Aerts (1996), who applied it with a moderate success to the  $\beta$  Cep star 12 Lac. Unfortunately, in the case of  $\delta$  Sct stars with rich oscillation spectra we do not have a single mode with reliably determined  $l$  and  $m$  values. The situation is somewhat better in the case of  $\beta$  Cep but, in fact, none of the suggested identifications could be regarded as fully reliable.

## 7. Seismic Stellar Models

Even in best cases the number of modes detected in individual stars is far too small to determine the radial structure directly from measured frequencies, like it is done in helioseismology. The best we may hope for is using observed frequencies to determine global parameters characterizing the star and certain parameters of the theory. Basic equations for the seismic model may be written in the following form

$$\nu_{j,obs} = \nu_{j,cal}(l_j, m_j, n_j, \vec{P}_S, \vec{P}_T),$$

where  $j$  labels measured frequencies,  $\vec{P}_S$  gives the set of parameters characterizing the model, and  $\vec{P}_T$  the set of free parameters of the theory.

The parameters given by  $\vec{P}_S$  are those characterizing the evolutionary sequence like initial mass,  $M,$  chemical composition,  $X_0$  and  $Z_0,$  and the angular momentum and a single parameter identifying the model, for which age is always a good choice. Alternatively, one may use the initial equatorial velocity  $V_{rot,0}$  instead of the angular momentum and, in application to objects in the expansion phase of the Main Sequence evolution,  $\log T_{eff}$  instead of the age. In any case there are five parameters in  $\vec{P}_S.$  The quantities one may include in  $\vec{P}_T$  are mixing length parameter  $\alpha,$  overshooting distance  $d,$  as well as parameters describing angular momentum evolution and mass loss. In applications to Upper Main Sequence objects  $\alpha$  is rather unimportant. Adopted defaults in our projects are  $\alpha = 1, d = 0,$  uniform rotation, and a global angular momentum conservation.

The model must be consistent with photometric and spectroscopic data on the star. However, the accuracy of such data is usually much poorer than

that of the frequency measurement and they allow several different identifications for the modes. In our project a preliminary fit of the frequencies is obtained by adjusting values of  $M$ ,  $V_{rot,0}$ ,  $\log T_{eff}$ , using the tabulation of model frequencies for all modes with  $l \leq 2$  in the specified frequency range. Identification of the excited modes is based on the fit quality, measured by

$$\chi^2 = \frac{1}{J} \sum_{j=1}^J \left( \frac{\nu_{obs} - \nu_{cal}}{\sigma_{obs}} \right)_j^2,$$

where  $J$  is the number of modes in the data set. In the example shown in Fig. 4 we see that a unique assignment of modes to all measured frequencies is not possible. At such point one may either continue the search in the parameter space or consider an identification with higher degree modes. A formal determination of all parameters by a least square method is the goal, which we hope to achieve at some point. At present, we still do not know whether this procedure will lead to a unique mode identification.

For seismic models of the Sun (Dziembowski *et al.* 1995, Basu & Thompson 1996) we have  $\chi \sim 1$ . We are very far from such good fits in asteroseismology. Therefore, all published models may only be regarded as approximate seismic models. Perhaps the most advanced are the models of the white dwarfs PG1159-035 (Kawaler & Bradley, 1994) and GD 358 (Bradley & Winget, 1994). Models of the  $\delta$  Scuti stars: GX Peg (Goupil *et al.*, 1993) and FG Vir (Guzik & Bradley, 1995) as well as that of the  $\beta$  Cep star EN Lac (Dziembowski & Jerzykiewicz, 1996) are crude and based on inadequate treatment of the effects of rotation on oscillation frequencies. Typical equatorial velocities of rotation in  $\delta$  Sct and  $\beta$  Cep stars are in the range 50-200 km/s. At such velocities the effect of rotation on oscillation is not reduced to a simple Zeeman-like frequency splitting. Accurate treatment of rotation is essential for the construction of seismic models of these stars.

## 8. What We May Expect from Space Asteroseismology

The amplitude resolution in observations from space is between 1 and 10  $\mu\text{mag}$  which means between two and three orders of magnitude better than in ground-based photometry. If everything goes as planned the EVRIS instrument installed on board of MARS96 will soon start providing us with data. It is likely that the more ambitious, devoted mainly to asteroseismology, COROT mission will follow (Baglin in these proceedings). Observations from space will open a new epoch in this field. We will certainly have much more frequency data. It is, however, less clear whether we will be able to make good use of them.

Almost certainly solar-like oscillations in a number of stars will be detected and two parameters known as large,  $\Delta\nu_l = \nu_{l,n+1} - \nu_{l,n}$ , and small,

$D_0 = 0.5(\nu_{0,n} - 0.5(\nu_{1,n+1} + \nu_{1,n}))$ , frequency separations will be determined. These measurements will allow us to locate the objects in the Christensen-Dalsgaard (1988) seismic H-R diagram and constrain their global parameters (e.g. Gough, 1995). It is less certain whether oscillatory features in the  $\Delta\nu_l(\nu)$  dependence will be detectable. These features as Gough (1990) showed probe the lower boundary of the convective zone.

Another safe prediction is that many new modes will be detected in  $\delta$  Scuti and in other types of known pulsating stars. The obvious candidates are nonradial modes with  $l > 2$ . Goupil *et al.* (1996) argued that with data from space a detection of modes up to  $l = 7$  is likely. Further, they considered a conjectured set of unstable modes with  $l = 1$  to 7 in  $\delta$  Scuti star models and showed that if we had measurements of the rotational splitting for a fraction of these modes we could quite precisely determine the rotation rate behavior in the interior, including the chemically inhomogeneous zone surrounding the convective core. The latter possibility follows from the presence of a number of g-modes in the conjectured set. The most optimistic aspect of that exercise is that we will be able to identify the modes. We should have no illusion that this will be an easy task.

### Acknowledgements

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