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Parametric Instability of a Broad-band Alfvén Wave: Nonlinear Evolution and Saturation

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Abstract. The nonlinear evolution of parametric instability of a non-monocrhomatic Alfvén wave is studied using numerical simulations. After a linear stage the instability saturates. For $\beta \sim 1$ the initial mode remains dominant, except at large scales, where an inverse cascade of backscattered Alfvénic modes is present. A comparison with solar wind data gives a qualitative agreement, indicating that parametric instability could play an important role in the evolution of solar wind turbulence.

1. Introduction

Low frequency fluctuations in the high-latitude solar wind are mainly Alfvénic, i.e., they have a high degree of velocity-magnetic field correlation and a low level of density and magnetic field intensity fluctuations. However, Alfvénicity tends to decrease with increasing distance r from the Sun. The normalized cross-helicity $\sigma = E^+ - E^-/(E^+ + E^-)$, where E^+ and E^- are the energies of outward and inward propagating Alfvénic fluctuations, decreases with increasing r. A possible mechanism of decorrelation is given by the parametric instability, which can affect an Alfvén wave, generating both oppositely-correlated Alfvénic fluctuations and compressive waves. It has been studied in various circumstances (Galeev and Oraevskii 1963; Goldstein 1978; Viñas & Goldstein 1991; Malara & Velli, 1996). Here we focus on: (i) the nonlinear evolution of the instability for a broad-band initial wave; (ii) the saturation level; (iii) the final fluctuation spectrum. We built up a numerical model (Malara et al. 2000) which describes the nonlinear evolution of a broad-band large-amplitude Alfvénic fluctuation. The results are compared with measures in the high-latitude solar wind.

2. Numerical model and results

We numerically solved the full set of nonlinear MHD equations . All quantities depend on one space variable x and time t. Periodicity boundary conditions have been used. The initial magnetic field is defined by

$$\mathbf{b}(x, t = 0) = B_0\mathbf{e}_x + B_1 \cos[\phi(x)]\mathbf{e}_y + B_1 \sin[\phi(x)]\mathbf{e}_z \tag{1}$$

where $B_0 = 1$ and $B_1 = 0.5$ are the amplitudes of the equilibrium field and of the initial wave, respectively. The above expression gives a uniform magnetic field intensity. The phase is given by

$$\phi(x) = k_0 x + a \sum_{k=1}^{k_{max}} k^{-1} e^{ikx+\delta_k} \tag{2}$$

For $a = 0$ the wave is monochromatic at wavevector k_0; with increasing a the spectrum becomes wider, approximately following a power law. The initial velocity is given by $\mathbf{v}(x, t = 0) = \mathbf{v}_\perp = \delta \mathbf{b}_\perp / \sqrt{\rho}$. Here δf indicates the fluctuating part of any quantity f. The initial temperature T_0 is uniform, as well as the initial density ρ_0, except for a small density perturbation, which has been added in order to initiate the instability.

We consider the evolution of the following physical quantities: the energy $E^\pm = \langle |\delta \mathbf{Z}^\pm|^2 \rangle / 2$ of the Elsässer variables $\mathbf{Z}^\pm = \mathbf{v} \pm \mathbf{b}/\sqrt{\rho}$; the levels $r = \sqrt{\langle (\delta\rho/\rho_0)^2 \rangle}$ and $m = \sqrt{\langle (\delta |\mathbf{b}| / |\mathbf{B}_0|)^2 \rangle}$ of density and magnetic field intensity fluctuations, respectively; the cross-helicity σ.

At the initial time, $\sigma = 1$ and compressive fluctuations have a very small amplitude. During the first stage of the time evolution, the quantities E^-, r and m grow, following a nearly exponential growth. The growth rate γ decreases with increasing a and it is maximum for a monochromatic wave. However, even for larger spectral widths, it remains of the same order of magnitude as in the monochromatric case, in accordance with the results of the linear theory by Malara & Velli (1996).

When the unstable perturbations have reached a sufficiently high level, non-linear effects stop the growth and the instability saturates. We considered the case $\beta = 1$, which is realistic for the solar wind. At saturation the level of $E^-(t)$ is comparable but it remains lower than $E^+(t)$ (Fig. 1, left, upper panel). Compressive fluctuations saturate at $r(t) \simeq 0.03$, $m(t) \simeq 0.01$ (Fig. 1, left, middle panel). Cross-helicity decreases in time, but the final level keeps positive ($\sigma \simeq 0.6$) (Fig. 1, left, lower panel). Thus, at intermediate β the instability reduces but it is unable to completely destroy the initial Alfvénic correlation.

Before saturation the spectra e_k^- and e_k^ρ of the two Elsässer variables have a similar shape (Malara et al. 2000). At saturation we found $e_k^- \ll e_k^+$ in all the spectral range, except at the smallest wavenumbers, where $e_k^- \sim e_k^+$. This behaviour is due to the particular coupling between unstable modes, which is strongly non-local, i.e., $k^- \ll k_0$, where k^- and k_0 are the dominant wavenumbers in the Alfvénic daughter and mother wave, respectively. After saturation, the spectrum e_k^- approaches e_k^+, and the two spectra become nearly parallel.

3. Comparison with solar wind data

Bavassano et al. (2000) calculated the behaviour of the energies $E^\pm$ of Elsässer modes with increasing the distance r from the Sun, using data of Helios 2 and Ulysses spacecrafts (Fig. 1, right).

In consequence of the solar wind expansion, both E^+ and E^- decrease with increasing r. The decrease of E^- is slower, but, at distances larger than ~ 2.5 AU the two curves become parallel. Correspondingly, the decrease of cross-helicity

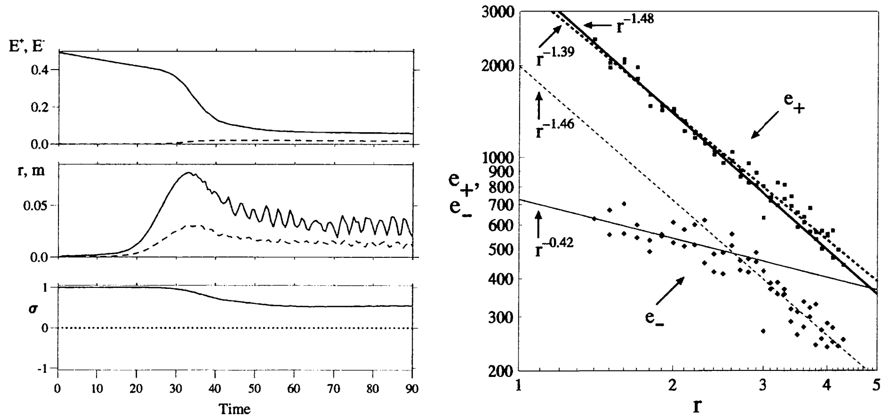

Figure 1. Left: in upper panel $E^+(t)$ (full line) and $E^-(t)$ (dashed line); middle panel $r(t)$ (full line) and $m(t)$ (dashed line); bottom panel $\sigma(t)$. Right: measures of E^+ and E^- vs. r (from Bavassano et al. 2000).

stops at the value $\sigma \sim 0.3$. This behavior is remenant of that found in our model. Concerning spectral quantities, at distances $r = 2$ AU, $e_k^- \ll e_k^+$ (except at large scales where $e_k^- \sim e_k^+$). At larger distances the two spectra approach each other, becoming nearly parallel (Goldstein et al. 1995). This behaviour is qualitatively similar to that found in the above-described simulations.

In conclusion, we showed that a non-monochromatic Alfvén wave is parametrically unstable, and the nonlinear evolution of the instability leads to saturation. For $\beta \sim 1$ the initial Alfvénic correlation is reduced but it remains positive. The spectra display an inverse cascade of $\mathbf{Z}^-$ fluctuations. These features are qualitatively in accordance with observations in the solar wind. Parametric instability can play an important role in the evolution of turbulence in the high-latitude solar wind.

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