Philip E. Seiden
IBM Thomas J. Watson Research Center
Yorktown Heights, New York 10598

## INTRODUCTION

Most approaches to explaining the long-range order of the spiral arms in galaxies assume that it is induced by the long-range gravitational interaction. However, it is well-known in many fields of physics that long-range order may be induced by short-range interactions. A typical example is magnetism, where the exchange interaction between magnetic spins has a range of only 10 Angstrbms, yet a bar magnet can be made as large as one likes. Stochastic self-propagating star formation (SSPSF) starts from the point of view of a short-range interaction and examines the spiral structure arising from it (Seiden and Gerola 1982). We assume that the energetic processes of massive stars, stellar winds, ionization-front shocks and supernova shocks, in an OB association or open cluster can induce the creation of a new molecular cloud from cold interstellar atomic hydrogen. In turn this new molecular cloud will begin to form stars that will allow the process to repeat, creating a chain reaction. The differential rotation existing in a spiral galaxy will stretch the aggregation of recently created stars into spiral features.

This process belongs to a class of phenomena called percolation and exhibits the phase transition characteristic of this kind of system (Schulman and Seiden 1982). It is only necessary that the short-range interaction be strong enough to cross the phase-transition point; its value after that is immaterial since, as shown below, the strong feedback between the gas and stars will control the process. In the rest of this paper I discuss only the statistical mechanics of the percolating process and not the details of the interaction itself.

THE MODEL
SSPSF is easily investigated by means of a computer simulation. We create a polar array of an appropriate number of rings and divide each ring into cells so that each cell in the array has the same area. The rings are allowed to rotate following a desired rotation curve. Three
arrays are included, one of stars, a second of atomic hydrogen and a third of molecular hydrogen.

The process proceeds as follows. If there is a young cluster created in a cell at time $t$, it can create a new molecular cloud in any one of its neighbouring cells at time $t+1$ with a probability

$$
\begin{equation*}
P_{e f f}=P D_{a}, \tag{1}
\end{equation*}
$$

where $P$ is a constant and $D_{a}$ is the density of atomic hydrogen. The molecular gas returns to its atomic form with a gas recycling time, $\tau$. The recycling time is not just the lifetime of some observed feature of a molecular cloud, but it is the length of time it takes for the gas to return to the cold atomic-hydrogen state suitable for shocking and repeating the process.

The feedback between the stars and the gas provides a close control of the amount of atomic hydrogen in the system (Seiden 1983). Figure 1 shows results for a model in which the total gas density was chosen to drop off exponentially with radius. The figure shows that, as long as the total gas density is large enough, the atomic hydrogen density is pinned at a constant value. This result is independent of the recycling, time, as long as it is great enough to provide effective feedback.


With this result in mind we can rewrite equation (1) as follows

$$
\begin{equation*}
P_{e f f}=D_{a} / D_{c}, \tag{2}
\end{equation*}
$$

where $D_{c}$ is a critical gas density at which cloud formation is assured (the largest possible value of equations (1) and (2) is unity, greater values are truncated to unity). We can evaluate $D_{c}$ from observations; Burton and Gordon (1978) found that the atomic hydrogen in our Galaxy is roughly flat over a wide range of radii with a value of about $0.35 \mathrm{H} \mathrm{cm}^{-3}$. Simulations show that feedback pins the effective nrobability at about 0.3 (Seiden 1933). Therefore, $D_{c}=1 \mathrm{H} \mathrm{cm}^{-3}$.

Since the atomic hydrogen is constant over the galactic disk, the molecular hydrogen will be given by the total gas content minus a constant. We don't know what the total gas content of a galaxy is in general, but we can argue that from the nature of the rotation curve it might be close to $1 / r$. Firstly, a flat rotation curve for a disk implies that the mass density of the disk is $1 / r$. For a spherical distribution it will be $1 / \mathrm{r}^{2}$, but if the disk is formed by the accretion of gas from the spherical distribution, the resulting gas disk will again be $1 / \mathrm{r}$. Observational evidence seems to indicate that the star-formation rate in the Galaxy has not changed appreciably over the lifetime of the disk, so the gas distribution may not have appreciably changed either. In any event the essential point can be illustrated with a $1 / r$ total gas distribution. This is shown in Figure 2: the $1 / \mathrm{r}$ line shows no trace of the rough exponential behaviour usually seen for both the luminosity of a galaxy and its molecular hydrogen (Young and Scoville 1982).


Figure 2. Origin of the exponential disk. The upper curve is $1 / r$; when a constant (C) is subtracted from this curve, the lower curve is obtained. The good fit to an exponential (superposed straight line) extends over a large fraction of the disk.

However, when a constant is subtracted from this curve, a clean straightline section appears - the so-called exponential disk (Freeman 1970). See Seiden, Schulman and Elmegreen (1984) for a more extensive discussion.

## THE MILKY-WAY SIMULATION

The SSPSF model described above will now be applied to a simulation of the Milky Way. The simulation was done with the following parameters. The array contains 90 rings and has a radius of 18 kpc so that the cell size is 200 pc . The position of the Sun is at 8 kpc . The rotation curve is rigid to 3 kpc and is then flat with a value of $220 \mathrm{~km} / \mathrm{sec} \cdot \mathrm{D}_{\mathrm{c}}=1 \mathrm{~cm}^{-3}$ as above and the total gas density is chosen to vary as $1 / r$ with a value of $0.3 \mathrm{~cm}^{-3}$ at 18 kpc . Inside of 3 kpc , where the rotation curve is rigid, the gas density is taken as flat, independent of radius. The time step is $10^{7}$ years and the gas regeneration time is $2 \times 10^{8}$ years.

Figure 3 shows typical radial dependencies for this simulation. The atomic hydrogen is roughly flat as expected, and the molecular hydrogen falls off roughly exponentially from 3 kpc to about 14 kpc . The blue luminosity shows the same approximate exponential behaviour. The more rapid fall-off at larger radii is due to the phase-transition nature of the process. Such a fall-off has been observed by van der Kruit and Searle (1982) for edge-on disk galaxies.


We can compare our results to observations in the following manner. First, we normalize the surface brightness by choosing $\mathrm{r}_{25}=11.5 \mathrm{kpc}$ as given by de Vaucouleurs and Pence (1978). Table I shows the values we get for a number of other parameters, along with the observed values.

TABLE I
calculated observed

| $\mu_{\mathrm{e}}\left(\mathrm{mag} / \operatorname{arcsec}^{2}\right)$ | 23.43 | 23.47 | (de Vaucouleurs and Pençe 1978) |
| :---: | :---: | :---: | :---: |
| $\mathrm{r}_{\mathrm{e}}$ (kpc) | 6.1 | 5.68,5.98 | (de Vaucouleurs and Pence 1978) |
| $\mathrm{r}_{\text {Holmberg }}(\mathrm{kpc})$ | 15.1 | 17.05 | (de Vaucouleurs and Pence 1978) |
| $\mathrm{r}_{\mathrm{D}}(\mathrm{kpc}){ }^{\text {c }}$ | 4.06 | 3.38,3.56 | (de Vaucouleurs and Pence 1978) |
| $\mathrm{r}_{\text {max }}$ (kpc) | 16.2,16.8 | 16-20 | (Chromey 1978) |
| $\mathrm{M}(\mathrm{HI})\left(10^{9} \mathrm{M}_{\square}\right.$ ) | $0.8 \mathrm{z}_{100}$ | 1.66 | (Baker and Burton 1975) |
| Arm/Interarm contrast: |  |  |  |
| HI | 3.6-5.2 | 4 | (Kulkarni et al. 1982) |
| $\mathrm{H}_{2}$ | 10-30 | 10-20 | (Stark 1983) |



Figure 4. Two time steps of the Milky-Way simulation. The outer circles are the limits of the array. The inner circle in the upper left-hand frame is the radius of the Sun.

One major difference we find from the de Vaucouleurs and Pence results is in the value for the Holmberg radius. Theirs is considerably bigger since they chose an exponential disk all the way out. The rapid fall-off of Figure 3 gives a smaller value for the Holmberg radius. The parameter $r_{\text {max }}$ is the van der Kruit and Searle parameter for the cut-off of the disk. We obtain it in two different ways. First, analysis shows that it is just $4 \mathrm{r}_{\mathrm{D}}$ (Seiden, Schulman and Elmegreen 1984). Second, we can do a fitting procedure to the data similar to van der Kruit and Searle (1982). The observed value for our Galaxy comes from the radial decrease of Chromey's (1978) blue-star counts for the anticenter. The atomic hydrogen is the amount within a $12-\mathrm{kpc}$ radius, $\mathrm{z}_{100}$ is the scale height in units of 100 pc . The observed value is the Baker and Burton (1975) result as corrected by de Vaucouleurs and Pence (1978) for 8 kpc .

Figure 4 shows the stars, atomic hydrogen and molecular hydrogen for two timesteps. The radius of the Sun is shown by the inner circle on the top left-hand picture. The spiral arms can be clearly seen in all three pictures. The stars and molecular hydrogen show the same structure but with different contrast. However, the atomic hydrogen is roughly a negative image of the stars and molecular hydrogen. The clearest arms are shown by a depletion of atomic hydrogen; nevertheless, one can still trace the arms by the ridges of dense atomic hydrogen, but these are displaced from the locus of the stellar and molecular-hydrogen arms by a few hundred parsecs. One should also note that there is atomic hydrogen surrounding each molecular cloud and this has not been included in Figure 4.

One feature that this simulation does not reproduce properly is the hole in the gas inside 3 kpc . This may be a three-dimensional effect, since in the inner region of the Galaxy the simple two-dimensional disk approximation may break down.

This one exception aside, the SSPSF model gives a reasonable description of the Milky Way with few adjustable constants. In fact the only constant that is really a free parameter unconstrained by observation is the gas recycling time.

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J.P. Ostriker: Several people have noted that cloud-cloud collisions will produce viscous transport of mass and angular momentum on a relevant time scale within a radius of a few kpc of the center of our Galaxy. If you had included dynamics: collisions and viscosity, you probably would have produced a central hole.
F.H. Shu: Could you list for us all the free parameters of your theory, what physical processes they are supposed to represent, and how you choose the numerical values for these parameters in your simulations?

Seiden: I can afford to be somewhat cagey about the physical processes, because $I$ am doing statistical mechanics of the array of molecular clouds and clusters being formed. Similarly, in statistical mechanics you do not have to know the molecular orbitals of the atoms scattering off each other; you only have to know that they scatter off each other so that they reach equilibrium. In this case all I have to say is that a process exists whereby massive stars can create new clouds. I do not have to specify the details of that process in order to make calculations and obtain results, because in statistico-mechanical language $I$ am doing the macroscopic theory of the phenomenon, not the microscopic theory (and "microscopic" here means 200 pc ). The free parameters are: rotation speed, cell size, molecular recycling time, and gas density distribution. Their values are given in the paper.
J.H. Oort: Can you make a barred spiral?

Seiden: No, a barred spiral is undoubtedly a dynamical effect. There is no dynamics in these calculations, there is no gravity (other than the rotation curve which comes from gravity).
U.J. Schwarz: Have you compared the morphology of your Galaxy with real galaxies? They make the impression of a mixture of irregular galaxies with spiral patches, a sort of fireworks.

Seiden: There are lots of galaxies which look like that in the Hubble Atlas.
R.S. Cohen: As far as we can tell in our own Galaxy, the molecular and atomic arm features agree very closely.

Seiden: To better than 200 pc ?
Cohen: Well, there are all these dangers of course in the (1,V) diagram. But as far as we can read off the diagram, they agree exactly.

Seiden: There should be a shift of a few hundred parsecs - that's all.
W. Renz: From your film I got the impression that the star-formation rate is rather constant in time. On the other hand, the poster paper by Feitzinger at this Symposium shows that for a certain choice of model parameters one can obtain a periodically variable star-formation rate. Have you performed numerical simulations about this, and what oscillation periods do you find?

Seiden: The original calculation was a collaborative effort by Feitzinger, Schulman and myself. The oscillations increase with the value of , the lifetime of the molecular clouds. For high enough values of , star formation occurs in pulses, with almost-zero values in between. Whether those values of are realistic or not, is an open question at this time. The oscillation period is related to , it is not quite linear in but it is of the same order.
A.A. Stark: A point of nomenclature. What you and others usually call the lifetime of molecular clouds, would be more properly called the "molecular recycling time".

Seiden: That's a wonderful word. I did not think of it, but I will use it from now on. (It is used indeed in the printed paper - Editor.)

Stark: I know, we people in AT\&T are always competing with IBM, but ...
Seiden: Hahahaha. Yes, but being the only industrials around, we should collaborate here.
M.L. Kutner: It seems that this recycling time is too long to allow a strong contrast between arms and interarm regions as we observe in CO.

Seiden: No - it depends on what you are talking about. I have not made a complete analysis of all the contrasts that are available in these models, $I$ just did a few to have some numbers here. But the highest contrast occurs around those cells that had clouds formed in them fairly recently. So the lifetime is not really important: the contrast is between cells that have not formed clouds for well over a timescale, and cells that have just formed clouds a few timesteps ago.
R. Beck: There is increasing evidence that the process of star formation is influenced by the surrounding interstellar magnetic field. Thus it seems important to consider the field strength and structure in your theory of star-formation propagation.

Seiden: That has been tried by Chiang and Elmegreen. They used an anisotropic, stimulated probability in the old-star model without gas. They found an effect in galaxies that do not have large rotation velocities, no large shears - for instance the Magellanic Clouds, or galaxies with even more rigid rotation than those. For large disk galaxies there was no effect: the strong rotation shear dominated everything.

