THE COMPARATIVE MERITS OF THE OLD AND NEW METHODS OF SOLUTION.

To the Editor.

SIR,—I select the enclosed from some old papers on which I have just put my hands. It was not prepared with any view to publication, and I send it you without the change of a word.

It appears to me deserving of notice from its bearing on the question of the comparative merits of the old and the new methods of solution—Barrett's and Milne's. The facility with which the result is obtained, and the *aptitude* (so to speak) with which the various elements combine to form an expression that so plainly tells its own tale, are very striking and instructive.

I am, Sir,
Yours obediently,

London, 17*th November*, 1865. P. GRAY.

PROBLEM.

A man, (x), and his wife (y), wish to provide an annuity of £a, to be enjoyed by (y) after the death of (x), if within n years; but to be entered upon in n years if both or either be then alive, and to continue till both lives are extinct. Required the annual premium, ϖ, payable till the annuity is entered upon.

The annuity consists of two distinct portions, the first during, and the second after, n years. The first is a temporary reversionary annuity on (y) after (x); and its present value, therefore, is $a\left(\frac{N_{y|n}}{D_y}-\frac{N_{xy|n}}{D_{xy}}\right)$. The second is a joint life and survivor annuity deferred n years, the present value of which is $a\left(\frac{N_{x+n}}{D_x}+\frac{N_{y+n}}{D_y}-\frac{N_{x+n\,y+n}}{D_{xy}}\right)$. The present value of the benefit therefore is

$$a\left(\frac{N_{y|n}}{D_y}-\frac{N_{xy|n}}{D_{x.y}}+\frac{N_{x+n}}{D_x}+\frac{N_{y+n}}{D_y}-\frac{N_{x+n\,y+n}}{D_{xy}}\right)=a\left(\frac{N_y}{D_y}+\frac{N_{x+n}}{D_x}-\frac{N_{xy}}{D_{x.y}}\right).$$

And since the premium will cease on the occurrence of either of the two events, the death of x or the lapse of n years, whichever should happen first, its present value will be $\frac{\varpi N_{x-1|n}}{D_x}$.

Equating this with the foregoing expression, we find

$$\varpi=\frac{aD_x\left(\frac{N_y}{D_y}+\frac{N_{x+n}}{D_x}-\frac{N_{x.y}}{D_{xy}}\right)}{N_{x-1|n}}.$$

Scholium.—From an inspection of the expression for the present value of the benefit, it appears that, agreeably to the method called by Professor De Morgan "The balance of annuities," the conditions of the benefit will be fulfilled by granting an annuity to both (y) and (x); that of the former to commence now, and that of the latter in n years; and to withdraw one of them so long as both (y) and (x) survive. And a little consideration shows that this is just. P. G.

London, 21*st September*, 1844.