Gravitational Fragmentation in Expanding Shells

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Abstract. We investigate the gravitational fragmentation in expanding shells by applying an instability 'thermometer' similar to the Toomre parameter for instabilities in self–gravitating disks. For Sedov–like evolving systems the onset of instability is mainly depending on the density of the ambient medium and the sound speed of the shell matter, whereas the energy injection rate is less important. Shells evolve towards gravitational instability, if the density gradient of the ambient medium is shallower than an isothermal profile, otherwise they become more stable. For density gradients flatter than $\propto r^{-1}$, the fragmentation becomes non–linear on the same time scale as the gravitational instability needs to start. In a homogeneous ambient medium the typical size of gravitationally unstable shells is 1 kpc for a gas density of $n = 1\,\mathrm{cm}^{-3}$ and decreases to 10 pc for $n = 10^4\,\mathrm{cm}^{-3}$.

1 Introduction

At the end of their life time massive stars strongly influence the interstellar medium (ISM) by the metal and energy input of supernova (SN) explosions. This produces supersonically expanding thin shells which sweep up the ambient gas. These shells might become gravitationally unstable, and therefore form fragments and finally stars, i.e. they could act as the agents of induced or propagating star formation. Many analytical and numerical calculations of SN induced shocks have revealed that they are unstable to the Vishniac or the Rayleigh–Taylor instability at some stages of their evolution (Vishniac, 1983; Mac Low & McCray, 1988; Mac Low et al., 1989; Vishniac & Ryu, 1989). On the other hand, in most studies the gravitational instability was considered to be practically negligible because of the low masses of the swept–up material for typical shell radii and ISM densities. However, this assumption fails when dealing with large superbubbles of the size of several 100 pc or with SN events in dense giant molecular clouds (GMC). In both cases, the mass in the shell reaches $10^4 - 10^6\,\mathrm{M}_\odot$ and gravitational fragmentation can become important. This paper investigates the gravitational instability and the fragmentation of shell matter in spherical systems applying a stability parameter – similar to Toomre's Q – to self–similar solutions for expanding shells. In an accompanying paper by Ehlerová et al. in these proceedings the implications for large–scale superbubbles in a realistic galactic environment including differential rotation and an inhomogeneous ISM are discussed.

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2 Gravitational Fragmentation

Onset of gravitational instability. According to Elmegreen's (1994) linear perturbation analysis the maximum growth rate ω of a transverse perturbation in a shell is given as

$$\omega = -\frac{v}{R}\left(-3+\sqrt{1+\frac{8}{\xi^2}}\right) . \tag{1}$$

An instability occurs, if $\omega > 0$ or the dimensionless stability parameter fulfills

$$\xi \equiv \frac{\sqrt{8}vc}{\pi G R \Sigma} < 1 . \tag{2}$$

R is the radius of the shell with a mass column density Σ. v denotes its expansion velocity relative to the ambient medium, c is the sound speed within the shell, and G is the constant of gravity. A second necessary condition is related to the finite size of the shell: The wavelength of the growing perturbation must not exceed the size of the shell. However, for the systems investigated here this criterion is always fulfilled, if $\xi < 1$ and, therefore, we can restrict the stability analysis to Eq. (2).

Fragmentation Time. The fragmentation starts at an 'instability time' t_b when Eq. (2) is fulfilled for the first time. In the linear stage of the growing instability we follow the evolution, until the fragmentation integral $I_f(t) \equiv \int_{t_b}^{t} \omega(t')dt'$ reaches unity. This defines the 'fragmentation time' t_f, when the fragments become strongly nonlinear, and the linear approximation is no longer valid.

3 Homogeneous Ambient Medium

Onset of instability. Using a dimensional analysis, Sedov (1959) derived a solution for supersonic, spherical expansion of a shell into the homogeneous medium in case of a single instantaneous energy input and vanishing external pressure. For the case of interstellar wind bubbles Castor et al. (1975) derived the solution which describes the structure and evolution of the shocked stellar wind region. This case is characterized by steady energy and mass input rates. The solution gives the relation between the radius of the shell R, the supernova rate $\dot{N}_{\rm SN}$, the mass density of the ambient medium ρ_0, and the expansion time t as follows ($E_{\rm SN}$ is the energy release per supernova):

$$R(t) = \left(\frac{125}{154\pi}\right)^{1/5} \left(\frac{\dot{N}_{\rm SN} \cdot E_{\rm SN}}{\rho_0}\right)^{1/5} t^{3/5} \tag{3}$$

where n_0 is the number density of particles in the ambient medium. The expansion velocity v of the shell is given by the time derivative of Eq. (3)

and the surface density Σ can be calculated assuming that the total mass $m(R) = 4\pi/3 \cdot \rho_0 R^3(t)$ inside the shell is swept up. Thus, we can calculate the instability parameter ξ at each time:

$$\xi(t) = \frac{24\sqrt{2}}{5} \left(\frac{18711}{64000\,\pi^4}\right)^{1/5} \cdot \frac{c}{G} \left(\frac{1}{\rho_0^4\ \dot{N}_{\rm SN}\ \cdot E_{\rm SN}}\right)^{1/5} t^{-8/5} \tag{4}$$

Eq. (4) shows that the system is always gravitationally stable prior to t_b and becomes unstable at later times.

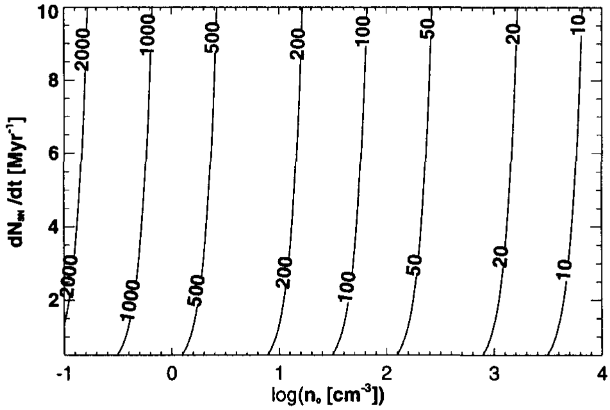

Fig. 1. The contour lines show the radius of the shell (in pc) at t_f as a function of the supernova rate $dN_{\rm SN}/dt$ and the number density n_0 for a homogeneous ambient ISM. The sound speed inside the shell was assumed to be $c = 1\,{\rm km\,s^{-1}}$.

Non-linear Stage of Fragmentation. In order to determine the fragmentation time t_f, we apply the relation $\xi(t) = (t/t_b)^{-8/5}$ together with the Sedov solution (3) to Eq. (1). From the condition $\int_{t_b}^{t_f} \omega(t)\,dt = 1$, we get

$$\frac{5}{3} = \int_1^{x_f} \frac{-3 + \sqrt{1 + 8x^{16/5}}}{x}\,dx \tag{5}$$

where $x_f \equiv t_f/t_b$. It should be noted, that the ratio of the fragmentation time t_f and the instability time t_b does neither depend on the density of the ambient medium nor on the energy injection rate and the sound speed in the shell. For a homogeneous medium a numerical solution of Eq. (5) gives $x_f \approx 2.03$. The time scales themselves are also almost independent on the energy injection rate, but depend on the density of the ambient medium. For a typical density in the solar neighbourhood of $n = 1\,{\rm cm^{-3}}$, the fragmentation time is of the order of (5-6)$\cdot 10^7$yr. The corresponding radii of the shells are shown in Fig. 1: For $n = 1\,{\rm cm^{-3}}$ we obtain radii of 600 pc to 1.1 kpc for sound speeds c within the shell between $1\,{\rm km\,s^{-1}}$ and $5\,{\rm km\,s^{-1}}$, respectively.

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4 Inhomogeneous Ambient Medium

In order to extend our analysis to inhomogeneous systems, we investigated spherical systems with a radial density gradient: $\rho(r) = \rho_0 \cdot r^{-\alpha}$. Again a self–similar solution for the expansion of the shell can be found for $\alpha < 3.5$, and we get the instability parameter

$$\xi(t) = \sqrt{\frac{8}{\pi^2}} \cdot \frac{c}{G\rho_0} \cdot \frac{3(3-\alpha)}{5-\alpha} \cdot K^{\alpha-1} \cdot t^{\frac{4(\alpha-2)}{5-\alpha}} \tag{6}$$

$$K^{5-\alpha} \equiv \frac{E_{\rm SN}\,\dot{N}_{\rm SN}}{\rho_0} \cdot \frac{(3-\alpha)(5-\alpha)^3}{6\pi(7-2\alpha)(11-\alpha)}. \tag{7}$$

Eq. (6) shows that shells evolve to gravitationally unstable systems for all density gradients flatter than an isothermal profile $\alpha = 2$. In case of steeper density gradients the dilution due to the expansion is not compensated by the agglomeration in the shell and the shells become gravitationally more stable. For an isothermal density profile ξ is constant, i.e. the gravitational stability of the shell is fixed by the initial conditions. It is interesting to note that the Rayleigh–Taylor instability starts to evolve for systems which are steeper than $\alpha = 2$. Therefore, expanding shells become unstable either to gravitational fragmentation or to the Rayleigh–Taylor instability (except for the marginally stable isothermal profile). The ratio t_f/t_b is almost constant for $\alpha < 1.2$, and goes to infinity for $\alpha = 2$, where t_b is either zero or infinity. Generalizing these results to non–spherical systems like exponential galactic disks, one expects that the whole shell is only gravitationally unstable, if it evolves into an ambient medium with a density gradient shallower than an isothermal profile. If the radius of the shell is larger than two scale–heights in z–direction before gravitational fragmentation starts, the over–pressure inside the shell will be transformed mainly into motion in z–direction, which decelerates the motion of the shell in the galactic plane and, therefore, stops further agglomeration of material there. Thus, the shell should be stable against gravitational fragmentation, if it has been grown to two scale–heights without gravitational fragmentation. However, then the Rayleigh–Taylor instability will develop.

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