SELF-SIMILAR GRAVITATIONAL CLUSTERING

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The nature of the distribution of galaxies poses a challenging problem for theorists. It seems reasonable, as a start, to suppose that galaxies and clusters arose from small perturbations by gravitational instability. However, one still has the problem of the choice of initial conditions, for example, the shape of the fluctuation spectrum and the cosmological density parameter Ω. A considerable simplification is to assume that the clustering pattern obeys some simple similarity scaling, so that the clustering at some early time, apart from a change in length scale, is statistically indistinguishable from the pattern observed today. The power-law shape of the two-point correlation function and the simple forms of higher order correlation functions (Peebles, 1980) have provided some evidence that such a simplifying assumption may be relevant - just how relevant is the subject of this article.

1. THE TWO-POINT CORRELATION FUNCTION

Self-similar gravitational clustering requires that the initial conditions do not possess any characteristic length scales and that the expansion of the universe does not present any characteristic timescales. Thus, A $\Omega = 1$ ($\Lambda = 0$, pressure = 0). B The initial power-spectrum of the matter distribution must be a power law $|\delta_k|^2 \propto k^n$. C The clustering must be due to gravity alone - non gravitational forces are ignored. With these assumptions the two-point correlation function obeys a scaling relation $\xi(x,t) \to \xi(s)$ with $s = x/t^{\alpha}$. The 'similarity' parameter α can be fixed from the initial conditions using the growing mode from linear theory, $\alpha = 4/3(3+n)$. The slope of the correlation function in the non-linear regime $\xi >> 1$ may be fixed by the requirement of small-scale stability (i.e. that clusters once formed are bound and stable with no subsequent evolution),

$$\xi(s) \propto s^{-\gamma}, \quad \gamma = 3(n+3)/(n+5), \qquad (1)$$

(Davis & Peebles, 1977). The observed non-linear slope $\gamma = 1.8$ implies white noise initial conditions, $n = 0$. Together with the stability

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assumption, the similarity solution for the three- and four-point correlation functions is $\zeta \propto s^{-2\gamma}$, $\eta \propto s^{-3\gamma}$ (Peebles, 1980, ¶ 73) and these relations are in excellent agreement with the observations.

Several approaches have been used in an attempt to derive the shape of the two-point function in the regime where $\xi \sim 1$. Davis & Peebles (1977) used the BBGKY equations truncated in a manner consistent with the observations of the three-point function ζ. Fry & Peebles (1980) have used a Monte-Carlo N-body technique, but the most widely used method has been the direct integration of Newton's equations in an expanding universe (e.g. Miyoshi & Kihara, 1975; Gott et al., 1979; Efstathiou & Eastwood, 1981; Frenk et al., 1982).

Figure 1 shows results for ξ from the N-body calculations of Efstathiou & Eastwood. The models give a correlation function which is much steeper than the observations and are in disagreement with eq. (1) because the stability assumption does not apply on scales corresponding to $\xi \lesssim 50$. Instead, one observes a radial streaming as clusters collapse in order to generate enough kinetic energy to satisfy the virial theorem.

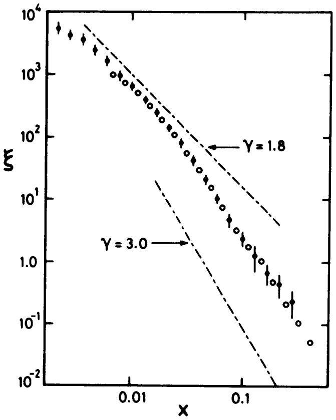

Figure 1. Results for the two-point correlation function using Poisson initial conditions and Ω = 1 after expansion by a factor of 9.9. Filled circles show results for 6 models with N = 1000 and the open circles show results for one 20000 body model.

Of course there are difficulties in the interpretation of the N-body results; one certainly does not expect the similarity solution to apply on scales smaller than the mean inter-particle separation and for small N systems this is uncomfortably close to the size of the whole system. The N-body results do deserve to be taken seriously because A different N-body schemes yield results which are in good agreement (Figure 1). B ξ does evolve in a manner consistent with a similarity solution (Efstathiou et al., 1979; Frenk et al., 1982).

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There is an additional important discrepancy between the N-body models discussed above and observations. If the models are scaled so that $\xi(r_o) = 1$ with $r_o = 4h^{-1}$ Mpc (h is Hubble's constant H_o in units of 100 km sec^{-1} Mpc^{-1}), the one-dimensional r.m.s. peculiar velocity between particle pairs of separation r_o is $\langle w^2 \rangle^{1/2} \sim 900$ km sec^{-1}. As discussed below this is much larger than the peculiar velocities between galaxy pairs.

2. PECULIAR VELOCITIES

The mean square peculiar velocity between galaxy pairs of separation r can be calculated from an integral over the three-point correlation function. For $\xi(r) = (r_o/r)^{1.8}$ the relation is (Peebles, 1976),

$$\langle w^2(r) \rangle^{\frac{1}{2}} \sim 870\ Q^{\frac{1}{2}}(r_o/4h^{-1}\mathrm{Mpc})^{0.9}(r/1h^{-1}\mathrm{Mpc})^{0.1}\Omega^{\frac{1}{2}}\mathrm{km\ sec}^{-1}, \qquad (2)$$

where Q is the ratio of the amplitude of the three-point correlation function to the square of the two-point function. There are two main assumptions necessary in deriving eq. (2) - the stability assumption and that the galaxy correlation functions measure the mass distribution.

In order to derive $\langle w^2 \rangle$ from redshift data I use a method similar to that of Peebles (1979). Consider two galaxies with observed radial velocities v_1 and v_2 separated on the sky by an angle θ_{12}. The approximate separation of the pair parallel to the line of sight is $\pi = (v_1-v_2)/H_o$ and the separation perpendicular to the line of sight is $\sigma = \sqrt{v_1 v_2}\theta_{12}/H_o$. The clustering pattern is distorted in the π direction because of peculiar velocities (and velocity errors) hence the two-point function ξ_v measured using the coordinates σ and π will differ from the true correlation function. This is modelled as,

$$\xi_v(\sigma,\pi) = \int \xi\{[(\sigma^2+(\pi-w/H_o)^2]^{\frac{1}{2}}\}\ f(w)dw, \qquad (3)$$

where w is drawn from the distribution function f(w).

Two recent magnitude limited redshift surveys have been analysed; the Kirshner et al. (1978) survey containing $\sim$ 160 galaxies and the Anglo-Australian survey (Peterson et al., 1982) containing $\sim$ 340 galaxies. Figure 2 shows estimates of $\xi_v(\sigma,\pi)$ for the combined sample. The method used to estimate ξ_v is described in detail by Bean et al. (1982). The distribution function f in eq. (3) is assumed to be

$$f(w) \propto \exp(-0.7966|w|^{3/2}\langle w^2 \rangle^{-3/4}), \qquad (4)$$

which has been found to be a good fit to the N-body models. The power-law model for ξ is assumed with $\gamma = 1.8$ and the best values of $\langle w^2 \rangle$ and r_o are found by a least squares fit of the model (eq. 3) to the observed histograms of $\xi_v(\sigma,\pi)$ over the interval $0 < \pi < 10\ h^{-1}$Mpc. These methods have been thoroughly tested using simulated catalogues similar to the AAT survey generated according to the prescription of Soneira & Peebles

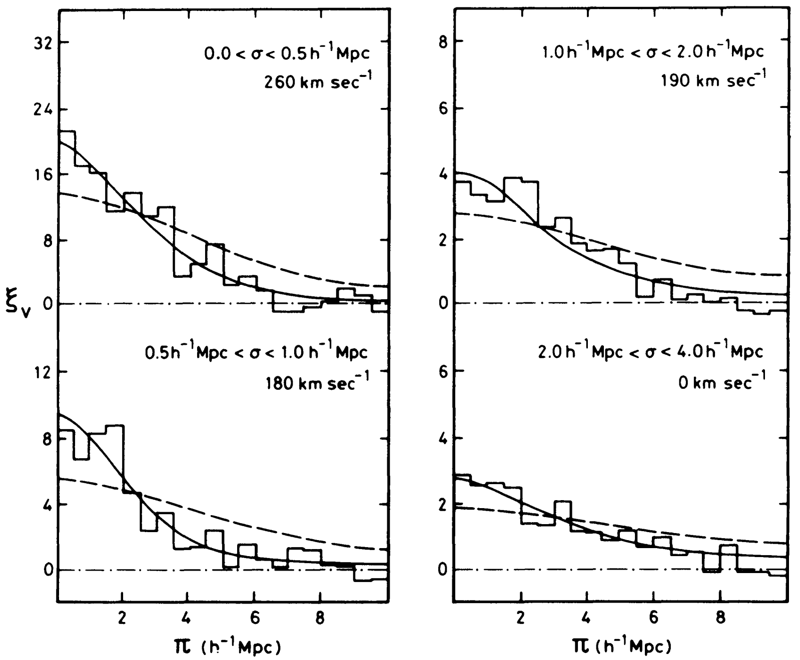

Figure 2. Estimates of ξ_v for the Anglo-Australian survey combined with the Kirshner et al. sample. The solid lines give the best least squares fit using eq. (3) and (4) corresponding to the quoted value of $\langle w^2\rangle^{\frac{1}{2}}$. The dashed line shows the best fit obtained using $\langle w^2\rangle^{\frac{1}{2}}$ = 500 km sec^{-1}.

Table 1. Monte-Carlo results and error estimates for $\langle w^2\rangle^{\frac{1}{2}}$

	200 km sec^{-1}		500 km sec^{-1}	
$\sigma(h^{-1}Mpc)$	$\langle w^2\rangle^{\frac{1}{2}}$	s.d.	$\langle w^2\rangle^{\frac{1}{2}}$	s.d.
0.0 - 0.5	210 ± 6	(40)	490 ± 13	(83)
0.5 - 1.0	203 ± 8	(50)	463 ± 13	(79)
1.0 - 2.0	205 ± 13	(85)	463 ± 15	(92)
2.0 - 4.0	189 ± 37	(233)	430 ± 34	(215)

The columns denoted by $\langle w^2\rangle^{\frac{1}{2}}$ give the mean values measured from two sets of 40 models, one set having $\langle w^2\rangle^{\frac{1}{2}}$ set to be 200 km sec^{-1} and the other with $\langle w^2\rangle^{\frac{1}{2}}$ = 500 km sec^{-1}. The columns denoted s.d. give the standard deviation of $\langle w^2\rangle^{\frac{1}{2}}$ expected from one model.

(1978) modified to include peculiar velocities. The results for two sets of 40 Monte-Carlo simulations are summarised in Table 1 which shows that the methods are essentially unbiased. From Figure 2 and Table 1, I conclude $\langle w^2 \rangle^{\frac{1}{2}} = 240 \pm 60$ km sec^{-1} at $r \sim 1\ h^{-1}$Mpc. The surveys also give $r_o = 4.1 \pm 0.3\ h^{-1}$Mpc and $Q = 0.6 \pm 0.1$, thus eq. (2) yields $\Omega = 0.1\text{x}(2^{\pm 1})$ (see also Davis & Peebles, 1982).

3. CONCLUSIONS

The results of Sections 1 and 2 suggest more complex initial conditions than those required for a similarity solution. The observed peculiar velocities imply an open universe unless the mass density is dominated by dark material which is much more uniformly distributed than galaxies on scales $\lesssim 5\ h^{-1}$Mpc. There are many other possibilities worthy of further work, e.g. schemes in which galaxies form before clusters but which give rise to non-power law fluctuation spectra (Bond et al., 1982; Peebles, 1982; Hogan & Kaiser, 1982) or schemes in which all fluctuations on scales smaller than clusters or superclusters are erased by Silk damping or the decay of neutrino perturbations (Bond et al., 1980). In the latter cases the main goal is to account for $r_o \sim 4\ h^{-1}$Mpc without obtaining large peculiar velocities; results from N-body simulations are discussed by Klypin & Shandarin (1982) and Frenk et al. (1982).

REFERENCES

Bean, J., Efstathiou , G., Ellis, R.S., Peterson, B.A. and Shanks, T.: 1982, in preparation.
Bond, J.R., Efstathiou, G. and Silk, J.: 1980, Phys.Rev.Lett., 45, 1180.
Bond, J.R., Szalay, A.S. and Turner, M.: 1982, Phys.Rev.Lett., 48, 1636.
Davis, M. and Peebles, P.J.E.: 1977, Ap.J.Suppl., 34, 425.
Davis, M. and Peebles, P.J.E.: 1982, preprint.
Efstathiou, G. and Eastwood, J.W.: 1981, M.N.R.A.S., 194, 503.
Efstathiou, G., Fall, S.M. and Hogan, C.: 1979, M.N.R.A.S., 189, 203.
Frenk, C.S., White, S.D.M. and Davis, M.: 1982, preprint.
Fry, J.N. and Peebles, P.J.E.: 1980, Ap.J., 236, 13.
Gott, J.R., Turner, E.L. and Aarseth, S.J.: 1979, Ap.J., 234, 13.
Hogan, C. and Kaiser, N.: 1982, in preparation.
Kirshner, R.P., Oemler, A. and Schechter, P.L.: 1978, A.J., 83, 1549.
Klypin, A.A. and Shandarin, S.F.: 1982, preprint and this volume.
Miyoshi, K. and Kihara, T.: 1975, Pub.Astr.Soc.Japan, 27, 333
Peebles, P.J.E.: 1976, Ap.J.Lett., 205, L109.
Peebles, P.J.E.: 1979, A.J., 84, 730.
Peebles, P.J.E.: 1980, "The Large-Scale Structure of the Universe", Princeton University Press, Princeton, N.J.
Peebles, P.J.E.: 1982, preprint.
Peterson, B.A., Bean, J., Efstathiou, S., Ellis, R.S., Shanks, T. and Zou, 1982, in preparation.
Soneira, R.M. and Peebles, P.J.E.: 1978, A.J., 83, 845.

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Discussion

Palmer: One of the most damning arguments against the self-similar clustering model is the velocity dispersion argument. The estimate of the observed dispersion depends upon your choice of the weight function, f(w). Could you say how sensitive your result is to the choice of this function, and why you chose the one you did?

Efstathiou: I chose a function which is a good fit to the distributions from the N-body models. If instead one uses a gaussian, then $\langle w^2 \rangle^{1/2}$ decreases by $\sim$ 10 km sec^{-1} and if one takes an exponential, the velocities increase by $\sim$ 10 km sec^{-1}. I would not seriously consider the possibility that the velocities are wrong by several hundred km sec^{-1} because of an inappropriate choice of the distribution function. Also, the quality of the model fit to ξ_v gives some indication of whether the distribution function is reasonable. Fitting a model to the observations is preferable to taking moments of ξ_v.

Shandarin: Did you estimate the two-point correlation function of clusters of particles in your simulations?

Efstathiou: Joshua Barnes has devised a fast algorithm for locating groups of particles. We found that the two-point correlation function for groups is the same as the two-point function for individual particles. Thus, rich groups are good tracers of the mass distribution. Apparently, this result is in conflict with the results of Hauser and Peebles which show that the two-point correlation function for Abell clusters has an amplitude $\sim$ 10 times larger than that for the galaxies. Recent work by Bahcall and Soneira agrees with the results of Hauser and Peebles.

B. Jones: Doesn't the fact that the cluster-cluster correlation is so much higher than the galaxy-galaxy correlation on the same scale simply suggest that either galaxy or cluster light is not a good tracer of the mass distribution?

Efstathiou: Perhaps neither galaxies nor clusters trace the mass distribution! I would have expected that the cluster-cluster correlation function should agree with the galaxy-galaxy correlation function, and this issue deserves more attention. One check comes from the way in which the peculiar velocities between galaxy pairs scale with pair separation. The results, particularly those from the CfA survey, are consistent with the scaling expected from the galaxy correlation functions.

Szalay: If one would invoke dissipation, wouldn't that change some of these results?

Efstathiou: Yes. The shape of the two-point function on small scales could be very different from that found in the purely gravitational N-body simulation of Klypin and Shandarin and Frenk, White and Davis. If the dissipation was recent, one might also obtain lower peculiar velocities than those expected from the virial theorem.

Now in the self-similar model, dissipation is ignored but if dissipative effects are important only on scales corresponding to individual galaxies, then the conclusions should be unaltered.

G. Burbidge (right) replying to a question. M. Schmist presiding.
(Courtesy, K. Brecher)

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